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The Shapes of Sacred Space: A Proposed System of Geometry Used to Lay Out and Design Maya Art and Architecture and Some Implications Concerning Maya Cosmology

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The Shapes of Sacred Space: A Proposed System of Geometry Used to Lay Out and Design Maya Art and Architecture and Some Implications Concerning Maya Cosmology

by

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Dedication

I dedicate this dissertation to my friend and mentor, Linda Schele. I did not know what a mentor was until she left us.

Acknowledgements

I gratefully take this opportunity to acknowledge and thank my family and friends for their love, help, and support during the many years of research that preceded this dissertation. I thank my wife, Alejandra, in particular for her faith and patience. I thank my mother and father for all they have given and for being the good examples forever set before me. I thank them, too, for their technical support. My mother taught me how to produce the graphics in this dissertation and my father helped me to learn the Adobe Indesign program that I used to assemble this final edition. With her artist's eye and his proclivities as a wordsmith, they both edited the content. I thank my friend and colleague Carol Karasik for her invaluable insights on how to present this material and for her many hours spent editing this manuscript. I thank my friend and assistant, Elizabeth Acosta Cano for organizing my files and library and editing some of the graphics. Without her help, I would surely still be working on this dissertation. I thank my friend Alfonso Morales Cleveland for many years of friendship and professional support and for so freely sharing his relevant ethnographic research. I thank Dr. Brian Stross for his early support of this work and for his unwavering encouragement. I thank Dr. Linda Schele for showing us clearly how much more we could accomplish by working together and helping one another. I thank Dr. Ed Barnhart for many years of collaboration and support, for the good works we've accomplished together, and for demonstrating over and again what friendship is.

Abstract

This dissertation explores the fundamental characteristics of a system of geometry and proportion currently used by Maya house builders and shamans to design vernacular architecture in indigenous Maya communities. An extensive examination of Pre-Columbian Maya art and architecture demonstrates how this system of geometry and proportion was also used by the Maya of the Classic and Post-Classic periods. The dissertation concludes with a brief discussion of how Maya geometry was, and is, an expression of Maya cosmology and religion.

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INTRODUCTION

This dissertation rests on the recognition of a simple set of repeated proportions found in the dimensions of Maya art and architecture. This set of proportions has been identified through a systematic analysis of Pre-Columbian and modern Maya architectural and artistic forms by this author. These proportions apparently were, in Pre-Columbian times, and certainly are today, intentionally incorporated into the designs of these artifacts.

I begin the introduction to this dissertation with a summary of the history of my investigation into Maya geometry and of my academic and professional career in order to provide the context for the results of this investigation and to expose some of my personal biases as a researcher before I presume to interpret the results of the ethnographical investigations included in this dissertation. I discuss evidence and ideas as to how and why Maya geometry was, and is, intimately entwined with their world view and with their religious beliefs. I present evidence here that points to the conclusion that the Maya developed a profound understanding of the geometry employed by the "Maker and Modeler" in creating the universe. This evidence also suggests that the Maya consciously emulated the shapes of the sacred space of their cosmos by using the very same geometry to lay out and design their own creations, the architecture of their physical environment, from the simple geometry used to lay out their vernacular houses, altars, and milpas to the complex geometrical formulae used to design the great pyramids and temples and the exquisite works of art in the long abandoned ruins of their magnificent ceremonial centers.

I was surreptitiously drawn into this study in 1989 during my last semester as an undergraduate student at the University of Arizona at Tucson. That semester dramatically changed the direction of my fledgling academic and professional career. My degree was in anthropology with a focus on Paleolithic archaeology. While finishing my courses and working half time as a field archaeologist for the Arizona State Museum, I was also moonlighting as a researcher for Dr. Jeffrey Goodman, who was writing a book on archaeology in the Americas. Part of my duties as a researcher was to read and summarize chapters of books and articles on the Pre-Columbian Maya. When I brought Dr. Goodman the results of my work I had an abundance of material on Maya architecture, iconography, epigraphy, cosmology, astronomy and mathematics. But I had found almost no information concerning Maya geometry. When I explained this to Dr. Goodman, he told me not to worry about it, because he knew from his own research that very little was to be found on this subject. But I remained bothered by this lack of information. The Maya were clearly accomplished mathematicians, and from my previous coursework concerning the histories of eastern and western civilizations and a course I had taken called "A Survey of Mathematical Thought," I knew that any study of the history and development of sophisticated mathematics invariably included a study of the history and development of geometry.

After I finished my work for Dr. Goodman, I became fascinated by Maya history and culture. I was particularity interested in the astronomical and mathematical content of the hieroglyphic inscriptions and, as a kind of hobby, I designed and carried out a simple strategy in an attempt to better understand Maya concepts of geometry.

I was not specifically searching for Maya units of measure or complex geometrical formulae. Instead, I was interested in determining whether the dimensions of rectangular edifices as well as carved stone and modeled stucco art panels, predominant at all Mesoamerican sites, were repetitive. If so, I would identify which proportions the Maya used most frequently to design public buildings and works of art.

I started by analyzing rectangular structures published in Ruppert's collection of measured drawings of the architecture at Chichen Itza (Ruppert 1952). These structures were chosen for two reasons: (1) The basic planar forms allowed statistical analysis of their relative proportions; and (2) Ruppert's plans represent unreconstructed structures and thus eliminate the potential for reconstructive error.

First I divided the lengths by the widths of both the interior and exterior dimensions of the selected structures and kept track of the proportional relationships. From this preliminary exercise a series of repetitive proportions emerged that were related to one another in an intriguingly simple manner. Beginning with a square, the length of each succeeding rectangle equaled the diagonal of the preceding one. These proportions are none other than the root rectangles (from the square roots of one to five) that so fascinated Pythagoras and other ancient geometers and that were intrinsic to the early development of planar geometry.

Two other rectangular proportions with ratios of 1 to 1.272 and 1 to 1.618 also were noted as repetitive. Although the diagonal of the first was found to equal the length of the latter, these rectangles did not initially appear to relate to the set of root rectangles previously identified. However, the proportion of 1 to 1.618..., called the Golden Ratio, or phi, in planar geometry (1.272... being the square root of phi), is intrinsically related to the root five rectangle. Apparently unrelated to the square root and phi rectangles, the Pythagorean 3,4,5 rectangle (with a width of three units, a length of four units, and a diagonal of five units) was also found to be repetitive (see Figure 1).

By this early stage in my extracurricular investigations of Maya geometry., I had completed my undergraduate program and had been invited to continue my studies in Paleolithic archaeology as a graduate student at the University of Arizona at Tucson. In addition to classes and field work in Paleolithic archaeology, I had been studying Mandarin Chinese and had taken Asian Studies courses in preparation for conducting Paleolithic field investigations in the northwestern deserts of China.

I was at a crossroads, but I knew that my newfound interest in the complexities of Maya history and culture and my virtual obsession with the initial results of my investigation into Maya geometry had superseded my passion for, and years of study of, Paleolithic archaeology.

I declined the offer to attend graduate school and spent the next four years working full time as a field archaeologist for the Arizona State Museum, then as a field supervisor with the New York State Museum, and ended up in San Francisco working for a private contract archaeological firm.

Between digs and during my spare time, I read widely about Maya history, culture, and archaeology. My foci in these studies alternated between attempts to better understand the mathematical structure and astronomical content found in Maya hieroglyphic books and inscriptions and in continuing my investigation of Maya geometry. I began to study the Spanish language, and I collected and analyzed measured drawings of Maya art and architecture.

In 1992 I decided it was time to pursue my academic interests formally by applying to graduate schools that offered Maya or Mesoamerican programs. I enjoyed living in San Francisco, so I applied to U.C. Berkeley, Stanford, and U.C. Santa Cruz. All three rejected my applications. I was disappointed but not surprised. Though I had graduated Magna Cum Laud from the University of Arizona and my G.R.E scores were good, I had not taken a single course in Maya or Mesoamerican archaeology and I had not studied Spanish as a second language.

I continued with my extracurricular studies, kept my job as a field archaeologist, and the next year applied to every university and college in the United States and Canada that offered a Maya or Mesoamerican Studies program. But this time I included a paper I had written on Maya math and astronomy entitled, "A New View on the Tun Ending Calendar Round."

I thought I would improve my chances of being accepted into a graduate program if I were to arrange interviews with the professors whom I hoped to study under before they made their final selection of candidates. I had the time and resources to travel to only one university. While trying to decide which university to visit, I was reading everything I could find written by Dr. Linda Schele. I was impressed with the results of her collaborative approach to research and by her honesty and daring to speculate where others feared to tread. More than anyone, Dr. Linda Schele's work brought the dry bones of Maya archaeology to life.

And so, I awoke one Friday morning in my apartment in San Francisco trying to remember a powerful dream I'd just had, and though I could not recall a single image of it, I felt strongly compelled to go to the University of Texas at Austin. I reached for the phone on the night stand and made plane reservations to fly to Austin at one o'clock that afternoon. (For the record, this sort of thing does not happen to me very often.) I then called information for the telephone number for the Anthropology Department at the University of Texas at Austin. I apparently misdialed the number because the response to my call was, "Hello, Maya Meetings." I was speaking with Peter Keeler, the director of the annual Maya Meetings, which I had never heard of. He told me that Dr. Schele was giving the keynote speech that evening, to be followed by three days of papers presented by various researchers and a weeklong intensive workshop on Maya epigraphy. I asked him to please sign me up. He also kindly gave me the correct number for the Anthropology Department and Dr. Schele's office number at the Art History Department. I made appointments to interview with Doctors Frederick Valdez and Brian Stross in the Anthropology Department and with Dr. Schele for later that afternoon. At one o'clock, due to a happy accident with my reservation, I was flying first class and sipping champagne on my way to Texas.

My interviews with Doctors Valdez and Stross were brief. They both politely informed me that they had accepted only three candidates that year and that I had not made the cut.

My interview with Dr. Schele was scheduled for late that afternoon. I did not expect much to come of it, because when I applied to the Anthropology Department at the University of Texas at Austin I had mistakenly assumed that Dr. Schele was a professor in that department. It turned out that Dr. Schele's students were either from the Art History Department or from the Institute for Latin American Studies.

I arrived early for my interview, and when Dr. Schele's secretary announced my presence, I heard Dr. Schele say, "Oh, that's the guy who wrote that astronomy paper," and I was shown into her office. I took that as a cue, and we spoke for a while about Maya astronomy. Dr. Schele was immersed in the subject at the time and was in the middle of writing the book Maya Cosmos, which was also the topic of her presentation at the Maya Meetings. I explained what I had been reading and studying over the past four years, but I did not mention my interest in Maya geometry. I also told her the mistake I had made by applying to the Anthropology Department and that I had applied to the University of Texas at Austin because I had hoped to study under her guidance.

Dr. Schele told my that I should be in the Institute for Latin American Studies because there I could arrange a multidisciplinary program of course work in both the

Departments of Art History and Anthropology. She made a couple of phone calls and had my application sent to the Institute for Latin American Studies. Then she asked me to rewrite my letter of intent and directed me to a Kinko's copy shop where I could type it up. She told me to turn the letter in to her office by 8: A.M. the next morning. She also told me it was a good thing that I had talked to her that day because she was choosing her next crop of graduate students the following day. I asked her what my chances were of being accepted into the program and she told me that if my letter of intent was on her desk in the morning I was in. That letter of intent was, of course, on her desk at 8:00 A.M. sharp.

After attending the Maya Meetings, I returned to San Francisco, finished excavating at the Los Morteros site (which was to be my last Paleolithic dig), and moved to Austin, Texas. My first semester went well, and it was a pleasant shock to be immersed in course work and to be surrounded by motivated students and professors after studying in virtual isolation during the previous four years.

But I still had not shared my findings concerning Maya geometry. I had been taking a course on Maya epigraphy with Dr. Brian Stross, and I made an appointment with him to ask his opinion about my project. He understood what I was doing and approved of my approach to the subject matter. He was (and has been over the years since) very supportive. As luck would have it, his wife was an architectural historian, and a conference for the Texas Chapter of Architectural Historians was being held that week. Dr. Stross and his wife arranged for me to give a presentation to the group, which I did using hand-colored acetates and an overhead projector. I received encouraging and constructive comments from Dr. Eugene George, Professor Emeritus of Architectural Engineering at the University of Texas at Austin. Dr. George is an expert on the history of the uses of square root and phi proportions in architectural designs, and he seemed pleased that I was able to convince him that the Maya had used them as well. When we later spoke at his office, I learned that he had incorporated these proportions into several of the buildings he had designed over his long career and that he was also teaching architectural engineering students how to use these proportions to maximize the strength and minimize the weight of various structural forms.

Boosted by the positive responses I had received from Dr. Stross and the members of the Texas Chapter of Architectural Historians, I finally drummed up the courage to present my pet project to Dr. Linda Schele. Though I greatly admired her work and liked her very much personally, Dr. Schele was such a powerful personality that, frankly, during my first year as a student, I was intimidated by her.

I made an appointment to see her, carefully prepared my presentation, and showed up with a thick three-ring binder full of my best examples of Maya geometry.

No sooner had I begun to explain my project when she told me that she did not want to see it or to hear about it. She told me that if I could demonstrate that what the Maya were doing today with their measuring cords to lay out and design their houses, altars, and milpas, had anything to do with what I thought they were doing in Pre-Columbian times, then come back and we would talk about it.

Then she told me I might get started in that direction by tagging along with the Studio Mexico Program, headed by Professor Sinclair Black of the Architecture Department and by Logan Wagner (now Dr. Logan Wagner), an architect and a fellow graduate student at the Institute for Latin American Studies. They were to lead a group of graduate students in architectural history to the Yucatan Peninsula to document and produce measured drawings of early Colonial and Pre-Columbian architecture.

Thus, I spent my second semester as a graduate student in the Yucatan Peninsula, where I helped the graduate students with their architectural documentation projects. They, in turn, helped me to produce measured drawings of two different styles of modern Maya houses. In the Yucatan, I also had the good fortune to meet the director of the Fundación Cultural Yucatán, Liticia Roche (for whom I would end up working two years later). She arranged for me to interview two Maya house builders, as well as Francis Faller, a retired engineer with an interest in and great knowledge of Maya house building. Liticia Roche also directed me to the library at the Universidad de Arquitectura in Merida where I found and copied architectural studies and measured drawings of vernacular Maya architecture.

The results of these and subsequent ethnographical investigations of Maya vernacular architecture and geometry are described in some detail in Chapter 6. From my initial investigation it was clear that at least some of the Maya were still using the square root and phi proportions to lay out and design their houses.

I was in the midst of what Dr. Schele used to call an "Ah ha! experience." I was thrilled to discover that the principles of geometry that I suspected had been used in Pre-Columbian ceremonial art and architecture had survived five hundred years of foreign invasion, inquisitions, and religious conversion.

When I returned to talk with Dr. Schele, she suggested that I develop these findingd into a Ph.D. dissertation and that I should also conduct more ethnographical investigations. Which I did.

The next ten years were so full of activity and went by so quickly that as I now reflect upon them they seem more dreamlike than real. I had arranged with Dr. Schele to take my course work in the fall semesters and to conduct field work in the spring semesters and summer breaks. In the summer of 1993, I worked as a field instructor for the Project for Belize at Cerros, a Pre-Classic Maya ceremonial center, under the direction of Dr. Frederick Valdez of the Anthropology Department at the University of Texas at Austin. In 1994 I went to the Highlands of Guatemala to study Spanish intensively and to conduct ethnographical investigations of Maya house building and related ceremonies. Between 1994 and 1996 I worked for three field seasons, supervising and conducting tunnel excavations at Copan, a Classic Maya ceremonial center in Honduras, for the Early Acropolis Project directed by Dr. Robert Sherer of the University of Pennsylvania Museum.

During those years, I also managed to complete my Master's thesis on the mathematical and astronomical content of Maya hieroglyphic books and inscriptions, to finish my course work for my Ph.D., and to conduct six months of ethnographical investigations on Maya house building and house-building ceremonies as part of my duties during my brief stint as Director de Investigaciónes Antropologicos for the Fundación Cultural Yucatan.

In 1997 I was invited by my good friend and colleague Alfonso Morales Cleveland to take the position of Field Director for the Projecto Grupo de Las Cruces at the Classic Maya ceremonial center of Palenque, in Chiapas, Mexico. The project was headed up and sponsored by Merle Greene Robertson and the Pre-Columbian Art Research Institute and directed by Alfonso, who held the position of Principal Investigator. We worked six days a week, year round, taking brief breaks each winter to write up the year-end field reports and the proposals for the next year's field work.

The Projecto Grupo de Las Cruces was a lucky project from the start. We unearthed marvelous examples of Maya architecture, art, and hieroglyphic texts that will keep researchers busy for many years to come. It was the apex of my career as a field archaeologist.

I held my position as field director for the project until the end of 2001 and continued to work at Palenque for another year, supervising excavations and restoration efforts as an employee of the Instituto Nacional de Antropologia y Historia for the Projecto Espescial Palenque, directed by site archaeologist Arnoldo Gonzalez Cruz.

In 1997 I had the great fortune to meet Alejandra Merino Trujillo, a molecular biologist who is presently in charge of medical investigations for the State of Tabasco and is the editor of a professional medical journal. We married in the year 2000, and by 2003, when we decided to have children together, I resigned from my position at Palenque and became Senior Research Associate at the Maya Exploration Center, a non-profit educational and research institute, founded and directed by my good friend and colleague Dr. Ed Barnhart. This is a position I still hold today, and my duties include instructing curriculum development courses for the National Science Foundation's Chautauqua Short Course Program, as well as study abroad courses for the Mathematical Association of America and various universities and colleges. These courses focus on Maya (and more recently Andean) history, culture, mathematics, geometry, astronomy and art. We also conduct archaeo-astronomical investigations at Maya and Andean ceremonial centers in Mexico and Central and South America, write and publish reports and articles, present papers at professional conferences and lecture at universities, colleges, and other venues.

My association with the Maya Exploration Center has finally provided me with the time and resources to write this dissertation and to produce the final versions of the numerous diagrams that form the bulk of it.

It is now the year 2010 and I am adding the final touches to this dissertation, including this summary of the history of my investigation. Twenty years have passed since I first analyzed the collection of measured drawings published in Ruppert's "Architectural Notes and Plans of Chichen Itza" (Ruppert 1952). For most of these years I have considered myself a fairly accomplished field archaeologist with some interesting hobbies. In my case, the word "hobby" is a polite synonym for the word "obsession," and I admit that I am, and have been, virtually obsessed with trying to better understand what the Maya know and knew about mathematics, astronomy, and geometry. Almost all of my free time has been dedicated to theses pursuits. It is fortunate that I am often insomniac.

To further test for the presence or absence of the square root, phi, and

Pythagorean 3,4,5 proportions in Maya architecture and art, I conducted a complete analysis of the measured drawings of the temples, art panels, and piers at the Late Classic Maya site of Palenque. I obtained these measured drawings from the four volume series, The Art and Architecture of Palenque by Merle Greene Robertson, and from copies of an unpublished set of measured drawings of most of the temples at Palenque excavated before 1980, produced and generously given to my by the late George F. Andrews, Professor of Architecture at the University of Oregon, and his wife, Geraldine D. Andrews. My examination of these measured drawings demonstrated, to my satisfaction at least, that this set of proportions was used by the Maya to design and lay out virtually all of the buildings and art works at Palenque.

I have since collected, analyzed, and reanalyzed the majority of the measured drawings and rubbings of Maya art and architecture that have been published over the last 120 years, as well as a number of unpublished measured drawings, some of which I have produced myself. The square root, phi, and Pythagorean 3,4,5, rectangles that were briefly described above, and that are described in detail below, were found to be clearly expressed in almost all of them.

In this dissertation I present selected sets of representative examples with a wide regional and temporal range, including a few examples from cultures beyond the Maya area. In part, this dissertation is intended as a kind of practical handbook on the basics of how Maya geometry is expressed in a wide variety of artifacts and in a wide variety of contexts.

These, and the rest of the examples that I have accumulated and have stored in hard copies and hard drives, contain examples from non-Maya cultures in Mesoamerica, a few of which are included in this dissertation, particularly in the section that analyzes the layout and geometry of Pre-Columbian codices. In the future I plan to analyze all of them in groups, site by site, region by region, from the Pre-Classic to the Post-Classic periods, in an attempt to better understand the details of how these physical expressions of Maya and Mesoamerican geometry developed and changed over time and space. If I can find the time, and if I live long enough, I plan to publish the results of these analyses as a series of companion articles to this dissertation.

I also recommend that a detailed regional study of the geometry of modern Maya and Mesoamerican vernacular architecture be conducted — the sooner the better — because traditional Maya and Mesoamerican houses are rapidly being replaced by cement structures. In too many cases, the knowledge that is traditionally passed on to the younger generations by the elders is dying when they die.

CHAPTER 1

METHODOLOGY

My methodology for defining, evaluating, and presenting Maya geometry and how it is expressed in the layout and design of Maya artifacts is simple. I begin with ethnographical investigations (Chapter 6) that clearly demonstrate how the Maya use geometry today to lay out and design their vernacular houses and altars. I explain in the Introduction, and show graphically in Chapter 5 (Figure 1), how the repeated proportions used by the Maya today form a coherent and interrelated set of rectangular proportions; that is, beginning with a square, the length of each succeeding rectangle is equal to the diagonal of the preceding one. I discuss the phi and Pythagorean 3,4,5 rectangles as well, and their relationships to this basic set of rectangular proportions. Then I rely on numerous diagrams showing these same proportions overlaid on measured drawings, rubbings, and photographs of various Pre-Columbian artifacts, in order to demonstrate that this same set of proportions was used pervasively in Pre-Columbian times. The Pre-Columbian artifacts examined are primarily examples of ceremonial architecture, bas-relief stone and stucco art panels, and stelae, as well as pages of Pre-Columbian codices.

The methodology for my efforts to better understanding the relationships between Maya geometry and cosmology is not quite as simple or straightforward. Using a structural and comparative analytical approach, I begin, in Chapter 3, by examining ceremonies that include the uses of measuring cords, as described in Contact Period documents. In Chapter 4 I briefly discuss some modern Maya ceremonies that involve measuring cords. In Chapter 6 I include the sparse but illuminating information about how the shapes of native flowers are related to the geometry of houses, information that a Maya shaman shared with me during one of my brief sojourns as an ethnographer.

Near the end of this dissertation, I speculate on and discuss how the methods that the Maya probably used to track the rising and setting positions of the sun and the moon on the horizon would have produced obvious examples of the regular polygons, the equilateral triangle, the square, and the pentagon, which share the same square root and phi proportions found in the set of rectangles used by the Maya today (and that are the shapes of flowers as well). I further speculate about circular Maya calendars and their subdivisions, and how they, too, would produce regular equilateral triangles, squares, and pentagons.

I conclude by discussing previous research on the cosmology expressed in Maya ceremonial centers and attempt to interweave the new information and ideas presented in this dissertation with the existing body of theories in order to add a new perspective to our incomplete view of Maya cosmology.

The methods that I used to examine and analyze the measured drawings, rubbings and photographs included in this dissertation and how I produced the diagrams that overlay them should also be discussed here.

In the early years of this study, all measurements were made on high-quality Xerox copies of the measured drawings and checked against the published drawings with a stainless steel ruler divided into 100ths of an inch.. This same ruler was used to measure the width and lengths of the measured drawings of the rectangular artifacts to determine their proportions, and all of the diagrams were drawn with a compass and a straght edge.

Some years later my mother, an accomplished artist and graphic artist, taught me to use the Adobe Illustrator program. The Adobe Illustrator program is ideally suited to both measure the dimensions of scanned measured drawings, rubbings, or photographs and to create extremely accurate geometrical diagrams. With this program, I do not draw the geometrical diagrams or formulae, I simply use a scientific calculator to input the dimensions of the rectangles, the precise angles of the diagonals and the lengths of radii of the arcs and circles used to create them, and the program magically produces these diagrams with an accuracy to the nearest thousandth of a point.

I often convert the original scans into a transparent bit-map format so that I can underlay them with colored shading to highlight various portions of the diagrams to facilitate describing them.

To incorporate these Adobe Illustrator images into a written document, I convert them into high-density bit-map images and import them into an open file in the Adobe Indesign program, which is ideally suited for writing books, particularly books with numerous diagrams, images, or charts. The images stay where they are supposed to, and the text wraps when it is supposed to, and that is more than I can say for other desktop publishing programs I have tried to use in the past.

To share the final document with others, the final version is converted to PDF format and copied on to disks or thumb drives.

CHAPTER 2

A REVIEW OF THE LITERATURE ON MAYA GEOMETRY

Several scholars, whose investigations are either mentioned or summarized below, contend that knowledge of geometrical concepts was widespread in Mesoamerica. The idea is practically commonplace, yet no one to date has discovered substantive evidence of a coherent system of measurement and proportion.

Francine Vinette concludes her superb survey of the topic, “In Search of Mesoamerican Geometry” (Closs 1986: 387), by stating, “There is little doubt that further investigations directed toward an evaluation of Mesoamerican knowledge will include geometry as a part of the amalgam of Mesoamerican science and religion.”

Most studies concerning geometry and architecture conducted over the past three decades concentrate on inter-structural geometrical relationships, particularly in the context of archaeo-astronomical observations. These investigations are most valuable for the light they shed on ancient astronomy, structural alignments, and overall site plans. Clearly, ancient knowledge of geometric principles anticipated, or went hand in hand with, astronomical observations, but that knowledge is generally inferred from current archaeo-astronomical data.

Because this dissertation focuses on the geometry of individual structures and works of art, I will only briefly mention and describe a few of these. Scarborough (1982) was among the first researchers to remark upon the general north-south alignments of ball courts. Hartung (1977) analyzed maps of Tikal and Copan and found what he considered non-coincidental east-west and north-south base lines and right angles as well as equal-sided, right-angled isosceles triangles between prominent pyramids, temples, and monuments. In the Nunnery Complex at Uxmal, right angles at the center of the courtyard (centered on offset doorways) and parallel sight lines were suggested as a cleverly hidden site plan by Aveni (1992). Dr. Anthony F. Aveni (1980, 1992), more than any other scholar, has written and compiled articles from dozens of other researchers to produce an enormous corpus of information concerning astronomical alignments at Mesoamerican ceremonial centers.

Treatises on the relationship between geometry and art have dominated Western art criticism since the Renaissance, providing a rigorous language on perspective, composition, and symmetry that has guided the course of painting and sculpture for hundreds of years.

In comparison, research on geometric measurements in Mesoamerican painting and sculpture is scant. However, studies by Arthur Miller (1973), Robertson (1977; 1983), and Sanders (1977) present evidence that templates were laid out in advance of the final execution of murals and bas-reliefs. These scholars contend that a master draftsman laid out the arrangements and subdivisions of the work surfaces before outlining the compositional content. Arthur Miller (1973: 42) suggests that "a kind of farmer's compass" was employed for this purpose. Vinette, citing Guerra (1969: 43), mentions a list of Aztec construction tools, including the compass (*tlayolloanaloni*), the plumb (*temetztepilolli*), the level (*quamniztli*), and the square (*tlanacazanimi*). Similar tools, she proposes, may have been used to engrave fifteen circles on the mural of substructure 3, zone 2 at Tepantitla. Separated by 7.5 cm. intervals, the first thirteen circles are 32 cm. in diameter. The last two engraved circles are 33.5 and 34.5 cm. with a space of 11.5 cm. between them. Once painted, the last two circles became 35 and 38 cm. in diameter respectively. Commenting on the lack of precision in final execution, Vinette (1986) concludes that after the masters set down the template, apprentices completed the work. Even in the absence of discernible markings underneath the plaster, Miller (1973:33) states that the profile figures in room 2 at Tepantitla show evidence of being carefully measured, "otherwise we cannot account for the fact that the figures fit so well in the allotted space, that they are roughly the same size and that they have similar spacing between them" (Vinette 1986:388).

Compositional symmetry also has been observed in Maya works of art. In her study of the murals of Coba, Fettweis Vienot (1980) records several examples of intentional bilateral symmetry. One of the examples she mentions includes a drop of blue paint that divides exactly in half the capital of a column at Coba. She also notes that the midpoint of a number of structures is clearly defined by a dash or square. Examples of this bilateral division were found at Paalmul, Playa Del Carmen; Structure IIIB of the Grupo Del Ray in Cancun; the interior doorways at the Castillo at Tulum; Structure 62 at Tancah; and Structure B at San Miguel de Ruz. The same observation holds for other structures at Cacaxtla, in central Mexico (Fettweis Vienot 1973). These intended bilateral symmetries, the author notes, lead to "a better understanding of the organization of the composition, and of the eventual repetitions of elements on both sides of a central axis" (translated from Fettweis Vienot 1980: 27-28 by Vinette 1986: 389).

Clancy's unpublished analysis of the monuments at Tikal (1977), summarized by Vinette (1986:389-392), also suggest evidence for ancient knowledge of geometry. Her study posits a pre-established compositional structure that corresponds with the placement of various motifs on carved stelae. This proposed structure is composed of intersecting and overlapping isosceles and Pythagorean triangles. Clancy constructs these triangles by drawing two sets of diagonals of 54 and 67 degrees from the lower horizontal base of a stele. The central vertical line of the composition is determined by the intersections of these diagonals. A horizontal line across the fifty-four-degree angles completes the basic compositional structure of the stelae. Middle Classic stelae, Clancy observes, exhibit a simpler compositional structure than the more complex angles seen in Late Classic stelae. Throughout her analysis, the critical interstices of the triangles were determined by the positions of certain motifs, the frequencies of which are summarized in Table 1 published by Vinette (1986:389).

The criteria Clancy used to determine that her observations were intentionally produced by Maya artists were:

a) to be significant, the angular and linear relationship must be found to repeat on at least three different monuments;

b) angular relationships were considered similar when there were no more than two degrees of variation between them; and

c) a basic repeatable structure that would maintain the above criteria must be common to a least fifty percent of the monuments tested.

Although Vinette considers Clancy's criteria as "insufficient to infer conclusive results," she concedes that Clancy's approach is open to further development which may, in the future, ". . . confirm her hypothesis of a prescribed compositional structure for Maya stelae as well as Maya cognizance and application of Pythagorean triangles."

CHAPTER 3

ETHNOHISTORICAL ACCOUNTS OF THE USES OF THE MEASURING CORD

Perhaps the most telling use of the measuring cord with regard to this discussion is recorded in the opening passages of the Popol Vuh, which begins with a description of the Maya creation (Tedlock 1985:71):

> And here we shall take up the demonstration, revelation, and account of how things were put in shadow and brought to light by the Maker, Modeler, named Bearer, Begetter . . .
> It takes a long performance and account to complete the emergence of all the sky-earth:
> the fourfold siding, fourfold cornering,
> measuring, fourfold staking,
> halving the cord, stretching the cord
> in the sky, on the earth,
> the four sides, the four corners,
> as it is said,
> by the Maker, Modeler,
> mother-father of life . . .

In this passage "the Maker, Modeler" create the cosmos by laying out a square or rectangle with a measuring cord. Then the gods halve and stretch the cord at least one arc into the sky and onto the earth. Herein lies a geometrical formula for creation. Given the two-thousand-year-old oral tradition of the Popol Vuh, this passage represents the source or ancient distillation of geometric knowledge for both Pre-Columbian and modern architects. As will be shown, these "divine instructions" constitute the basis for the repeated proportions deciphered in my analysis of Maya art and architecture as well as the formulae incorporated into the designs of modern Maya vernacular architecture.

Two ceremonial uses of the measuring cord, one a "rite of passage" ritual and the other a Maya New Year ceremony, were recorded by Fray Diego de Landa in his sixteenth century Relaciónes de las Cosas de Yucatán. Another sixteenth-century chronicler, Fray Diego Duran (Hayden and Horcasitus, ed. 1977:162-163), reports a similar use of the measuring cord in an Aztec ceremony dedicated to the god Tlaloc.

Fray Diego de Landa, in his Relaciónes, describes two rituals in which a stretched cord plays a prominent role. In the first (Tozzer 1941:104), Landa translates the name of the ceremony literally as "to be born anew." The ritual primarily served to mark the transition of adolescents into adulthood. The ceremony took place in a newly swept courtyard and was presided over by a "priest" or shaman and four elders who, after three days of fasting, were designated as "Chacs." In this regard, Tozzer

(1941:104) notes that four Chacs holding a cord may be depicted on page 19 of the Codex Tro-Cortesianus. He also notes that Lothrop (1936:28) illustrates a carved vase found at Zacualpa that depicts four seated figures, each holding a piece of twisted rope. Then, quoting Landa:

> ...they placed four stools in the corners of the court, on which the four Chacs sat down with a long cord held from one to the other, so that the children remained shut up in the middle or inside the cord; after which all the fathers of the children who had fasted, passing over the cord had to enter inside the circuit. Afterwards or before, they placed in the middle another little stool on which the priest sat down with a brazier and with a little ground maize and their incense. Then the boys and girls came in order, and the priest put into their hands a little ground maize and incense, and they threw it into the brazier, and this they all did. And these censings being over, they took the brazier in which they made them, and the cord with which the Chacs had surrounded them, and they poured a little wine into a vessel, and gave the whole to an Indian to be carried out of the town, enjoining upon him that he should not drink nor look behind him as he came back.

The second relevant ceremony described by Landa (Tozzer1941:151-152), the New Year ceremony, was considered a very solemn occasion and began with considerable fasting. Next, all utilitarian objects such as plates, vessels, old clothes and even “the stuffs with which they wrapped their idols were thrown out and renewed.” Again quoting Landa:

> All having come together with the presents of food and drinks, which they had brought, and also a great quantity of wine, which they had made, the priest purified the temple, seating himself in the middle of the court, clothed like a pontiff. The Chacs seated themselves at the four corners, and stretched from one to the other a new cord, within which were to enter all those who had fasted, in order to drive out the evil spirit…Once having expelled the spirit, all began to pray with great devotion and the Chacs kindled a new fire, and lighted the brazier for use in the feasts in which all joined in common, they burned incense to the idol with new fire and the priest began to throw this incense into it…and this was their new year and a service very acceptable to their idols.

Fray Diego Duran (Hayden and Horcasitus, ed. 1977:162-163), describes the use of a measuring cord in an Aztec ceremony dedicated to the god Tlaloc. In this ceremony, five trees, one large and four small, are carried to a court in front of a temple dedicated to the god Tlaloc. The large tree is called Tota, which means "Our Father." The following is Duran's description of this portion of the ritual:

"Once the great tree and the four small ones had been set up in the form of a square with Tota in the center, from each of the small trees emerged a twisted straw rope, attached to the large one in the center. From the small trees, therefore, emerged four ropes and all four were tied to the central tree called Tota."

Neither Landa nor Duran describes a Maya ceremony that occurs at the end of the fifty-two year Calendar Round cycle. Tozzer (1941:151), however, notes that among the Aztec a "renovation," similar to that described by Landa above, did take place at the end of each fifty-two-year period. He further speculates that the same would be true for the Maya. He then quotes Vaillant (1938:552): "One such rite involved the destruction of old household furniture and equipment in order to make new utensils when the new cycle began. A second ceremonial observance, after kindling the new fire, was to embellish their temples."

In each of the three ceremonies described above the most basic Mesoamerican cosmogram, the quincunx, is laid out with measuring cords, the centers of which are formed by shamans in the Maya examples and by the Tota tree in the Aztec. The quincunx is the foundation of Maya cosmograms and is generally depicted as square or rectangular with four dots in the corners and a somewhat larger dot in the center. The four corners and four edges represent the four directions on the flat plane of the surface of the earth. Trees are often depicted in this quincunx arrangement, the central tree representing the *axis mundi* located at the center of the world. This axis mundi, with its roots in the underworld and its branches in the sky, bridges the three levels of the Maya universe: the sky, the earth and the underworld.

In support of the above interpretations is my previous structural and comparative analysis of ten "ordering rituals" depicted in the Mixtec Vendobonensis Mexicanus I (Powell, 1993). This codex is among the relatively small group of pre-

Hispanic and early colonial pictorial manuscripts that have survived to the present from the Mixtec-speaking region of southern Mexico.

Most researchers agree that the obverse of this codex is a pre-Hispanic, Mixtec depiction of the creation and ordering of the universe (Furst 1978, Troike 1978 and Nowotny 1948). At some point during the codex's history in Europe, the fifty-two pages were erroneously numbered from left to right with Arabic numerals. The observe is correctly read from right to left, that is, from page 52 to page 1. The Vendobonensis Obverse depicts ten rituals that are similar in structure and content. These rituals have been interpreted as prerequisites to the ordering of landforms and places in the Mixtec world (Furst 1978: 309; Troike 1978:555; and Nowotny 1948:194). Immediately preceding the ordering of earthly landscapes and places, two men can be seen stretching a taut cord before a series of architectural elements. These images are followed by a fire-drilling ceremony. Note that four of the ceremonies from the ethnohistoric accounts and all of the examples from the Vendobonensis Obverse involving the measuring cord were also associated with architecture. In the ethnohistoric accounts, these associations include: performance in courts in front of temples (creating a quincunx or sacred space in front of or near the temple); purification of temples; or refurbishing of temples. Of these, three were immediately followed by a fire drilling and/or the ritual use of fire. Thus, even the sequence of the primary symbols of the “ordering ritual” (measuring cord, architecture, then fire drilling) is identical to the ethnohistoric examples. In the ten “ordering rituals” of the Vendobonensis Obverse, it is proposed that the two males are using a measuring cord to lay out the shapes of sacred architecture. Assuming that the architecture serves as a cosmogram for sacred landforms, the males would then be partaking in a symbolic or perhaps actual act of creation of the natural world (i.e., the *lienzo* depictions of landforms following the “ordering rituals”). The subsequent fire-drilling ceremony may in effect sanctify or give life to their creation.

CHAPTER 4

ETHNOGRAPHICAL ACCOUNTS OF THE USES OF THE MEASURING CORD

Ceremonies similar to those described during the Contact Period are practiced today among indigenous cultures in Mesoamerica. Over the years, I have witnessed, and occasionally participated in, a variety of Maya rituals, from private ceremonies held in individual households to elaborate public ceremonies conducted on mountain tops, in caves, in cemeteries, in the atriums and interiors of Catholic churches, and in the central plazas of towns and villages. Many of these ceremonies begin by using a measuring cord to lay out a temporary altar on the ground in the form of a square with its center demarcated (a quincunx). The corners of these temporary altars are often aligned to the cardinal directions, and among the variety of offerings that are invariably placed at the corners and centers are colored candles, preferably made of tallow rather than wax. Red candles are placed in the eastern corner, black candles to the west, white to the north, yellow to the south, and blue and green candles in the center; a color scheme that can be traced back to Pre-Columbian times.

The Cha Chac ceremony, as practiced by modern Yucatecans, displays geometrical constructions that are remarkably similar to those described by Landa and Duran. In this ceremony, four large posts are erected in the form of a square. These posts are called “the four corners of the earth” (Nikolai Grube, 1993, personal communication). Placed at the center of this square is a smaller square (or double square) altar, consisting of a table with four legs. Cords or ropes are attached diagonally along the ground from the four legs of the altar to the four larger corner posts. At the four corners of the central altar stand four vertical posts. Two arched boughs are attached diagonally to the tops of each post, crossing at the upper center of the altar. At the center of this elaborate quincunx, on top of the altar and beneath the arched boughs (representing the sky and possibly the ecliptic), food and other offerings are laid. This becomes the focus of the ceremony conducted by a shaman, the purpose of which is to bring forth rain. Sometimes young boys are tied by their ankles to the four corners of the altar, and they imitate the croaking of frogs to help bring on the rain.

Dr. Brian Stross (1992, personal communication) once observed an interesting use of a measuring cord in the northern Maya Lowlands. A cord was used to lay out the dimensions of a grave as well as to measure the exact placement of grave goods. When the burial was covered, the cord was rolled into a ball and placed “like a seed” at the center of the fill-dirt.

A measuring cord is also traditionally used today by many Maya people to lay out and parcel their quadrangular *milpas*, or maize fields. It should be noted here that the Maya metaphorically compare their milpas to the surface of the four-sided earth (Miller and Taube 1993:83-84). The four corners and center of milpas are still demarked today with stacked stones and/or buried offerings.

Ethnographer Dr. Duncan Earl, during his investigations in the Tzotzil Maya community of Chamula, in the Highlands of Chiapas, Mexico, witnessed a solemn annual ceremony where the male heads of households would carefully lay out their measuring cords side by side, not only to determine that they were of equal length, but also to determine that the single knot that divides these measuring cords precisely by a ratio of one to the square root of two, were also aligned. Dr. Earl observed that the Chamulans used these cords to turn right angles and to create squares when they were parceling out their milpas and laying out the designs of their houses (Duncan Earl, 1994, personal communication). The short section of these measuring cords are used to determine the sides of a square and the longer section to determine its diagonal and to insure a true right angle.

When we discussed our mutual investigations, Dr. Earl was disappointed that he did not record the geometry that the Chamulans used to lay out their houses.

The following is a description of a house-building ceremony that I observed and documented in 1995 as part of my duties as Director of Anthropological

Investigations for the non-profit institute Fundación Cultural Yucatán. The ceremony, and subsequent house building, were conducted near the town of Muna in the state of Yucatan, on the property of Hacienda Tabi, the oldest hacienda in the Yucatan, dating back to the late sixteenth century. Don Aurelio Hernandez Mucul, also from Muna, conducted the ceremony with the aid of an apprentice, his eleven year-old great-great-great-granddaughter. Don Aurelio was born on June 12, 1904 and was 101years old. This was to be his last public ceremony, as he passed away six months later.

Because the chosen building site had been built upon, farmed, and abandoned over one hundred years before, Don Aurelio decided it prudent to conduct a simple preliminary ceremony to seek permission from the souls of ancestors and of various nature spirits that might still dwell in and upon the land. Though I witnessed this ceremony, it was conducted in Yucatec, a Maya language that I am not conversant in, and I did not feel comfortable interrupting the ceremony to ask for regular translations into Spanish. In summary, the ceremony lasted about twenty minutes, during which Don Aurelio offered balche (a honey-based liquor), copal incense from a brazier, and prayers to the four cardinal directions and to the sun. He then asked permission of the ancestors, nature spirits, and *duendes* (a Maya version of trickster elves or "little people") to build upon and to farm the land in question. Next, Don Aurelio cracked a raw chicken egg into a glass of water provided by his apprentice. He then removed a quarts crystal from a small leather pouch and held it up to his left eye. Through the crystal, he peered into the glass, which was backlit by the sun, and studied the contents. I do not know what he saw in that glass, but in the end he was pleased that the permissions he had asked for were granted.

The master builder of the future house, Don Lucio Gonzales Kan Te, his apprentice, Jose Conteras, and I spent the next several days clearing the trees, roots, and brush from a plot extending approximately thirty meters around the house-building site, which had been determined by Don Aurelio during the preliminary ceremony described above. Upon completion of this task, Don Aurelio and his apprentice returned to the cleared land, briefly consulted with Don Lucio, and promptly staked out a square on the ground with a measuring cord. The corners of the square were approximately aligned to the cardinal directions, with the proposed doorways facing north/east and south/west. It was important, according to Don Aurelio, *not* to align the doorways due east/west. I failed to ask why this was so.

As is common to most of the Maya houses I have examined, the sides of this initial square are two *uinics* in length. The Yucatecan word “uinic” translates as “human being” and is also a unit of measure roughly equal to the height of a human being. The Maya are aware that when a person stretches his arms perpendicular to his body the distance between extended fingertips is virtually equal to his height. Thus, the uinic unit of measure is accomplished by grasping the end of a measuring cord with one hand, the length of the cord with the other, stretching it perpendicular to the body, and grasping the long end of the cord with the fingertips. The arms are actually bowed behind the back a bit so that the uinic is slightly longer than the individual's height. This is done because the height of the spring line of the house, and thus the height of the doorway, are measured as one half the width of the house and should be slightly taller than the height of the individual measuring it, so that he, at least, does not have to bow his head when entering the doorway.

The four sides of the square were laid out by dead reckoning at first and then lightly staked into place at the four corners. Then a second measuring cord was used to measure the diagonals of the square, and the lightly staked corners were adjusted until the diagonals were of equal length, thus assuring true right angles at the four corners of the square. Then two cords were stretched diagonally from the corners of the square and where they crossed, a fifth stake was pounded into the ground to demarcate the center of the square.

Don Aurelio then instructed us to excavate post holes at the four corners of the square, approximately forty centimeters in diameter by eighty centimeters deep, and to excavate a hole in the center of the square some sixty centimeters wide by eighty centimeters deep. Because the soils in this area were less than twenty centimeters deep and were underlaid by solid limestone bedrock, this task required the use of heavy, sharpened iron bars, some two meters long, and a day's worth of labor.

We spent the following day preparing for the principal house-building ceremony. A *pip*, or cooking pit, was excavated approximately ten meters east of the house site, firewood was gathered, cut, and stacked, and many items to be used in the ceremony were purchased at the market in Muna. These items included: a large ceramic jar of balche, two live white turkeys, corn meal, sugar, sewing needles, safety pins, nopal cactus pads, copal incense, a large aluminum pot, and a dozen *jicara* bowls made from the dried, gourd-like fruits of the calabash tree.

On the day of the ceremony, about an hour before sunrise, Don Aurelio, his apprentice, and some forty to fifty men, women, and children from surrounding farms and hamlets arrived at the house-building site. Working together, they placed stones at the bottom of the pip and laid a fire on top of them. Three hearthstones were set west of the house-building site, and a fire was lit there as well. One of the turkeys was killed, plucked, chopped into pieces, and then set to boil in a large aluminum pot over this cooking fire. Dozens of plain corn tamales, decorated with equal-sided crosses made of cut corn husks, were wrapped in banana leaves and placed on the hot stones inside the pip. The tamales were covered with fragrant leaves and buried under earth and stone.

Meanwhile, Don Aurelio and his apprentice made a small altar on a plank of wood, which rested on several flat stones just to the southwest of the house-building site. Long, flexible branches were tied together and anchored into the earth, forming an arch running lengthwise over the altar. Above the wooden plank, the eleven-year-old apprentice carefully placed the items to be used in the ceremony, including equal-sided crosses made of nopal cactus pads, five jicara bowls filled with balche, the sewing needles, safety pins, corn meal, etc., and a lit brazier for the copal incense. After speaking and praying before this makeshift altar, pouring a bit of balche to the earth in front of it, and adding copal incense to the coals in the brazier, Don Aurelio placed a perfectly square, four-legged plastic table, with a round Coca Cola logo in the center of it, directly over the excavated hole in the center of, and parallel to, the square that was laid out two days before.

The ceremony then began. The sun had risen over the tree line and was shining above the altars, and the men, women, and children were arrayed to the northeast of the excavated corners of the square that was to be the focal point of the ceremony.

The ceremony was, of course, conducted in Yucatecan Maya, most of which I was unable to understand. As a result, the following is a summary of what I witnessed.

Don Aurelio's apprentice placed a bowl of balche and the fragrantly smoking brazier onto the plastic table that had now become the central altar and principal focus of the ceremony. Don Aurelio held the brazier toward the rising sun and spoke briefly. He then censed each of the holes at the corners of the square, beginning with the eastern corner and moving clockwise to the southern, western, and northern corners. He returned to the center of the square, and falling to his knees, he censed the excavated hole underneath the plastic square altar. I was able to understand the Yucatec words for “red tree” while he was speaking or praying in the eastern corner, “yellow tree” in the southern corner, “black tree” in the western corner, “white tree” in the northern corner and “blue/green tree” in the center. In the same order, Don Aurelio poured balche and sprayed balche from his mouth into the corner holes and center hole of the square. Next, he poured balche into twelve jicara bowls, and all of the participants drank.

With great emotion and animation, Don Aurelio then narrated a dramatic story. He sang songs and chanted and spoke some more. Occasionally the participants sang and chanted along with him. It was a long and elaborate performance, lasting well past midday, at the end of which he took the chicken egg, the safety pins and sewing needles, the crosses made of nopal cactus, and a few items that I was unable to recognize from his leather pouch and placced them into the center hole of the square. Finally, he retrieved the live turkey that had been tied to a stake near the hearth. Facing east, he held the turkey above his head and said a few words. He then kneeled, held the turkey between his legs, removed a pocketknife from his trousers, and quickly slit its throat. Immediately he clamped one hand across the wound while wiping the blood from the knife onto the feathers of the turkey with his free hand and then returning the knife to his pocket. He carried the turkey to the eastern corner of the square, removed his hand from the neck wound, and poured blood from the still living turkey into the hole. In this manner, he offered blood to each of the corner holes, again moving clockwise from east to north and terminating with a blood offering to the center. He then placed the turkey into the center hole, wiped his bloody hands on a piece of red cloth, and placed the cloth into the center hole. Then he stood up and declared a two-hour siesta. For some reason, he asked me to move the plastic table to one side and to bury the offerings in the center hole using the original soil excavated from it.

Some of the participants who lived nearby returned to their homes, others rested under the shade trees nearby. A few women tended the boiling turkey soup, adding corn meal to thicken it to the consistency of gruel. A few men and boys uncovered the pip, removed the cooked tamales, and placed them on a folding table that had been placed adjacent to the pip.

I had finished burying the offerings and was sitting under a tree, scribbling notes, when I noticed a small commotion. Don Aurelio had collapsed to the ground and was unconscious. Two women, his apprentice, and myself carried him to the shade of a tree and laid his head in the lap of one of the women, who began to speak or pray to him. His apprentice brought a bowl of water and a cloth and began to bathe his face. Don Aurelio soon regained consciousness, but he appeared delirious and scared. He was speaking with the voice a frightened, agitated child. He was weeping too. He went on like this for fifteen or twenty minutes, then he just cried, bitter, bitter tears. Then he slept for about an hour.

While he slept, one of the women told me that Don Aurelio had been reliving a horrific incident from his childhood. When Don Aurelio was a boy, near the turn of the twentieth century, a series of bloody Maya rebellions, the Caste Wars, were raging across the entire Yucatan Peninsula. The Maya were rebelling against the brutal slavery imposed upon them by the Spanish hacienda owners who controlled and dominated most of the farmland and most of the Maya farmers and families in the region. Soldiers were brought in from the Valley of Mexico to quell these rebellions. By all accounts, the Maya, armed mainly with machetes, knives, and farming tools, fought bravely against well-armed troops, cavalry, and cannons. By all accounts, they were brutally slaughtered in great numbers.

The incident that Don Aurelio was reliving occurred during these turbulent times. Early one morning Don Aurelio's mother was bathing him some distance from the family compound that housed his brothers and sisters, his aunt, uncle and cousins, his grandparents, parents, and himself. Soldiers suddenly attacked the compound, and his mother told him to hide in the forest while she returned to the compound. He watched as the soldiers killed his entire family. He ran naked into the forest and eventually walked from the state of Yucatan to the Highlands of Chiapas. A family there took him in, and he apprenticed with a shaman. Years later he returned to Yucatan.

When Don Aurelio awoke, he drank three bowls of water and stood up. He composed himself and spoke softly to his apprentice and the women who had looked after him.

The rest of the afternoon resembled a fiesta more than a formal ceremony. The participants returned, Don Aurelio spoke some words over the turkey soup and tamales, and offered a bit of each to the earth in front of the pip and the cooking pot. Everyone consumed the food and chatted casually. Some folks drank water and others were tipping the jar of balche. When the sun was again above the tree line, but now setting in the west, Don Aurelio delivered a final short oration and the ceremony ended.

The next day Don Lucio, Jose, and I began to build the house. The wood from six different species of trees and the guano (palm thatch) had been cut at or near the full moon and stacked and dried for approximately three weeks. Most Maya house builders cut the wood and thatch for houses at, or a few days after, the full moon, because they believe that the sap rises in these plants at this time of the month and that the increased sap content serves as both an insect repellent (mostly to ward off termites) and to help prevent these materials from rotting.

Early that morning, as we were hauling the wood to the building site, Don Aurelio showed up (unexpectedly to me at least) to complete the ground plan of the house. With the help of Don Lucio, he stretched his measuring cord between the east and south corners of the house, then folded his cord in half to mark its center point. He asked me to hold my thumb and forefinger on the cord to mark the center as they re-stretched the cord to the east and south corners. I placed my thumb on the ground to mark the center point between the corners while Don Lucio cut a sharpened stake to mark this spot. We repeated this process on the opposite side of the square. Don Aurelio then tied his measuring cord to this central stake and pulled it taut to the eastern corner of the square, at which point he tied a short, sharpened stick. He then used this sharpened stick to etch an arc in the soil from the east corner to the south corner and repeated the process, etching an arc between the north and west corners. While we placed stakes at approximately thirty-centimeter intervals along these arcs to demarcate them permanently, Don Aurelio left us.

The following chapter entitled, “How It Works,” describes and diagrams in detail the square root, phi, and Pythagorean 3,4,5 rectangles and their dynamic subdivisions. This chapter is intended as a compendium of Maya geometrical formulae, geometry that is incorporated into various Maya vernacular house designs, and apparently incorporated into Pre-Columbian art and architecture as well.

CHAPTER 5

HOW IT WORKS: DYNAMIC PROPORTIONS AND THEIR PERFECT AND VIRTUALLY PERFECT SUBDIVISIONS

The square root and phi proportions in this study of Maya art and architecture have been considered a formal set of proportions for thousands of years. The unique and dynamic attributes of these proportions were examined and puzzled out by the great geometers and mathematicians of old, from Pythagoras to Da Vinci, and continue to intrigue modern geometers and mathematicians. They are called dynamic proportions in modern vernacular in part because of their unique capacity to subdivide and combine into smaller and larger but similar forms. (The word “similar” in this context means identically proportioned but larger or smaller than). Another dynamic aspect of this set of rectangles is that they are the rectangular expressions of the proportions inherent in regular polygons. These relationships are made clear by examining the most elementary set of regular polygons, the equilateral triangle, the square, and the pentagon and all other regular polygons that may be derived from these three; for example, the hexagon, octagon, decagon, etcetera (see Figure 2). The heptagon and nonagon are not included in this set of dynamic forms. In fact, the heptagon is not included in planar geometry at all. Unlike other regular polygons, there is no geometrical formula for creating a regular heptagon, and it cannot be drawn with a compass and a straight edge. It can only be approximated, its angles expressed in fractions of degrees. Though a regular nonagon can be drawn with a compass and a straight edge, its formula is complex, and the resulting proportions do not relate to the square root or phi proportions as do the regular equilateral triangle, square, and pentagon.

Thus, the repetitive proportions discovered in my analysis form a coherent and interrelated system of measurement and proportion. Because root and phi proportions are irrational, they cannot be divided into even units. (This in itself might help to explain why they have remained elusive to previous researchers.) As mentioned earlier, the root rectangles (from the square root of one through five) are created by first laying out a square and then using a compass and a straight edge (or a cord that can function as both a compass and a straight edge). The length of each succeeding root rectangle is derived from the diagonal of the preceding rectangle.

The phi rectangle is created by laying out a square and halving a measuring cord at the base of the square to determine its midpoint. Then a cord is stretched from this midpoint at the base of the square to either of the upper corners of the square and swung downwards until it is parallel to the base of the square to determine the length of the rectangle. If this procedure is performed from both upper corners of the square, a square root of five rectangle is produced (see Figure 1).

It should be noted that the terms “square root” and “phi,” and the irrational numbers they represent, are western terms and mathematical concepts. The use of these proportions by the Maya in no way implies, nor requires, Maya knowledge of square root or irrational numbers.

My findings do support the hypothesis that the Maya developed and applied a sophisticated system of planar geometry, which, by definition, does not require the use of mathematics. Planar geometry can be defined as any shape that can be drawn with a straight edge and a compass (or more simply, any shape that can be drawn using a cord).

With the aid of fairly simple mathematical equations that require the use of irrational numbers, the dynamic subdivisions referred to above (and that are drawn and diagramed below) can be shown to be mathematically perfect subdivisions. But along with these “perfect” subdivisions, I have found a handful of virtually perfect subdivisions that are expressed repeatedly in the designs of Maya art and architecture. I call these subdivisions “virtually perfect” because the extremely slight inaccuracies of these subdivisions would be virtually impossible to detect without the use of mathematical equations that utilize irrational numbers.

Maya mathematics apparently consisted of equations, theorems, and ratios that relied exclusively on the use of whole numbers. Because the margin of error of these virtually perfect subdivisions are too small to detect using the common tools of planar geometry — the straight edge, the compass, or the measuring cord — and because the Maya did not have the mathematical tools to prove these slight inaccuracies, these virtually perfect subdivisions, would have appeared to be, for all practical purposes, perfect subdivisions. Thus, the dynamic subdivisions of the square root and phi probably seemed even more dynamic to a Maya geometer.

The Pythagorean 3,4,5 rectangle can also be virtually perfectly subdivided by the root and phi rectangles and thus was probably considered an integral part of this coherent and interrelated system of measurement and proportion. These perfect and virtually perfect dynamic subdivisions of the square root, phi, and Pythagorean rectangles are drawn and diagramed below, beginning with the square. I present only the examples of dynamic subdivisions that the Maya appear to have intentionally incorporated into the layout and designs of their art and architecture.

An equiangular spiral can also be easily drawn from the subdivisions of a phi rectangle (or a phi triangle). Chapter 7 will present evidence that the Maya understood these formulae for drawing equiangular spirals of phi proportions and that they used them in the layout and designs of various works of Pre-Columbian art.

The diagrams included in this chapter are, in my opinion, the nuts and bolts of Maya geometry, and though I probably have yet to detect some of the virtually perfect subdivisions, I believe these diagrams represent a nearly complete collection of Maya rectangular proportions and their subdivisions. The diagrams and formulae below will be repeatedly referred to throughout the rest of this dissertation.

Though some of the formulae may seem complex, their constituent parts are remarkably simple. All of the Maya geometry described in this dissertation consist of various combinations of only five rectangles: the square, the root two, the root three, the phi and the Pythagorean 3,4,5 rectangles (six, if you include the root phi rectangle that must be derived from the phi rectangle). A few examples of concentric circles with root and phi proportions and a few examples of phi equiangular spirals are also noted and examined.

Figure 1

The Square Root, Phi, and Pythagorean 3,4,5 Rectangles

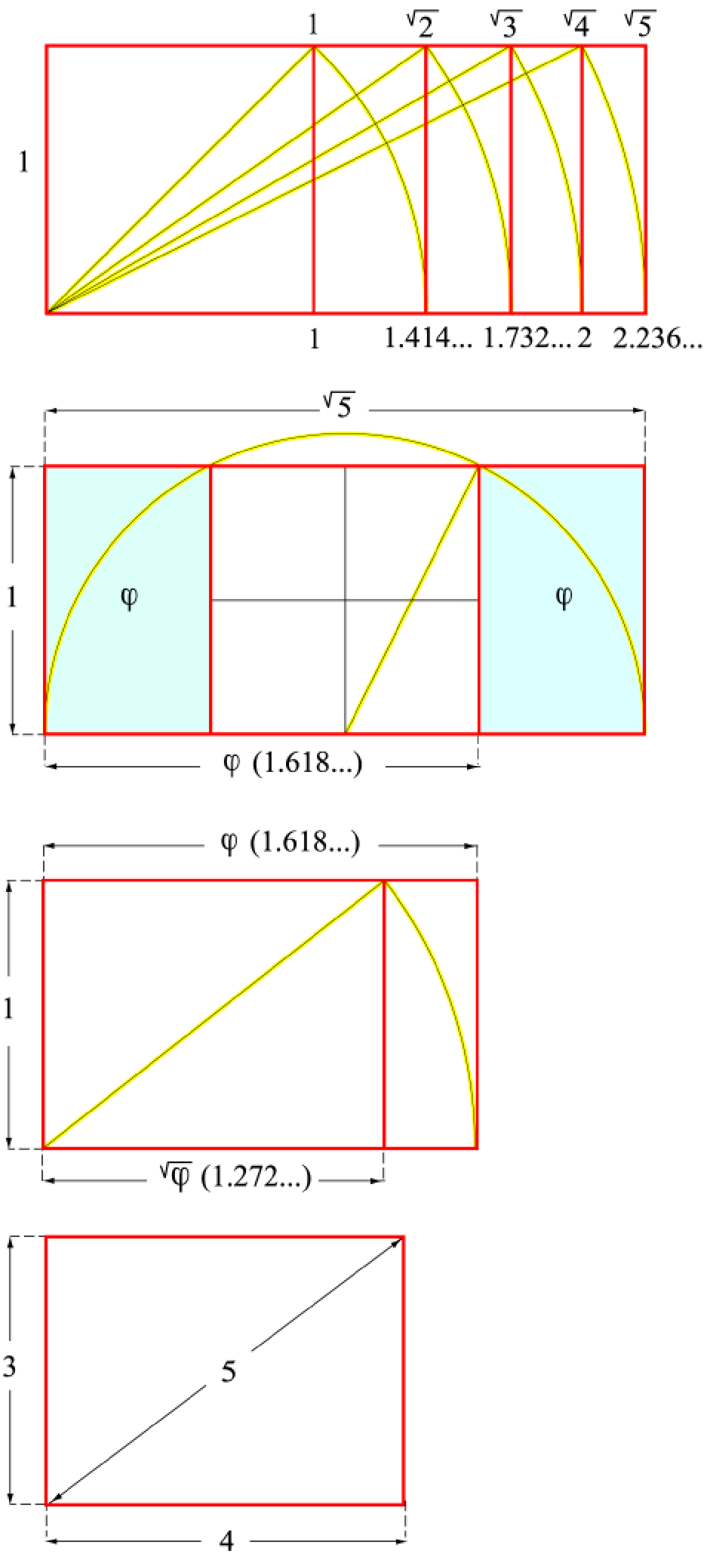


Figure 2

The Square Root and Phi Proportions and Regular Polygons

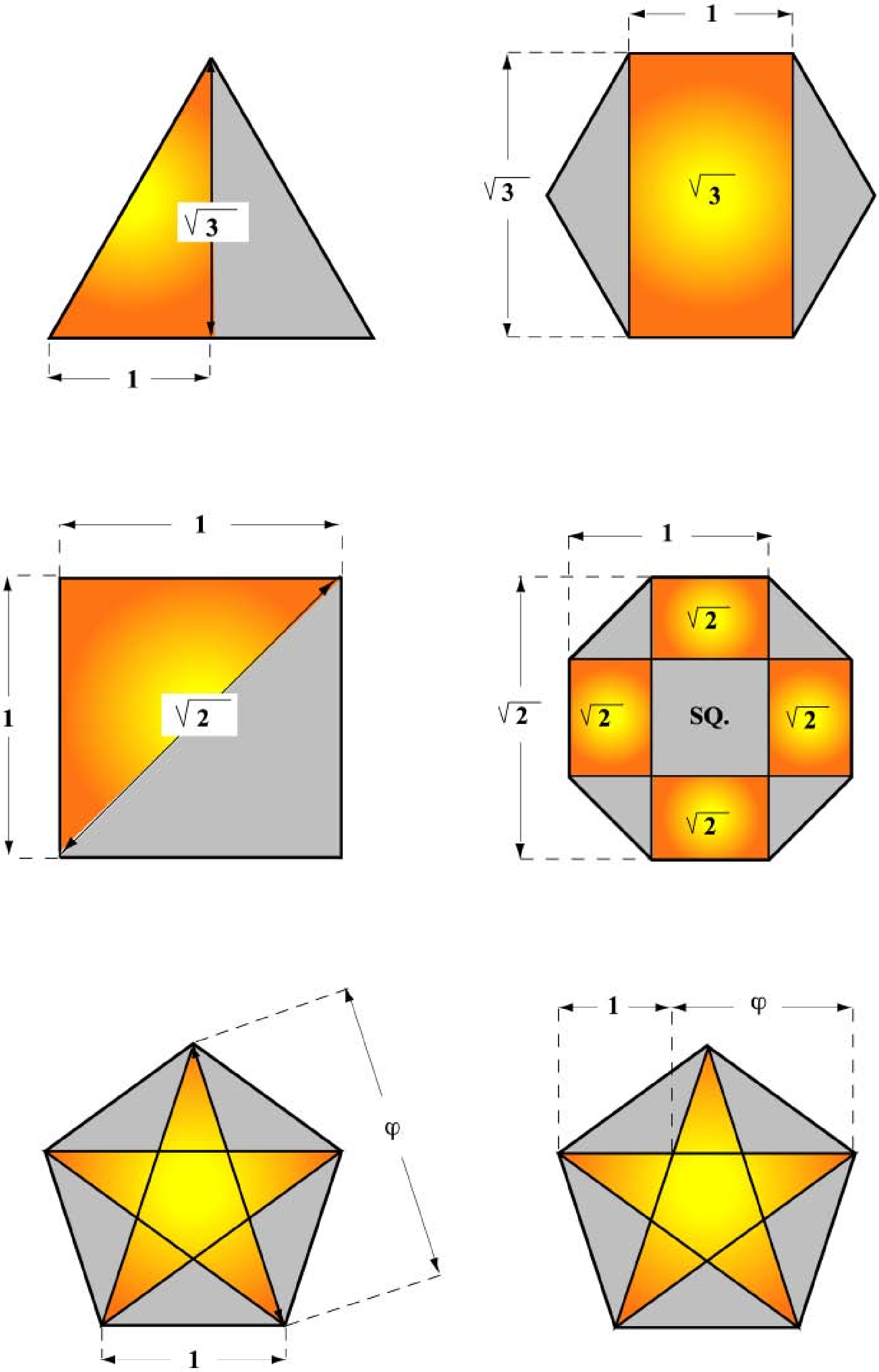


Figure 3

The Square Root of Two, the Square Root of Three, Phi
and the Shapes of Flowers and Shells

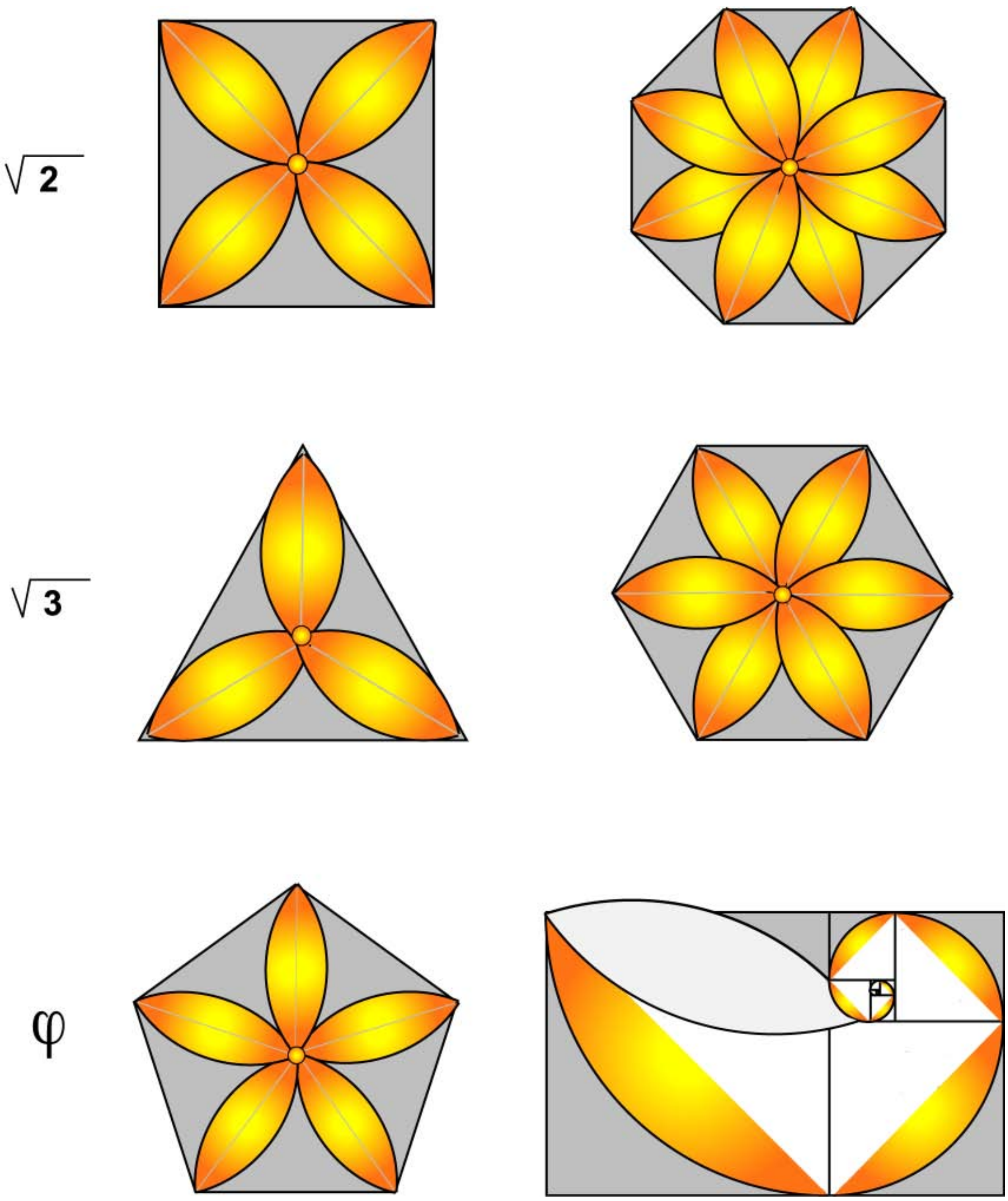
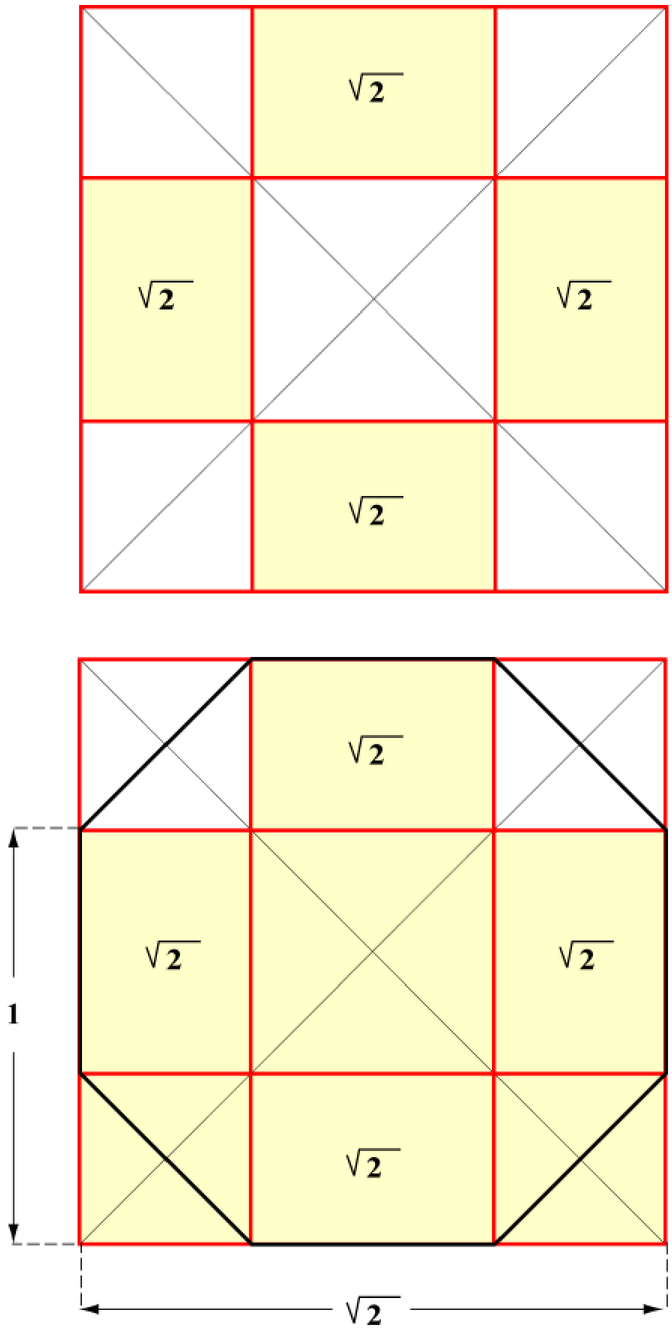


Figure 4

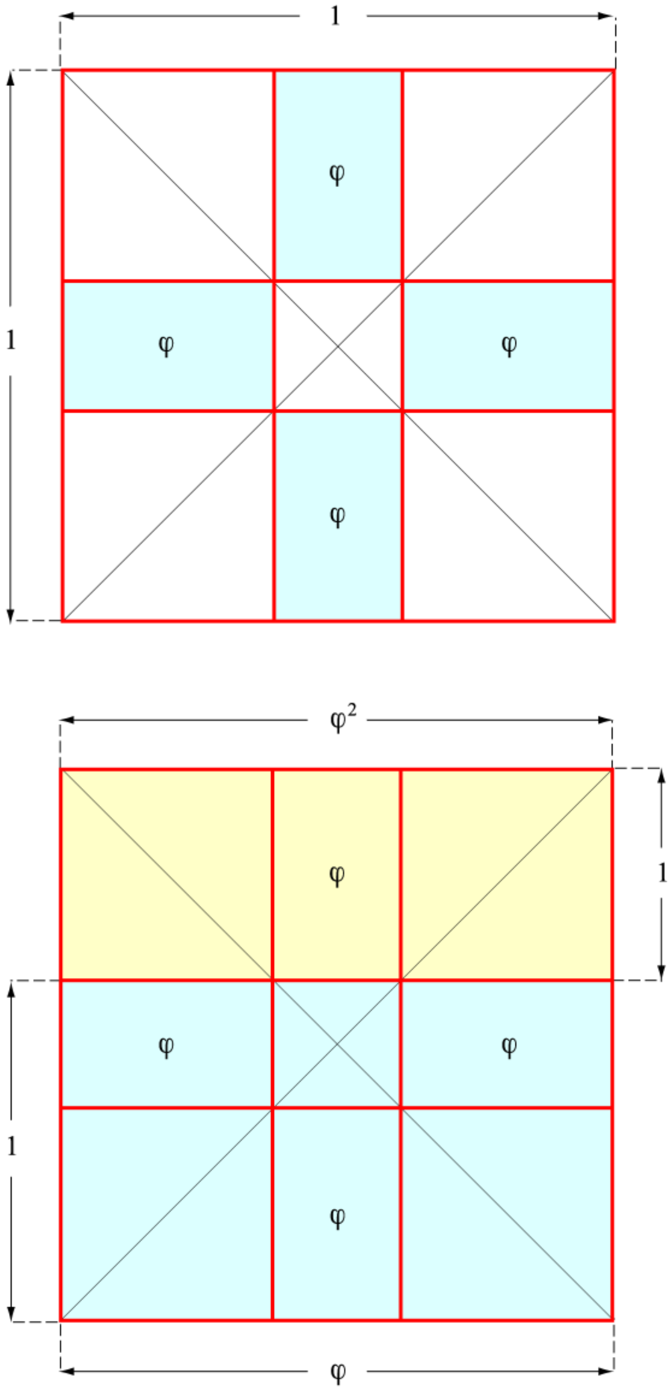
Formula Square-1



Formula Square-1 diagrams the subdivisions of a square by root two rectangles shaded in yellow and the remaining squares in white. The lower diagram illustrates the relationship of the root two subdivisions to a regular octagon (in bold black lines) inscribed in the principle square.

Figure 5

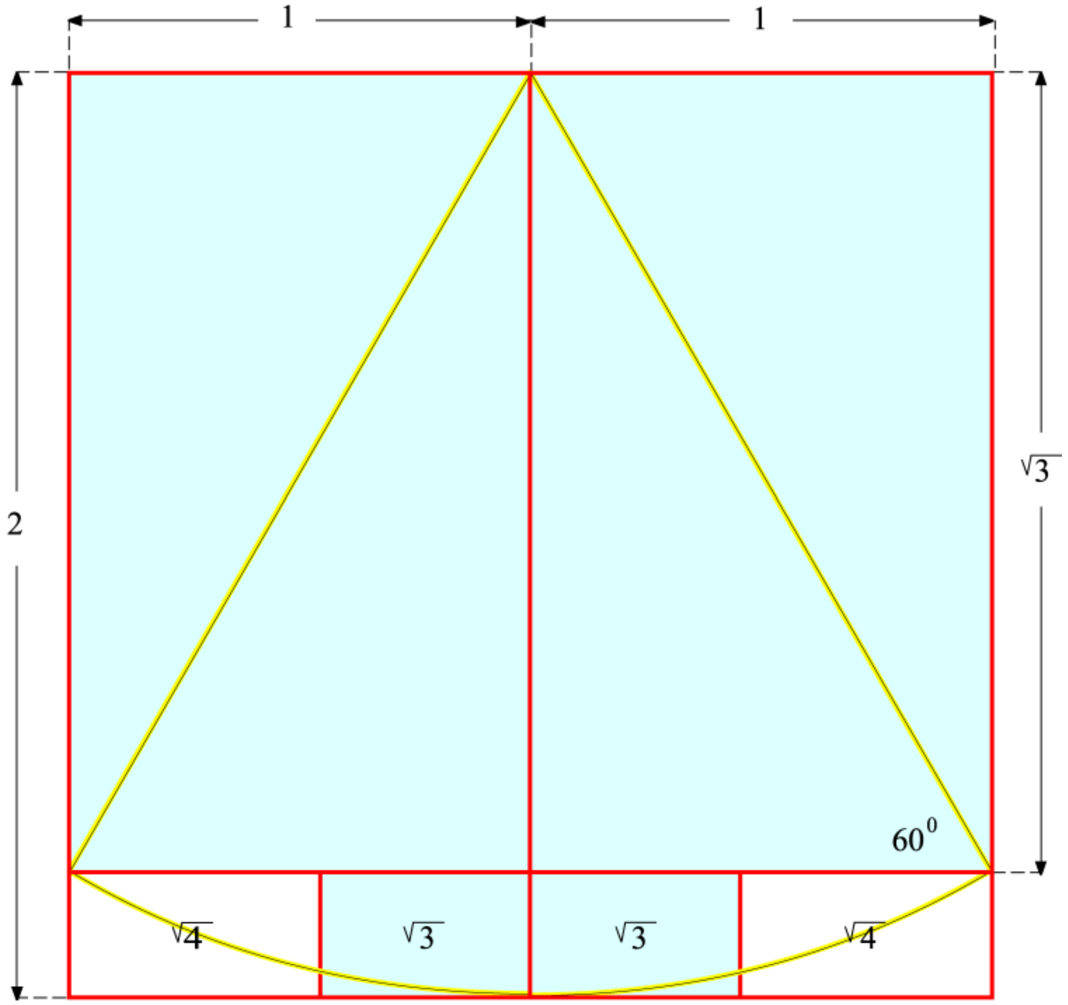
Formula Square-2



Formula Square-2 diagrams the subdivisions of a square by phi rectangles (shaded blue) and the remaining squares are in white. The lower diagram illustrates how these phi subdivisions also divide a square into a phi rectangle with the same length as the square (shaded blue) and a phi squared rectangle (shaded yellow).

Figure 6

Formula Square-3



Formula Square-3 diagrams the subdivisions of a square by root three rectangles (shaded blue) and root four rectangles (in white). The highlighted diagonals of the larger paired root three rectangles are the radii for the arc that terminates at the center of the base of the square. The remaining space at the base of the square is subdivided by root three and root four.

Figure 7

Formula Square-4

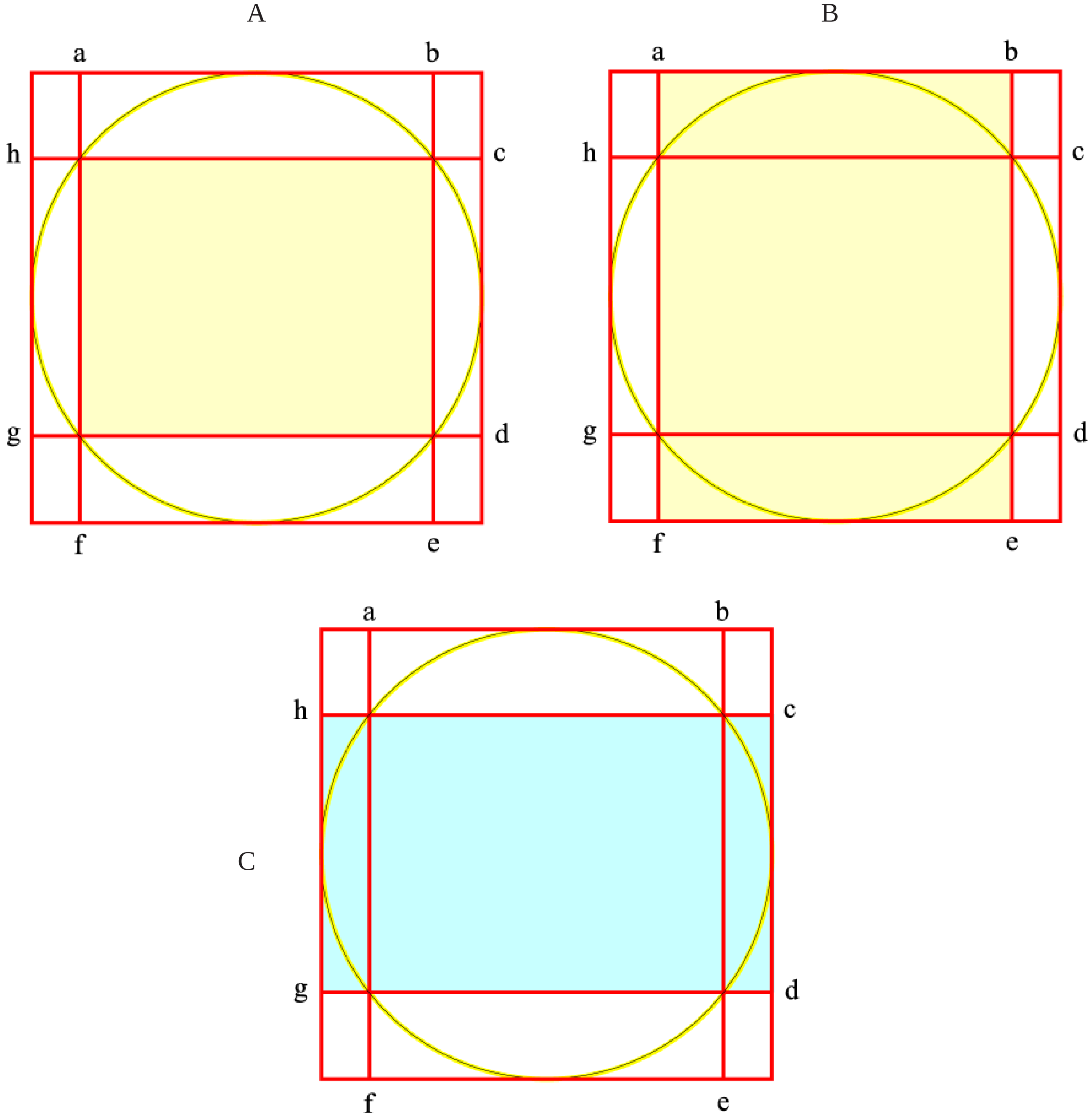
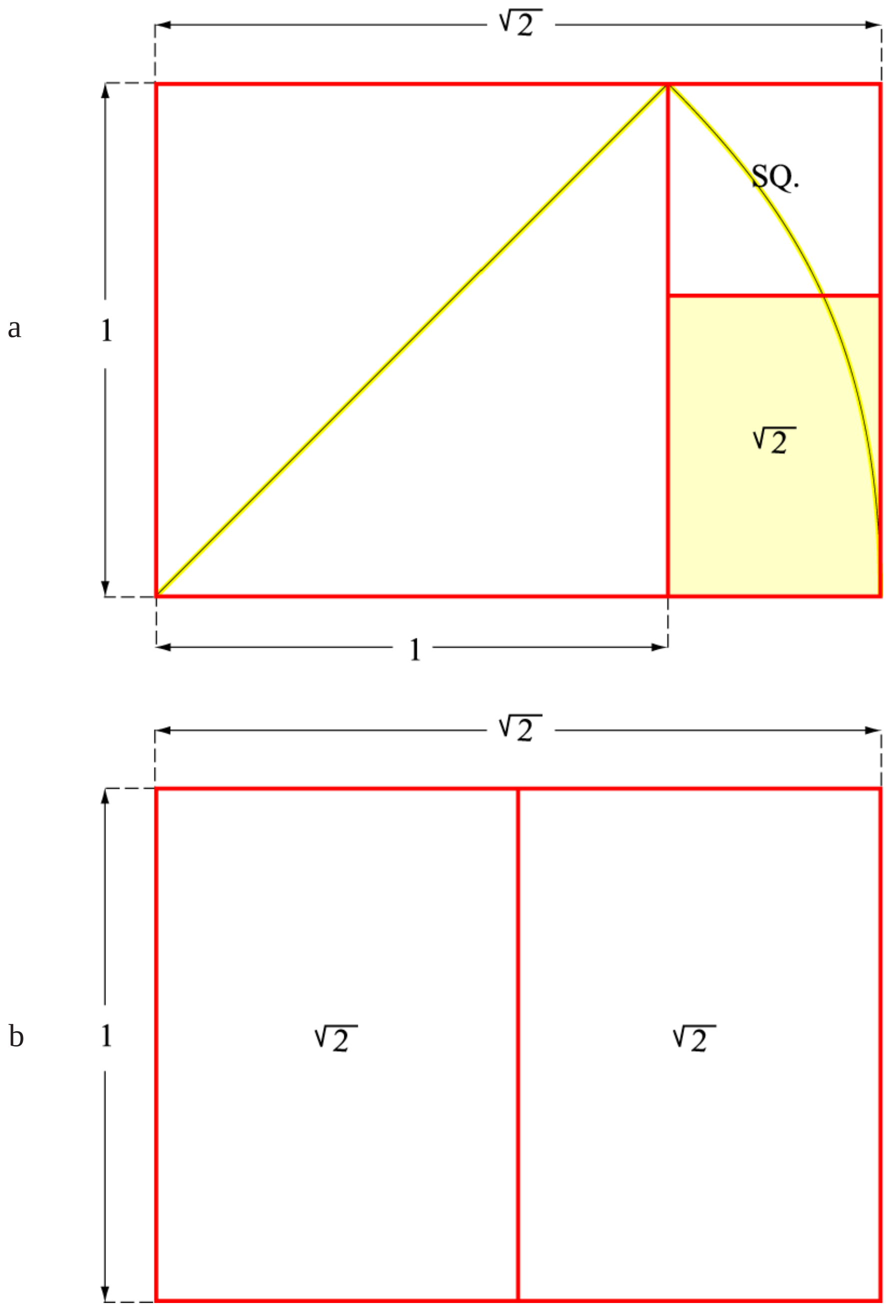


Diagram A shows a root phi rectangle (shaded yellow) inscribed within a circle that is inscribed within a square. Diagram B shows a root phi rectangle (a.,b,e,f) whose width is equal to the length of the root phi rectangle in diagram A and whose height is equal to that of the square. Diagram C shows a phi rectangle (shaded blue) (c,d,g,h) whose length is equal to that of the square and whose height equals that of the root phi rectangle in diagram A.

Figure 8

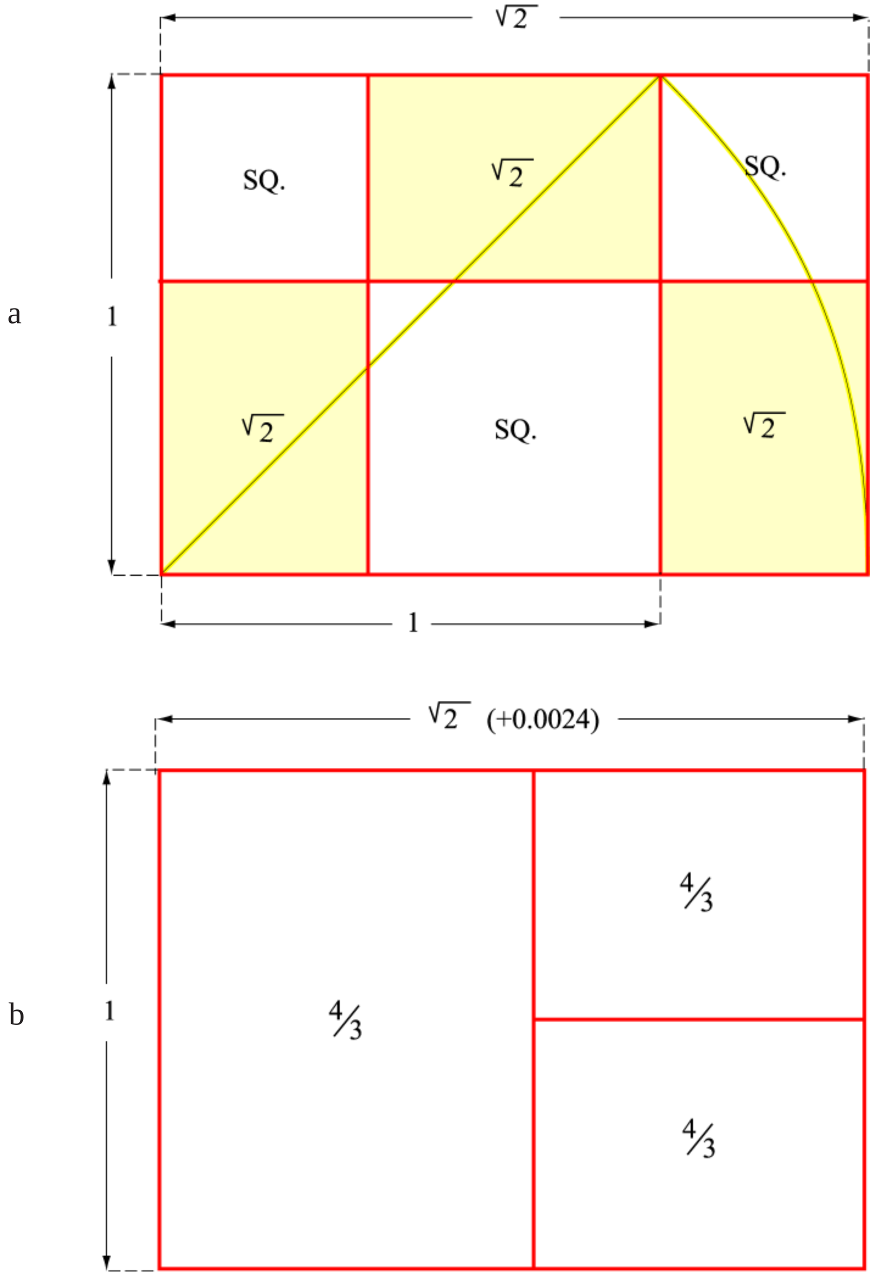
Formulae Square Root of Two-1and 2



Formula Square Root of Two-1 (a) diagrams the subdivision of a root two rectangle by two squares (in white) and a remaining root two rectangle (shaded yellow). The diagonal of the larger square is the radius that determines the length of the base rectangle.
Formula Square Root of Two-2 (b) diagrams the subdivision of a horizontal root two rectangle by two vertical root two rectangles. (Note that the root two rectangle is its own gnomon.)

Figure 9

Formulae Square Root of Two-3 and 4



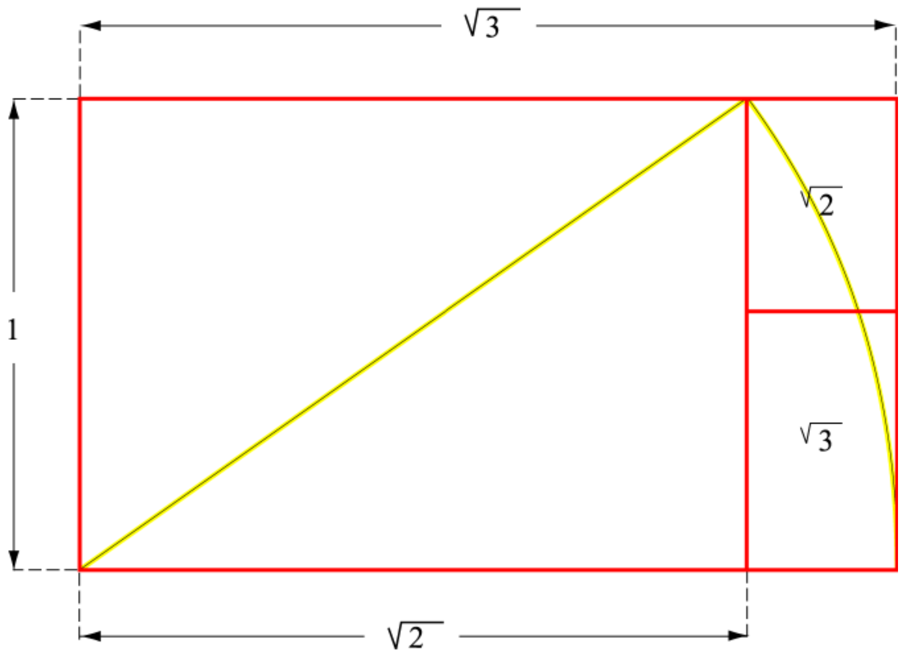
Formula Square Root of Two-3 (a) diagrams the subdivision of a root two rectangle by squares (in white) and root two rectangles (shaded yellow). The highlighted diagonal is 45 degrees and is the radius for the arc that forms the root two rectangle.

Formula Square Root of Two-4 (b) is a virtually perfect subdivision of a root two rectangle by a three Pythagorean 3,4,5, rectangle. (The margin of error for this formula is 0.0024.)

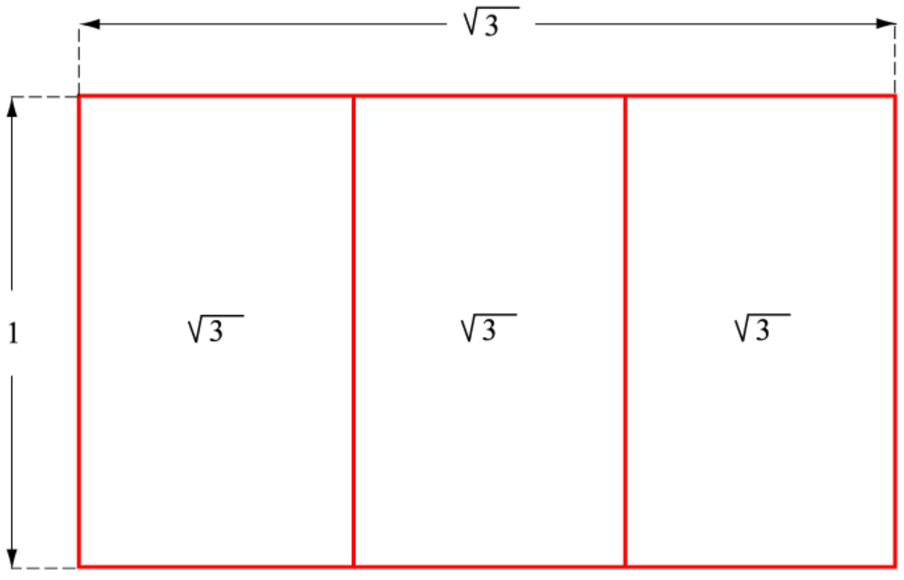
Figure 10

Formulae Square Root of Three-1 and 2

a



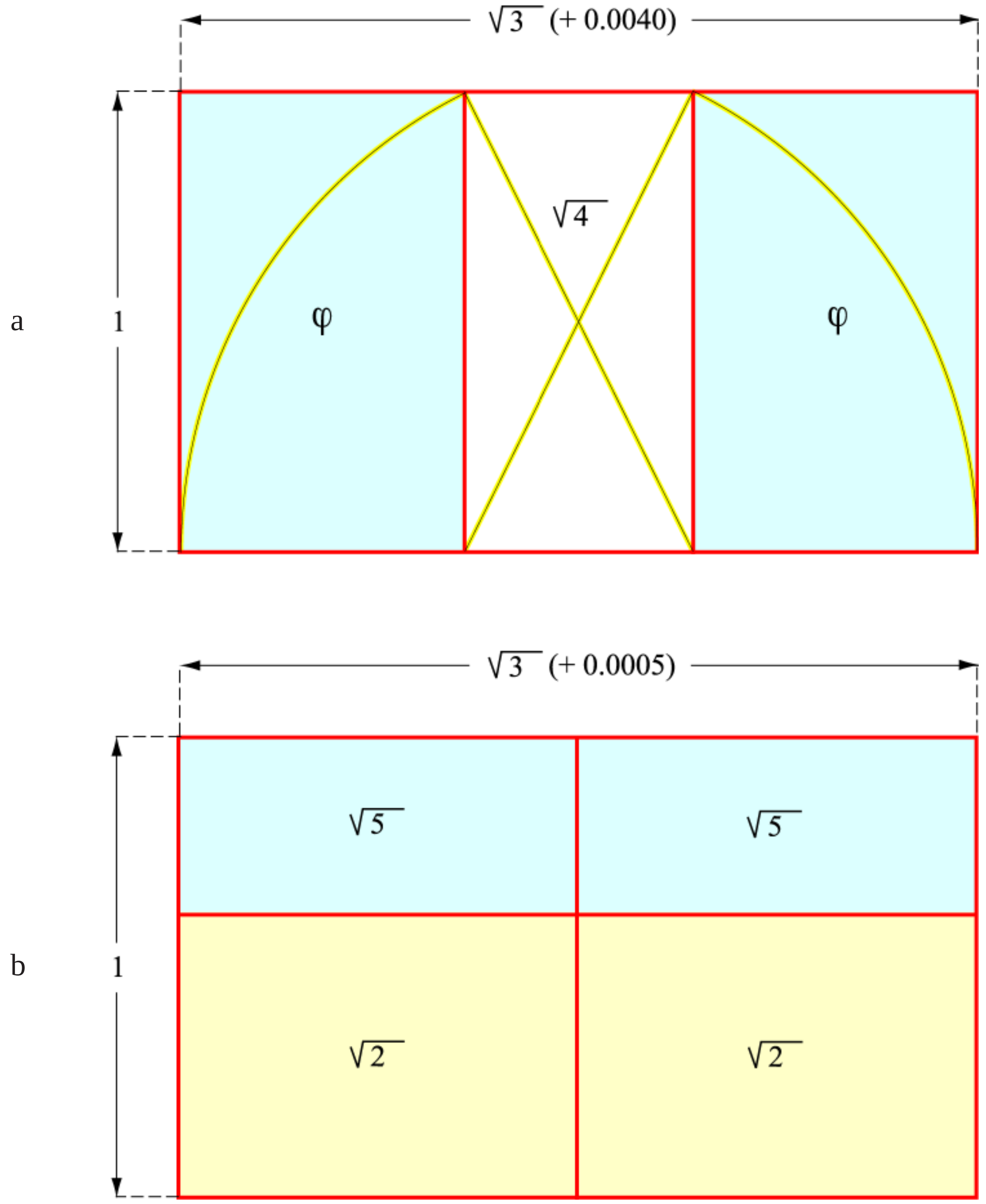
b



Formula Square Root of Three-1 (a) diagrams the subdivision of a root three rectangle by two root two rectangles and a root three rectangle. The diagonal of the larger root two rectangle is the radius of the arc that determines the length of root three rectangle (highlighted in yellow).
Formula Square Root of Three-2 (b) diagrams the subdivision of a horizontal root three rectangle by three vertical root three rectangles.

Figure 11

Formulae Square Root of Three-3 and 4

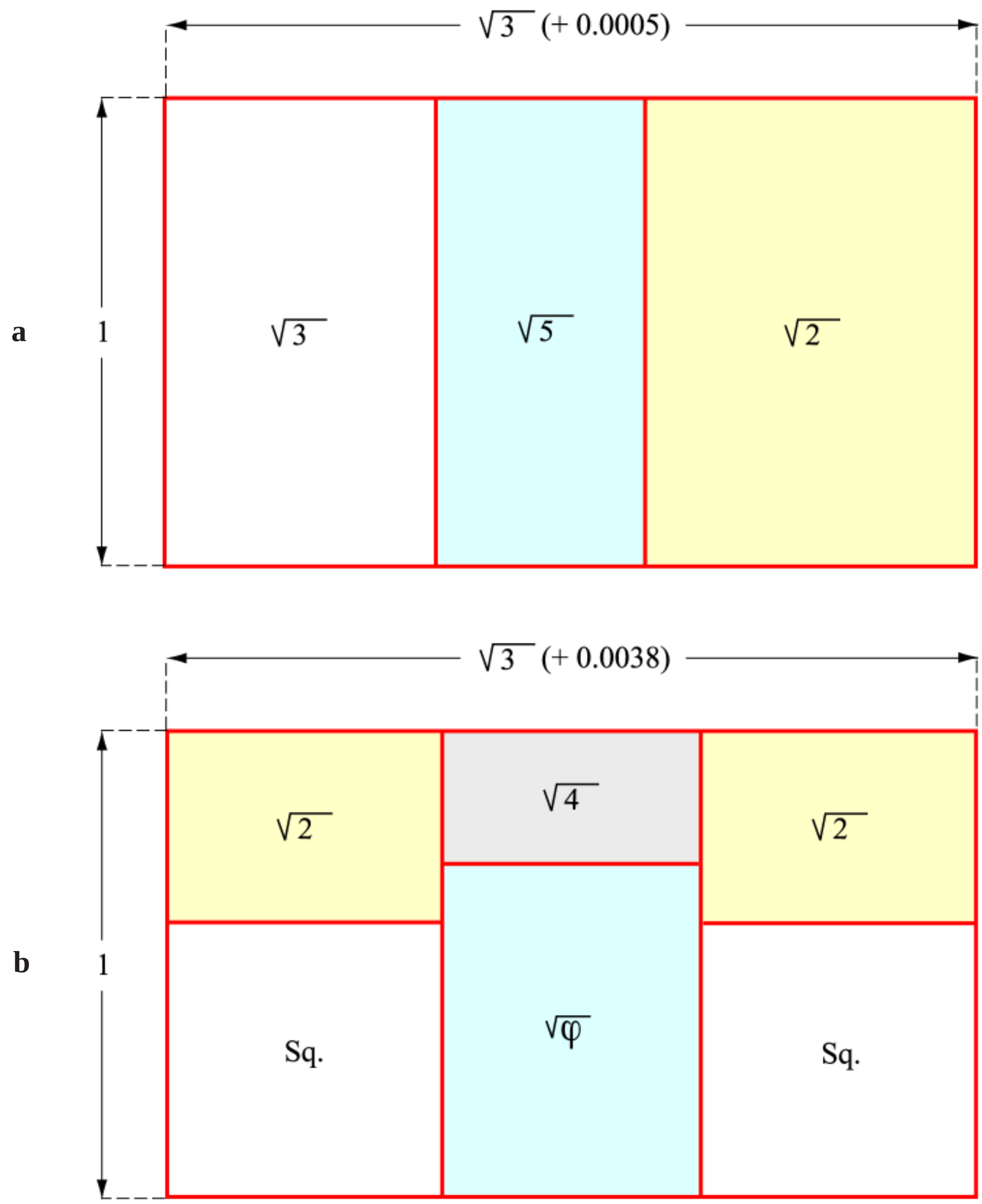


Formula Square Root of Three-3 (a) is a virtually perfect formula that subdivides a root three rectangle by two phi rectangles (shaded blue) and a root four rectangle (in white). The diagonals of the root four rectangle are the radii of the arcs that form the adjacent phi rectangles. (The margin of error is 0.0040)

Formula Square Root of Three-4 (b) is a virtually perfect formula that subdivides a root three rectangle by two root two rectangles (shaded yellow) and two root five rectangles (shaded blue). (The margin of error is 0.0005).

Figure 12

Formulae Square Root of Three-5 and 6

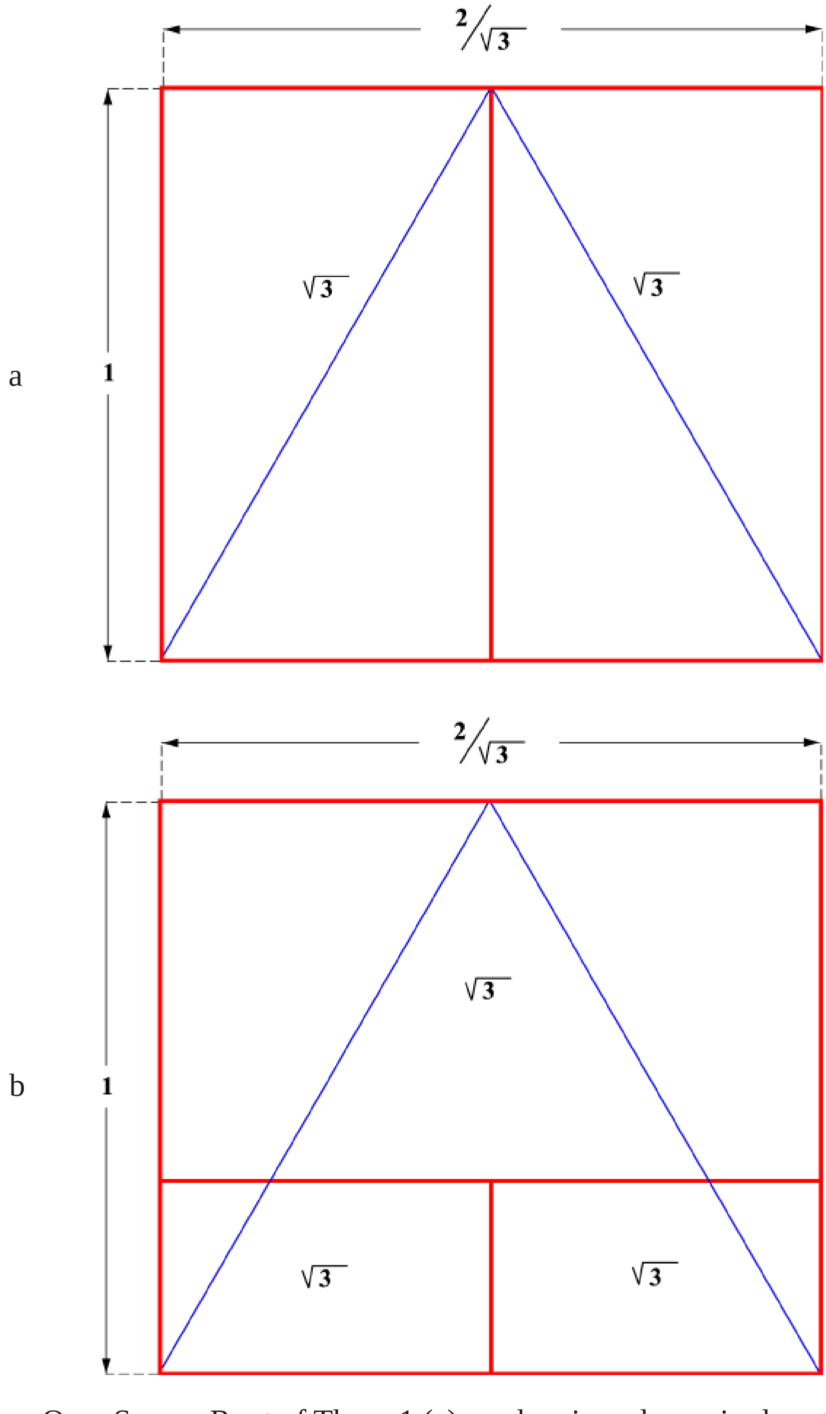


Formula Square Root of Three-5 (a) is a virtually perfect formula that subdivides a root three rectangle by a root three rectangle (in white) and a root five rectangle (shaded blue) and a root two rectangle (shaded yellow. (The margin of error is 0.0005)

Formula Square Root of Three-6 (b) is a virtually perfect formula that subdivides a root three rectangle by two root two rectangles (shaded yellow) and two squares (in white), a root phi rectangle (shaded blue) and a root four rectangle (shaded grey). (The margin of error is 0.0038).

Figure 13

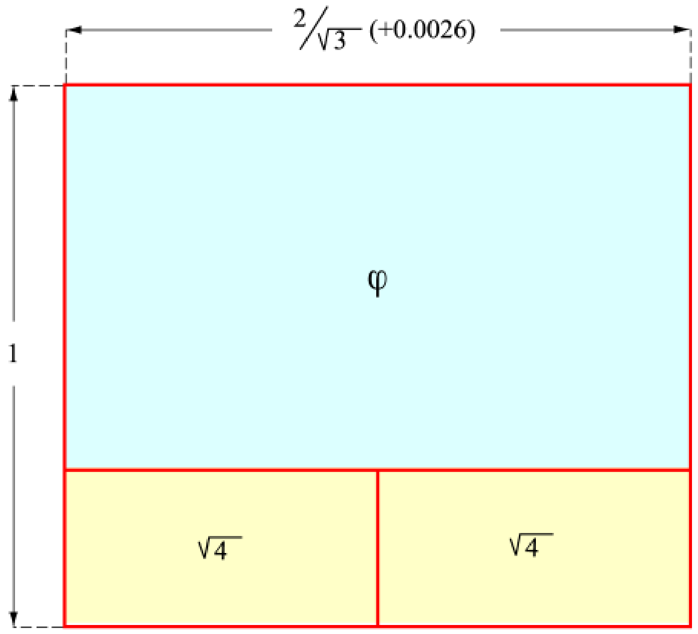
Formulae Two Over Square Root of Three-1 and 2



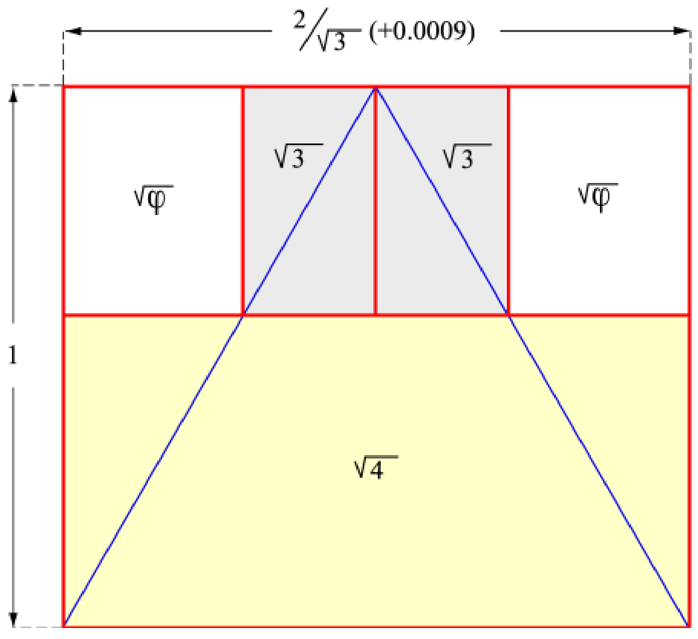
Formula Two Over Square Root of Three-1 (a) can be viewed as paired root three rectangles. This rectangle also inscribes an equilateral triangle (in blue lines).
Formula Two Over Square Root of Three-2 (b) is a two over root three rectangle subdivided by three root three rectangles

Figure 14

Formulae Two Over Square Root of Three-3 and 4



a



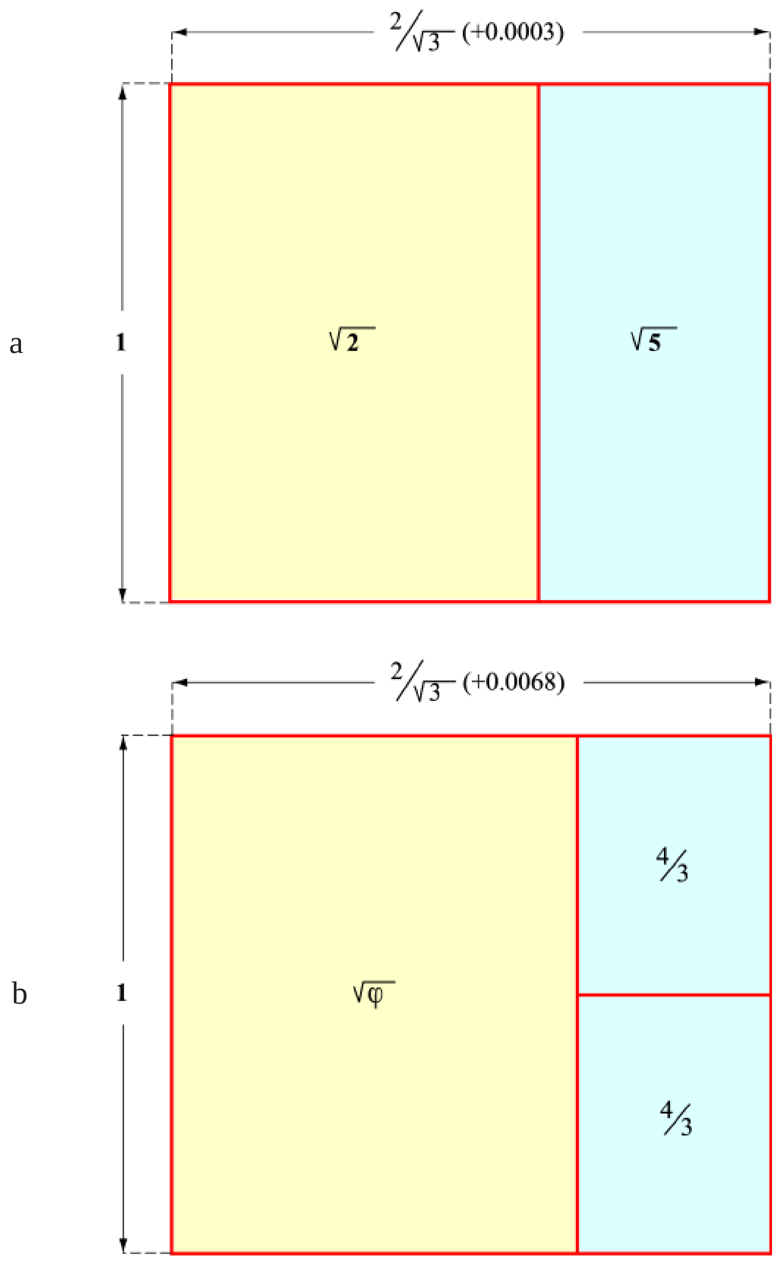
b

Formula Two Over Square Root of Three-3 (a) is a virtually perfect formula that subdivides a two over square root of three rectangle by a phi rectangle (shaded blue) and two root four rectangles (shaded yellow). (The margin of error is 0.0026).

Formula Two Over Square Root of Three-4 (b) is a virtually perfect formula that subdivides a two over square root of three rectangle by a root four rectangle (shaded yellow), two root phi rectangles (in white) and two root 3 rectangles (shaded grey) (The margin of error is 0.0009).

Figure 15

Formulae Two Over Square Root of Three-5 and 6

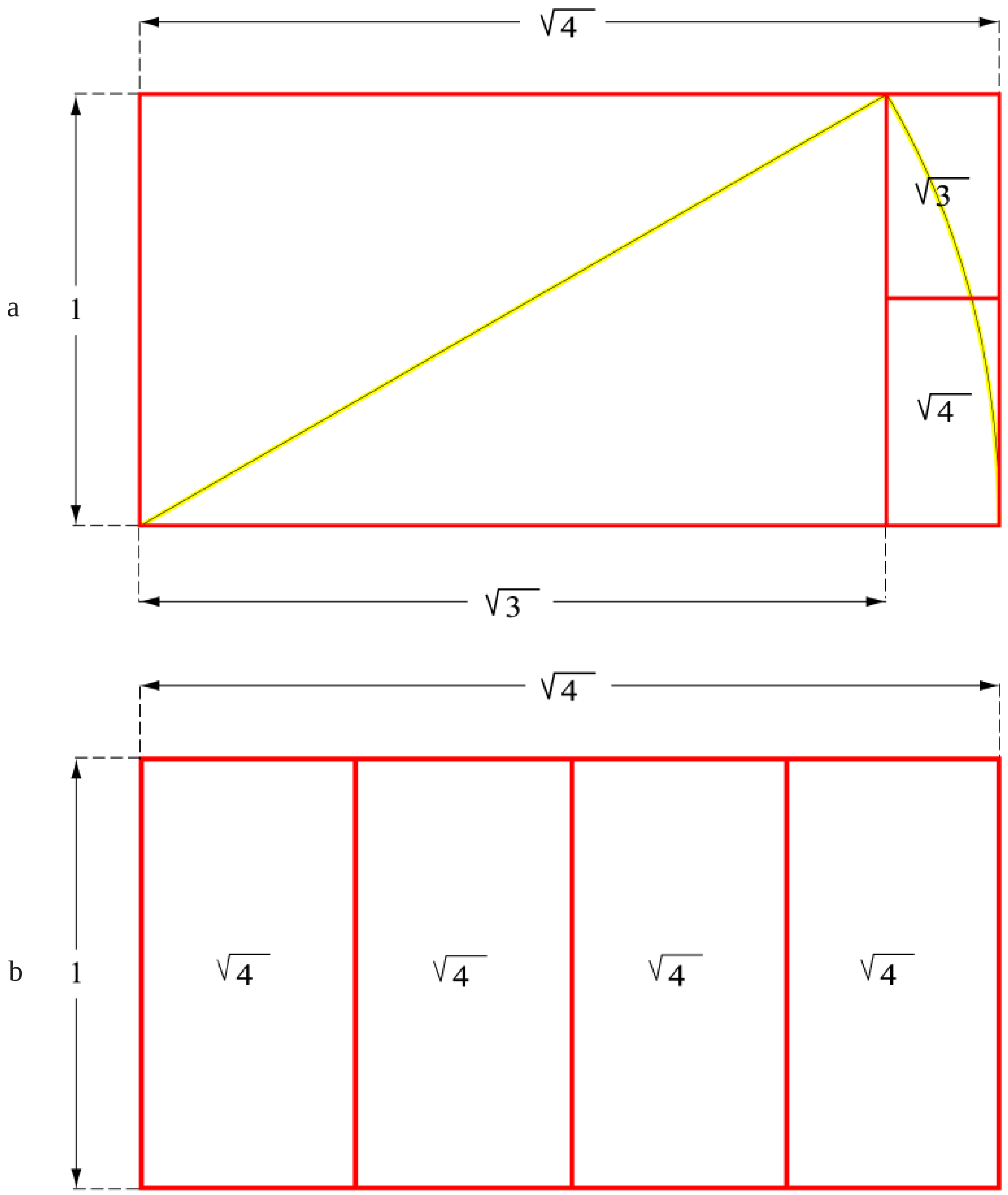


Formula Two Over Square Root of Three-5 (a) is a virtually perfect formula that subdivides a two over square root of three rectangle by a root two rectangle (shaded yellow) and a root five rectangle (shaded blue). (The margin of error is 0.0003).

Formula Two Over Square Root of Three-6 (b) is a virtually perfect formula that subdivides a two over square root of three rectangle by a root phi rectangle (shaded yellow), two Pythagorean 3,4,5, rectangles (shaded blue). (The margin of error is 0.0068).

Figure 16

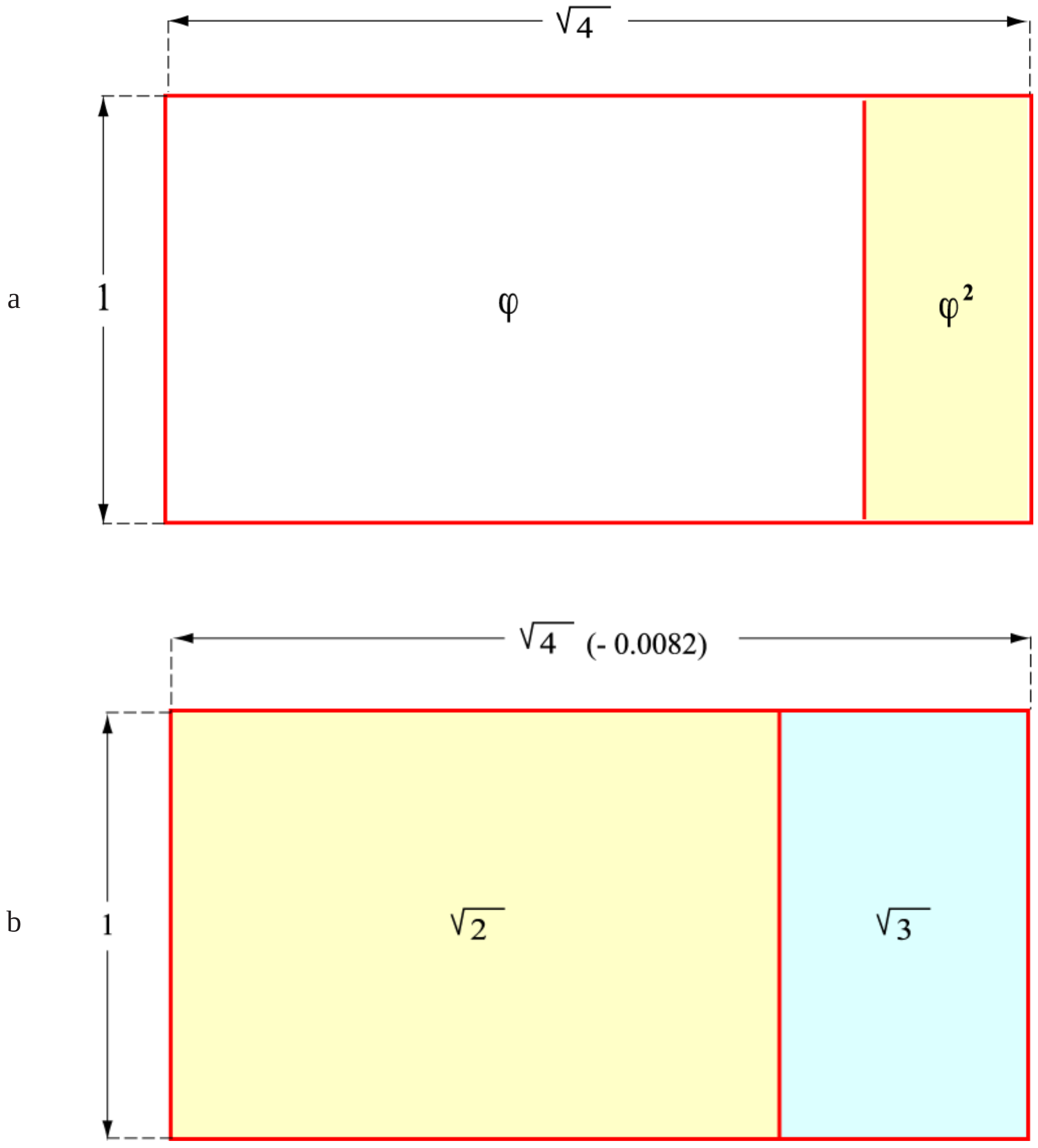
Formulae Square Root of Four-1 and 2



Formula Square Root of Four-1 (a) diagrams the subdivision of a root four rectangle by two root three rectangles and a root four rectangle. The diagonal of the larger root three rectangle is the radius of the arc that determines the length of root four rectangle (highlighted in yellow).
Formula Square Root of Four-2 (b) diagrams the subdivision of a horizontal root four rectangle by four vertical root four rectangles.

Figure 17

Formulae Square Root of Four-3 and 4



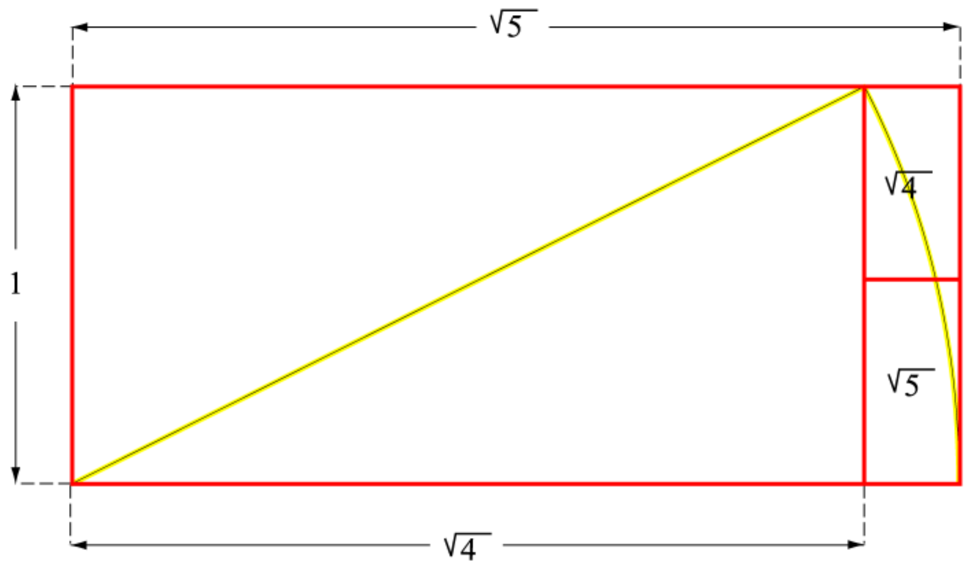
Formula Square Root of Four-3 (a) subdivides a root four rectangle by a phi rectangle (in white) and a phi squared rectangle (shaded yellow).

Formula Square Root of Four-4 (b) is a virtually perfect formula that subdivides a root four rectangle by a root two rectangle (shaded yellow) and a root three rectangle (shaded blue). (The margin of error is 0.0082.)

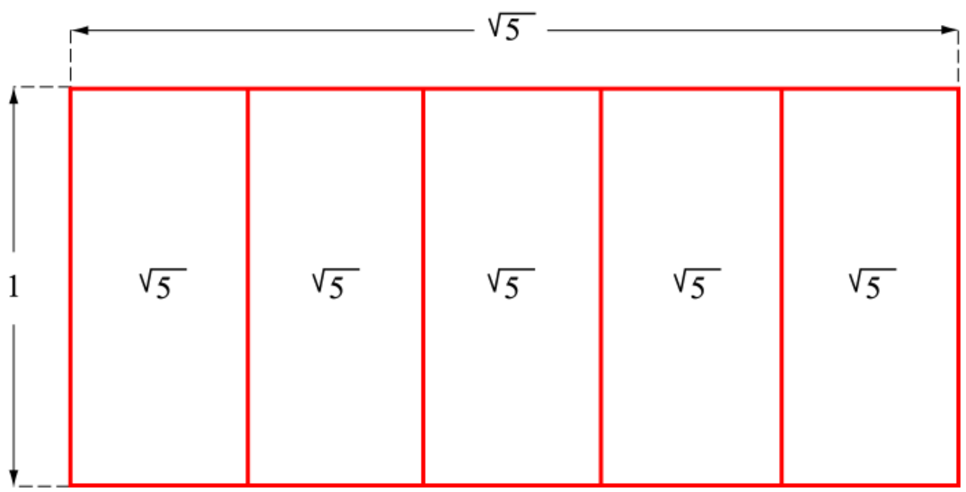
Figure 18

Formulae Square Root of Five-1 and 2

a



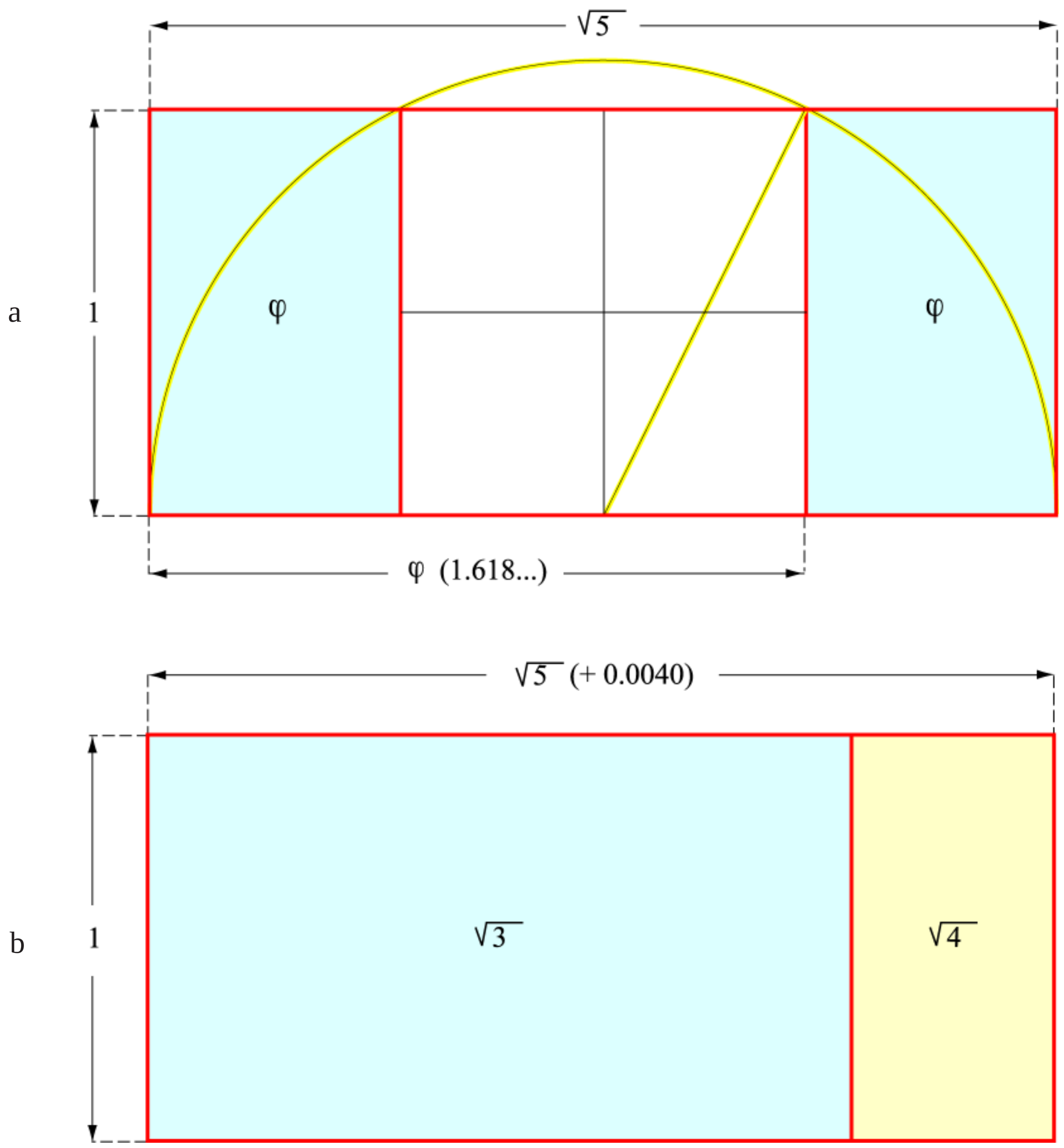
b



Formula Square Root of Five-1 (a) diagrams the subdivision of a root five rectangle by two root four rectangles and a root five rectangle. The diagonal of the larger root four rectangle is the radius of the arc that determines the length of root five rectangle (highlighted in yellow).
Formula Square Root of Five-2 (b) diagrams the subdivision of a horizontal root five rectangle by five vertical root five rectangles.

Figure 19

Formulae Square Root of Five-3 and 4

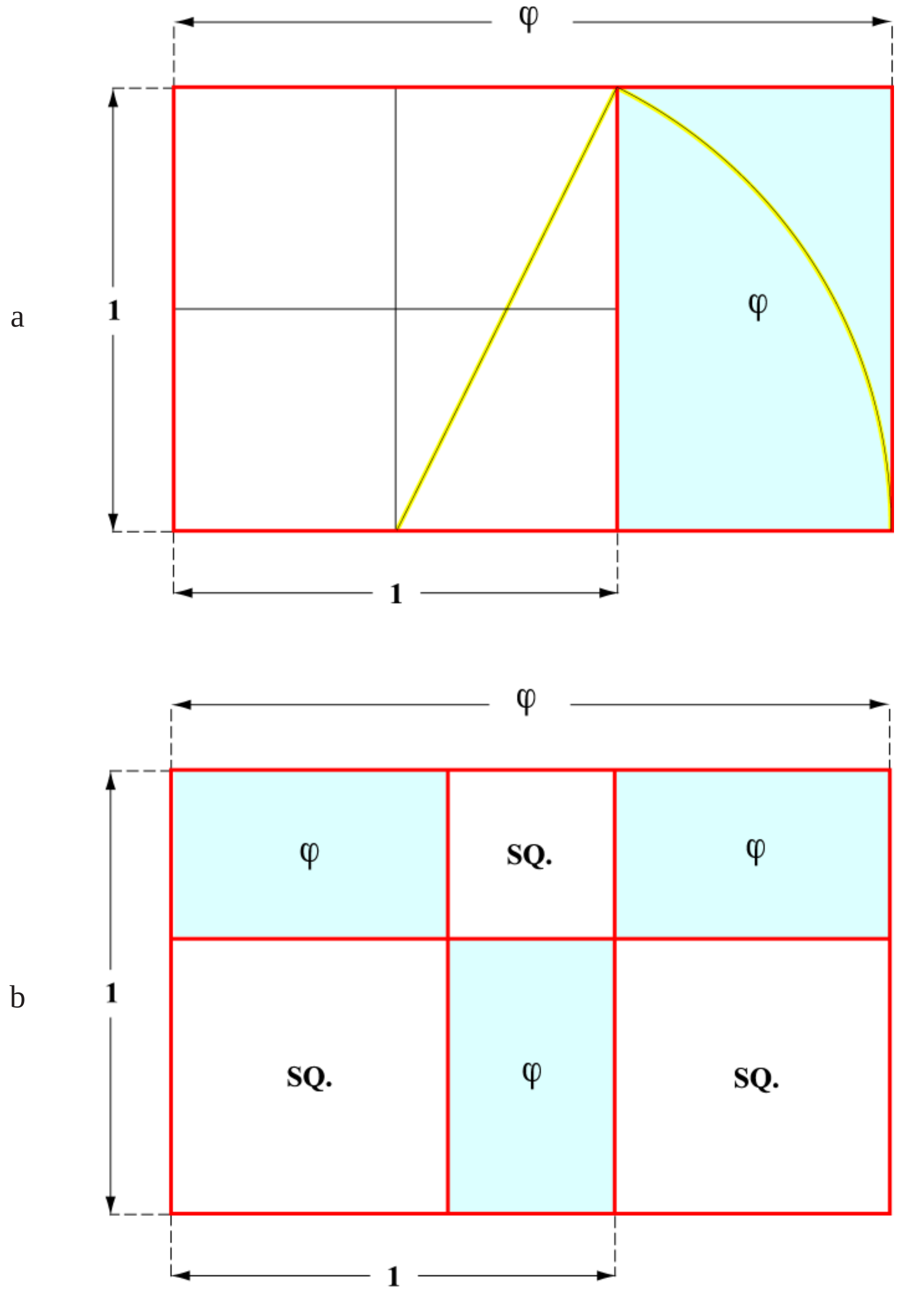


Formula Square Root of Five-3 (a) diagrams the subdivisions of the root five rectangle into a square (in white) and two phi rectangles (shaded blue). The highlighted diagonal from the center base of the square, to its upper corner, is the radius for the arc that forms the phi rectangles. Because the square is the gnomon of the phi rectangle, the square and either of the phi rectangles in blue form a larger, horizontal phi rectangle.

Formula Square Root of Five-4 (b) is a virtually perfect formula that subdivides a root five rectangle by a root three rectangle (shaded blue) and a root four rectangle (shaded yellow). (The margin of error is 0.0040.)

Figure 20

Formulae Phi-1 and 2

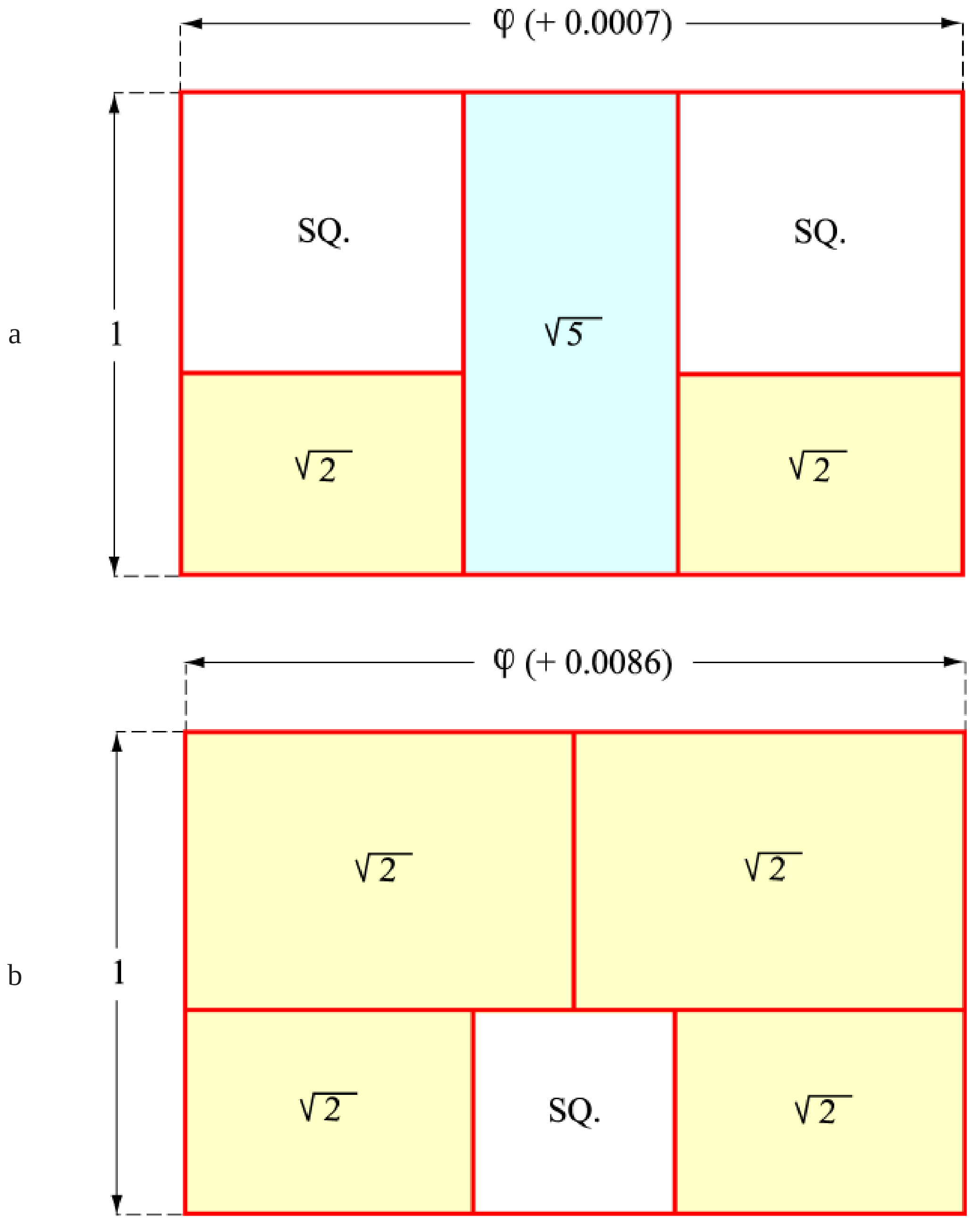


Formula Phi-1 (a) diagrams the subdivisions of a phi rectangle into a square (in white) and a smaller phi rectangle (shaded blue). The highlighted diagonal from the center base of the square, to its upper corner, is the radius for the arc that forms the phi rectangle.

Formula Phi-2 (b) subdivides a phi rectangle into squares (in white) and smaller phi rectangles (shaded blue).

Figure 21

Formulae Phi-3 and 4

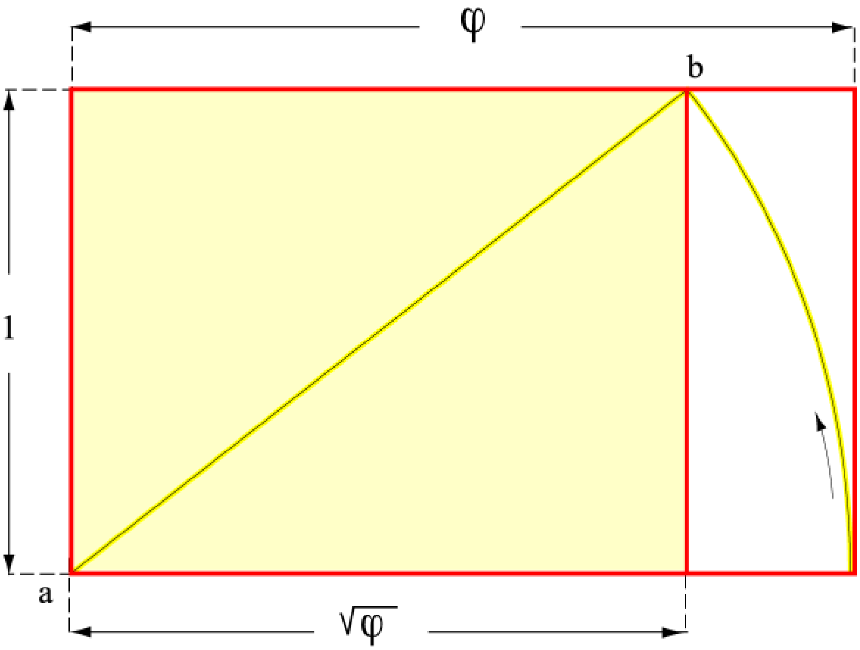


Formula Phi-3 (a) is a virtually perfect formula that subdivides a phi rectangle into two squares (in white), two root two rectangles (shaded yellow) and a root of five rectangle (shaded blue). (The margin of error is 0.0007).
Formula Phi-4 (b) is a virtually perfect formula that subdivides a phi rectangle into one square (in white) and four root two rectangles (shaded yellow). (The margin of error is 0.0086.)

Figure 22

Formula Square Root of Phi-1

a



b

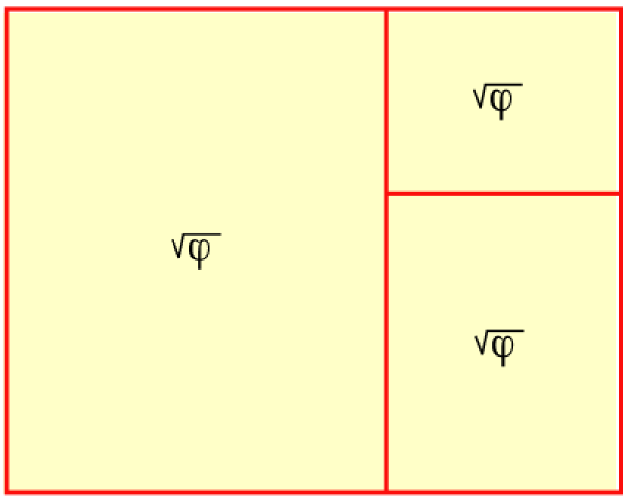
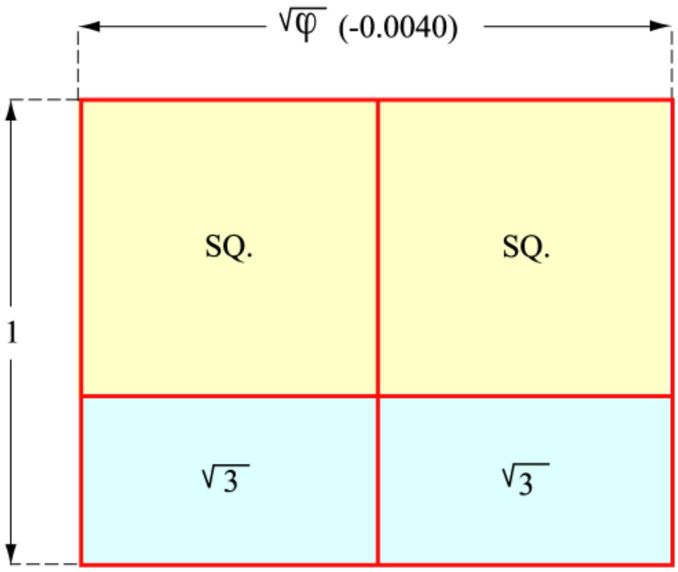


Diagram a illustrates the only simple way I know of to create a root phi rectangle. First, lay out a phi rectangle then stretch a cord or place the points of a compass across the length of the phi rectangle and use this segment as a radius to form an arc to the opposite side of the phi rectangle. The resulting rectangle (shaded yellow) has a length of root phi (1,272...) and a diagonal (a,b) of phi. Formula Square Root of Phi-1 (b) is a root phi rectangle subdivided into three smaller root phi rectangles. The diagonal of the smallest is equal to the height of the next largest, whose diagonal is equal to the width of the largest.

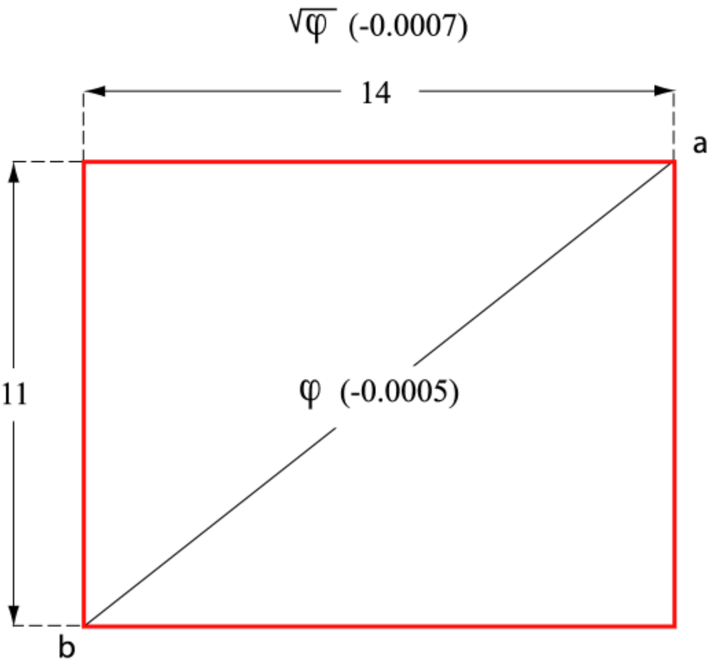
Figure 23

Formulae Square Root of Phi-3 and 4

a



b

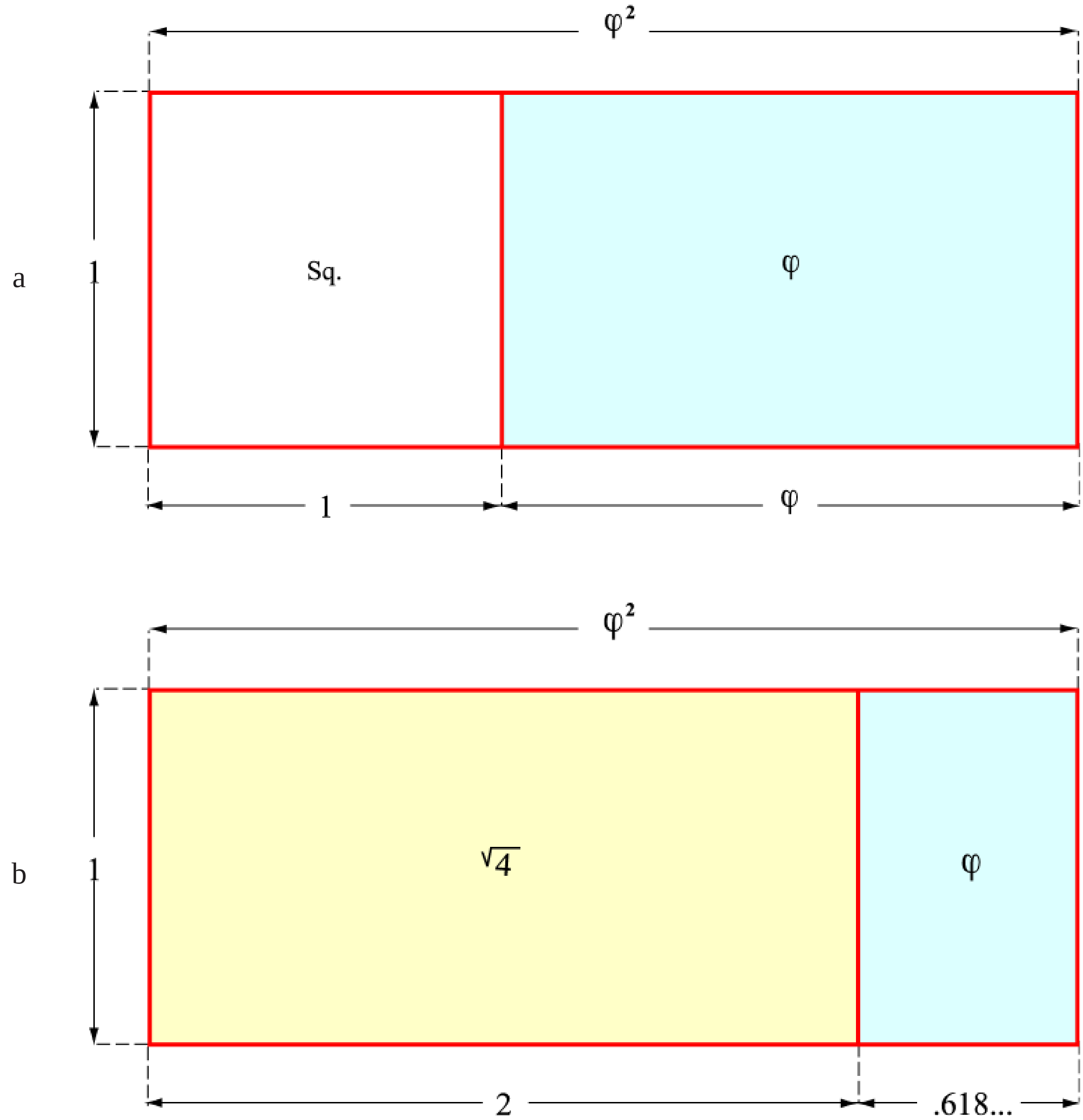


Formula Square Root of Phi-3 (a) is a virtually perfect formula that subdivides a root phi rectangle by a root four rectangle (shown as paired squares and shaded yellow) and two root three rectangles (shaded blue). (The margin of error is 0.0040.)

Formula Square Root of Phi-4 (b) is a virtually perfect root phi rectangle that is eleven units wide and fourteen units long. (The margin of error for the length is -0.0007, and the diagonal (a,b) equals phi -0.0005.)

Figure 24

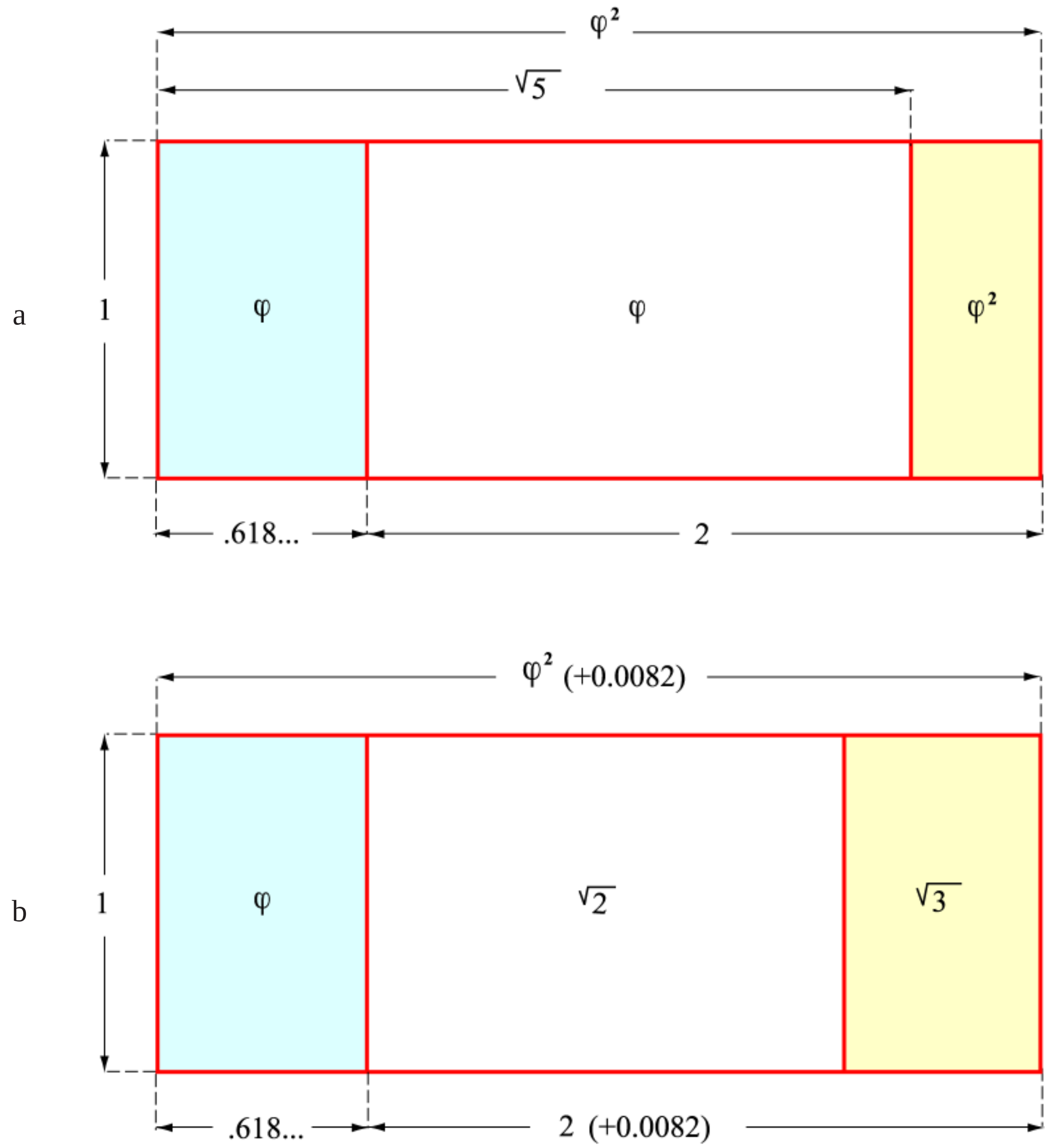
Formulae Phi Squared-1 and 2



Formula Phi Squared-1 (a) subdivides a phi squared rectangle by a square (in white) and a horizontal phi rectangle (shaded blue), a horizontal phi rectangle (in white) and a vertical phi squared rectangle (shaded yellow).
Formula Phi Squared-2 (b) subdivides a phi squared rectangle by a root four rectangle (shaded yellow) and a vertical phi rectangle (shaded blue).

Figure 25

Formulae Phi Squared-3 and 4



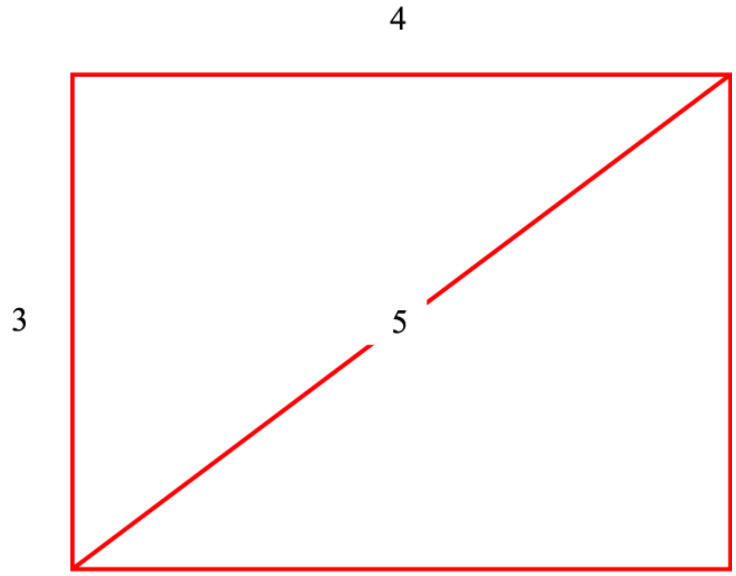
Formula Phi Squared-3 (a) subdivides a phi squared rectangle by a vertical phi rectangle (shaded blue), a horizontal phi rectangle (in white), and a vertical phi squared rectangle (shaded yellow). The two phi rectangles form a root five rectangle, and the horizontal phi rectangle and the vertical phi squared rectangle form a root four rectangle. (A unique aspect of the phi ratio is that phi squared (2.618...) equals phi plus one.)

Virtually perfect subdivisions of the phi squared rectangle can by created by substituting virtually perfect formulae for the root four and root five rectangles diagrammed previously. Formula Phi Squared 4 (b) is one example.

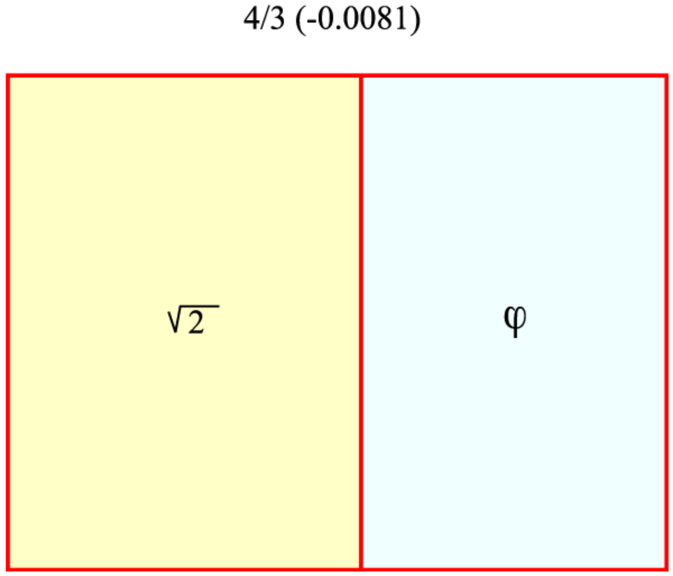
Figure 26

Formulae Pythagorean 3,4,5-1 and 2

a



b



Formula Pythagorean 3,4,5-1 (a) diagrams the relationship of the whole number subdivisions of the sides of this rectangle and its diagonal. Only Pythagorean rectangles have a diagonal that can be divided into even units relative to its sides.

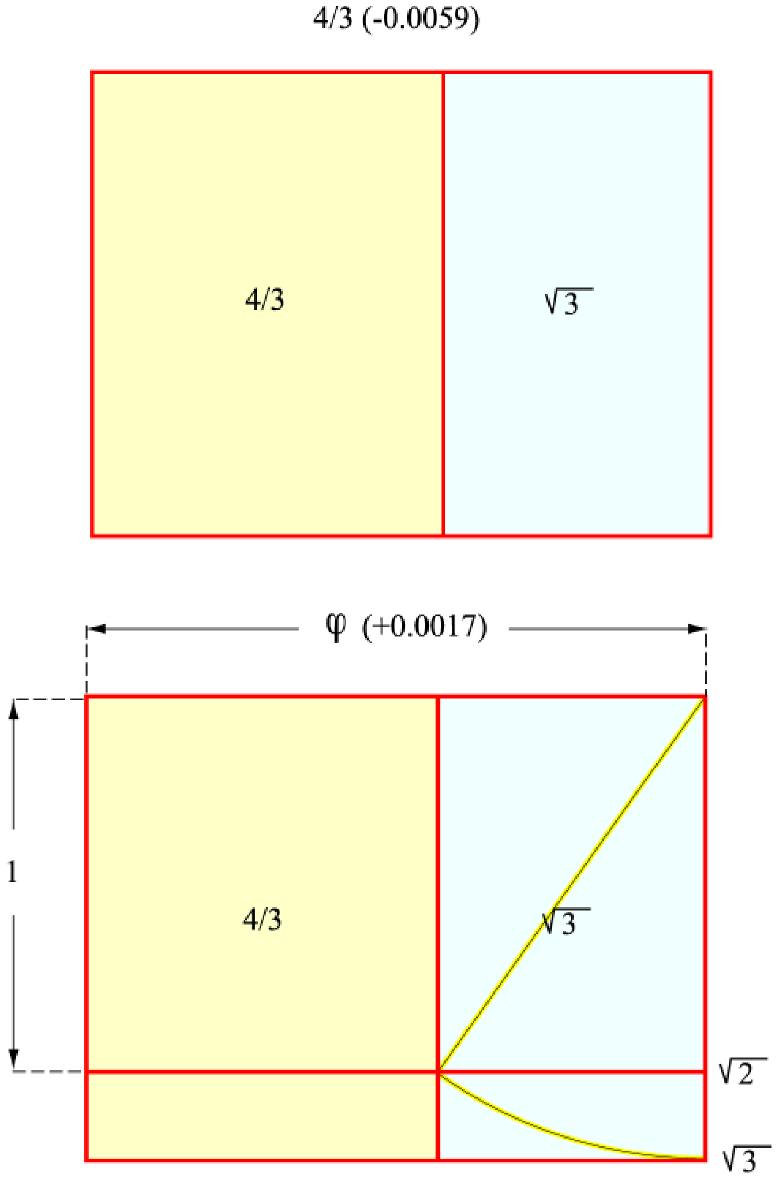
Formula Pythagorean 3,4,5-2 (b) is a virtually perfect formula that subdivides a Pythagorean 3,4,5 rectangle by a root two rectangle (shaded yellow) and a phi rectangle (shaded blue). (The margin of error is 0.0081.)

Figure 27

Formulae Pythagorean 3,4,5-3 and 4

a

b



Formula Pythagorean 3,4,5-3 (a) is a virtually perfect formula that subdivides a Pythagorean 3.4.5 rectangle by a vertical Pythagorean 3,4,5 rectangle (shaded yellow) and a root three rectangle (shaded blue). (The margin of error is 0.0059.)

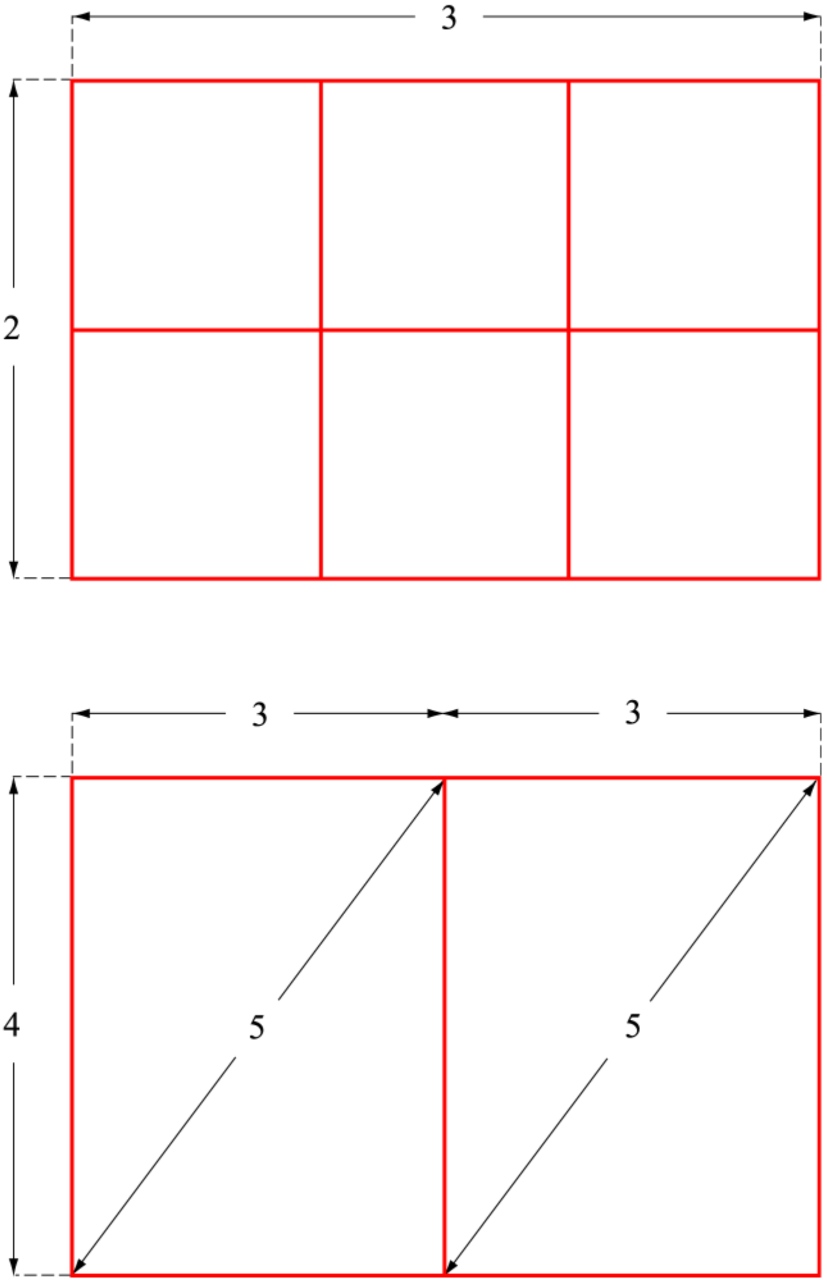
Formula Pythagorean 3,4,5-4 (b) is an elaboration of formula Pythagorean 3,4,5-3. The root three rectangle is subdivided by a root two rectangle (highlighted yellow diagonal) and the base of this root two rectangle is extended across the length of the entire rectangle and produces a virtually perfect phi rectangle with a margin of error of +0.0017...

Figure 28

Formulae 3/2-1 and 2

a

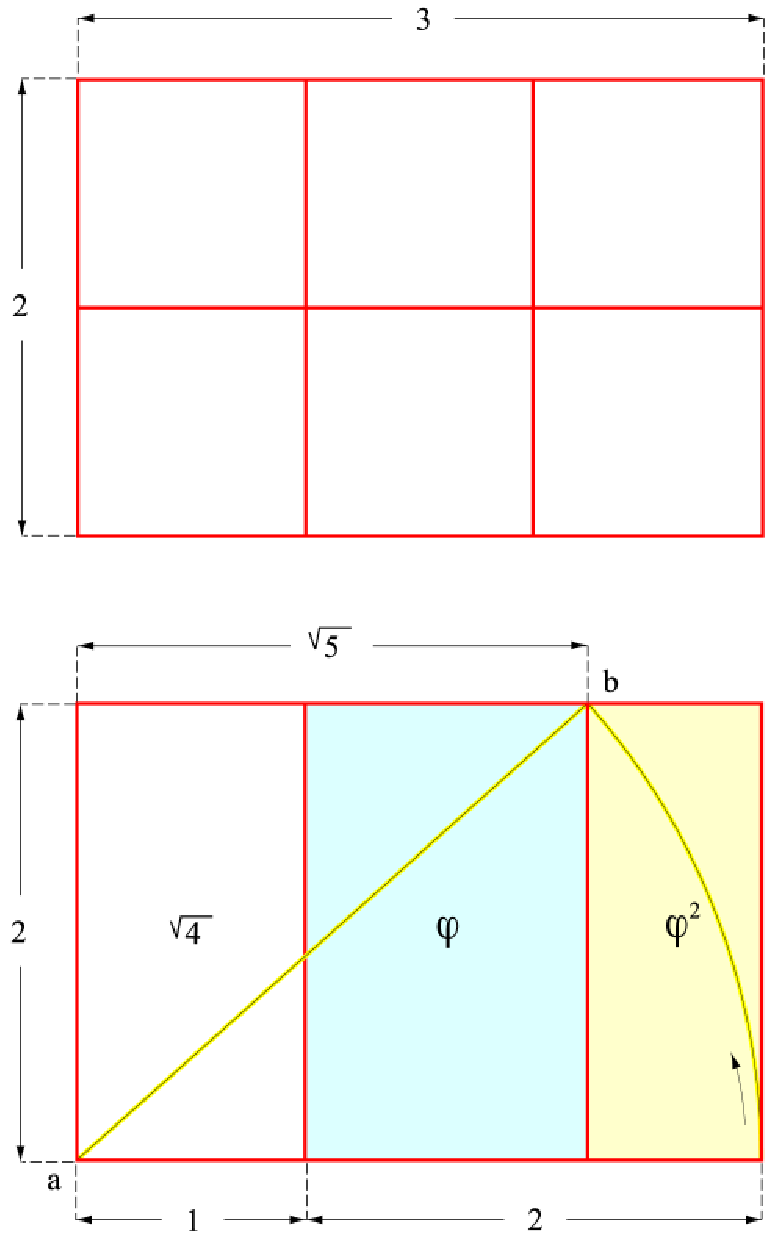
b



Formula 3/2-1 (a) subdivides a 3/2 rectangle by six equal sided squares or three root four rectangles.
Formula 3/2-2 (b) divides a 3/2 rectangle in half to produce two Pythagorean 3,4,5 rectangles.

Figure 29

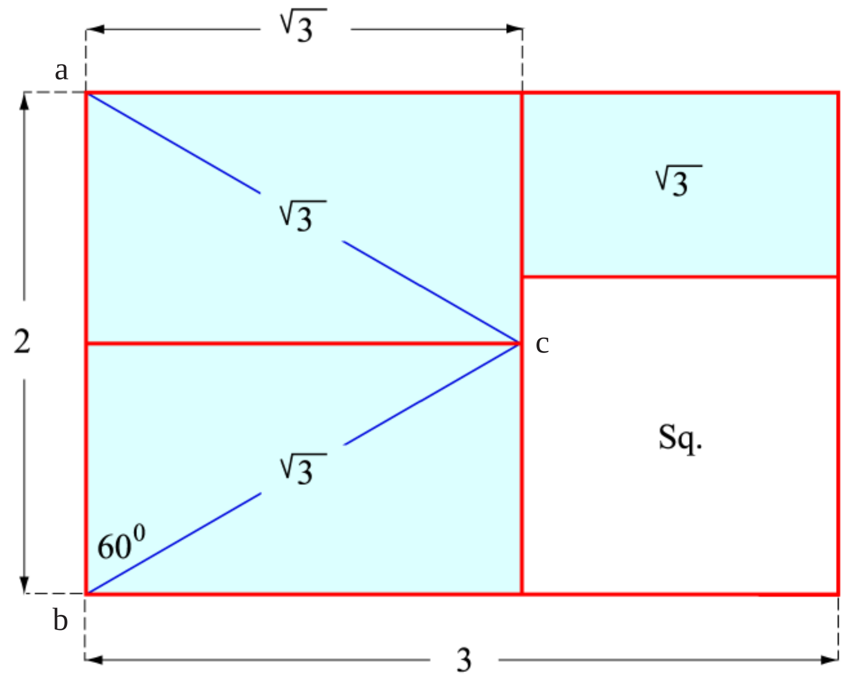
Formula 3/2-3



Formula 3/2-3 subdivides a 3/2 rectangle by vertical root 4, phi, and a phi squared rectangle and demonstrates an intrinsic relationship between the 3/2 rectangle (or paired Pythagorean 3,4,5 rectangles) and the phi and root five rectangles. This true formula is derived very simply. Place a compass or cord at the base of the length of the 3/2 rectangle and swing an arc upwards, pivoting at (a) until it crosses at the top of the rectangle at point (b) (highlighted yellow lines). The width of the rectangle is two units, its length is three units, and diagonal a,b is also three units.

Figure 30

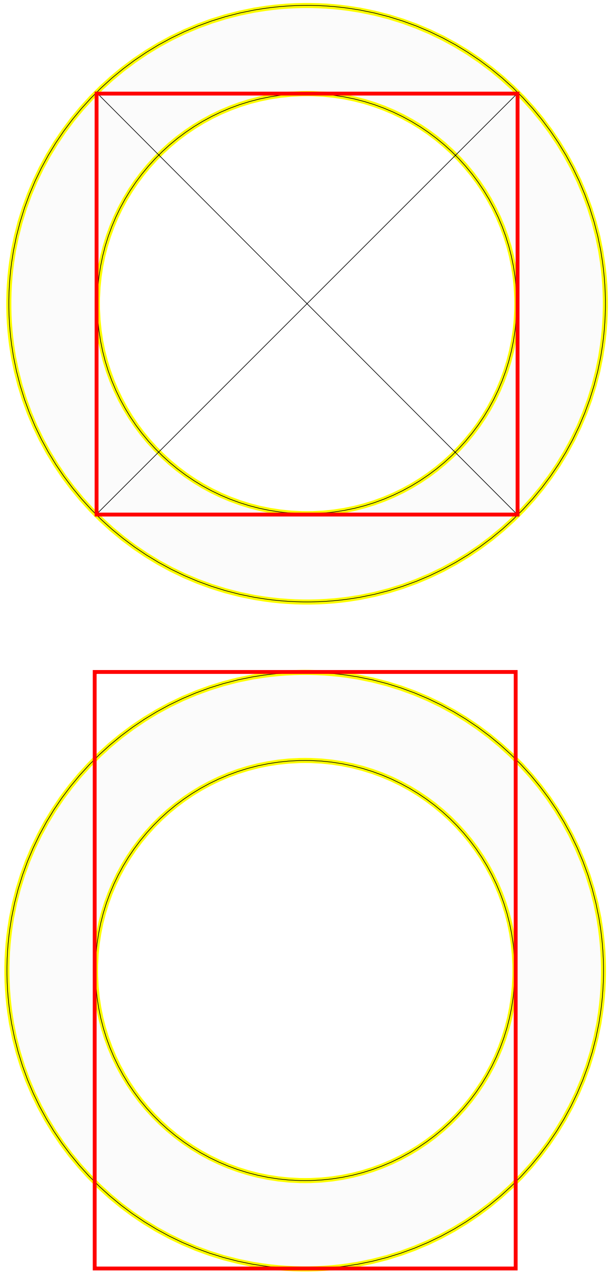
Formula 3/2-4



Formula 3/2-3 subdivides a 3/2 rectangle by three horizontal root three rectangles (shaded blue) and a square (in white). This is a true formula and demonstrates an intrinsic relationship between the 3/2 rectangle (or paired Pythagorean 3,4,5 rectangles) and the square root of three proportion. This formula is also derived very simply. First divide the 3/2 rectangle in half laterally and place a compass or cord at the corners of the side of the rectangle (a,b) and swing an arc (not shown) from either a,b or b,a until it crosses the center line at point c. Points a,b,c, form an equilateral triangle. The remaining portion of the 3/2 rectangle divides precisely into a square and root three rectangle.

Figure 31

Formula Concentric Circles-1

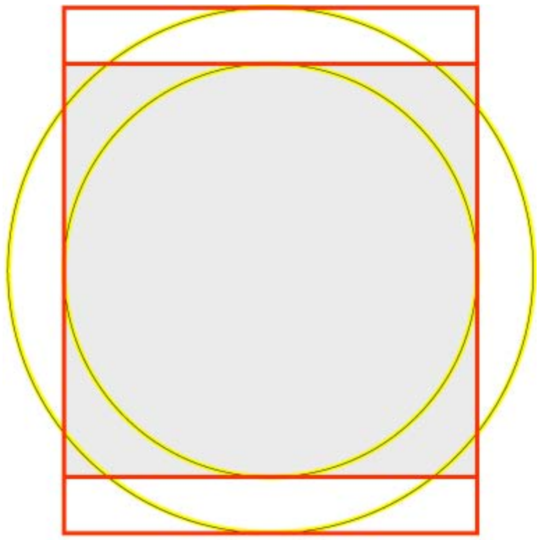


Formula Concentric Circles-1 diagrams two concentric circles, the diameter of the inner circle being one, and the diameter of the outer circle is square root two. Two simple methods for creating concentric circles with these proportions are shown: Above, a square is drawn and a circle is inscribed within it and another circle is drawn that inscribes the square. Below, a root two rectangle is drawn with concentric circles inscribing its height and width.

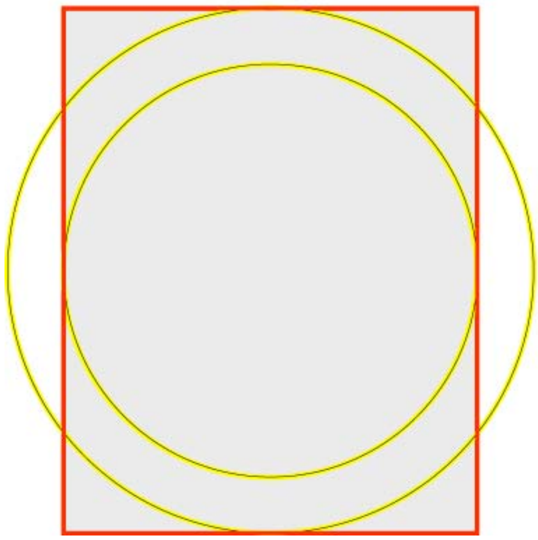
Figure 32

Formula Concentric Circles-2

a



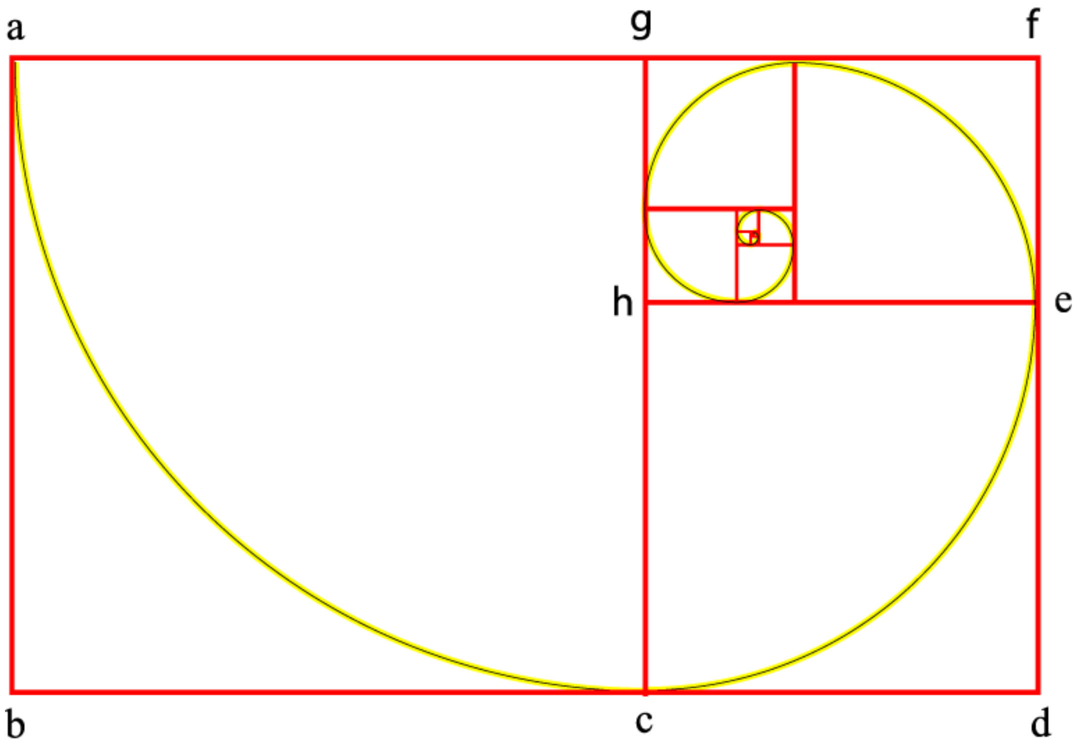
b



Formula Concentric Circles-2 diagrams two concentric circles, the diameter of the inner circle being one, and the diameter of the outer circle is square root of phi. This formula is easily created by drawing concentric circles inscribing the height and width of a root phi rectangle. Circular monuments of ten display this formula (particularly at Tonina) and, whether the Maya knew it or not, the perimeter of the square that inscribes the inner circle (a, shaded gray) is virtually equal to the circumference of the outer circle (with a margin of error of 0.0153), and the area of the root phi rectangle used to create both circles (b, shaded gray) is virtually equal to the area of the outer circle (with a margin of error of -0.0012).

Figure 33

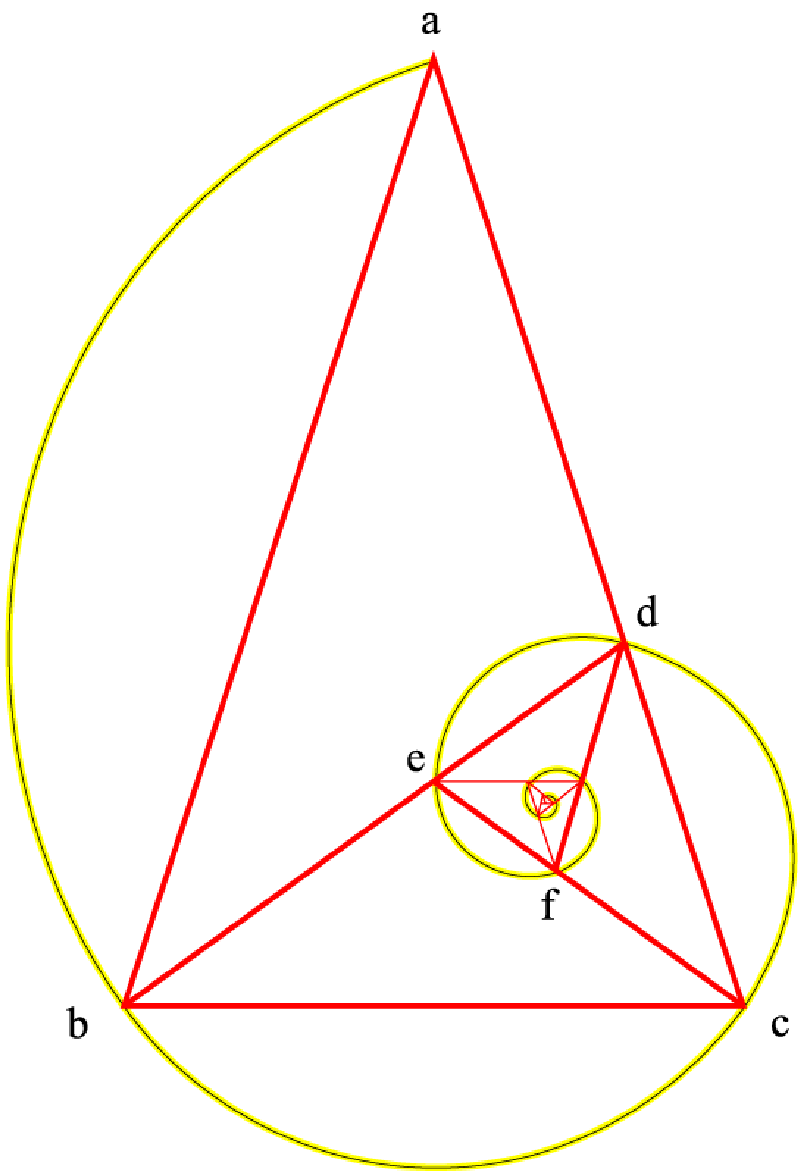
Formula Phi Equiangular Spiral-1



Formula Phi Equiangular Spiral-1 diagrams how a phi rectangle can be subdivided into smaller squares and phi rectangles add infinitum. Add a square (a,b,c,f) to a phi rectangle (a,b,d,f) and the remainder (c,d,f,g) is a similar phi rectangle. Add a square to this smaller rectangle (c,d,e,h) and the remainder (c,d,g,h,) is also a smaller phi rectangle and so on. These squares also provide the radii for the arcs of the equiangular spiral highlighted in yellow. Radii g,a and g,c are used to form the arc a,c; radii h,c and h,e to form arc c,e, etcetera.

Figure 34

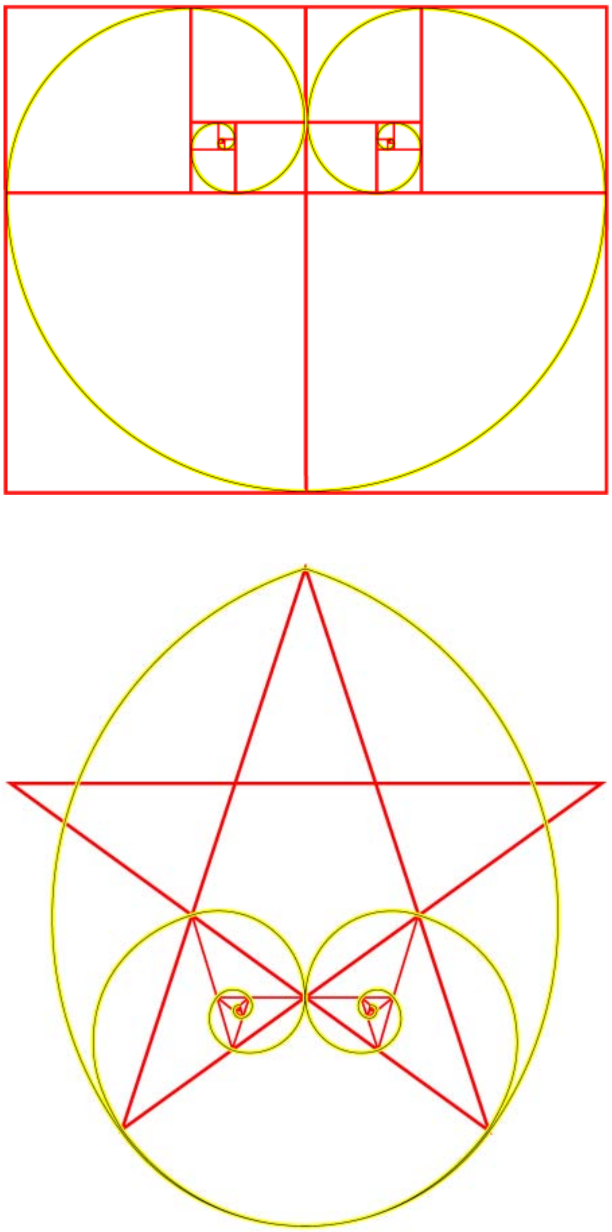
Formula Phi Equiangular Spiral-2



Formula Phi Equiangular Spiral-2. Begin with a triangle whose base (b,c) equals one and the slope (a,b) equals phi. Place a compass or cord at b,c, and make an arc pivoting from b until b,c,crosses the slope a,c (at position d). Draw the line d,b, and c,d,b is a similar triangle to a,b,c at a ratio of one to phi. Place your compass or cord at c,d and pivot from c until c,d crosses line d,b at e and draw line e,c, creating another similar triangle, etcetera. These divisions of a phi triangle also provide the radii to form the equiangular spiral highlighted in yellow. Radii d,a and d,b form arc a,b; radii e,c and e,b form arc b,c; radii f,c and f,d form arc d,c, etcetera.

Figure 35

Formulae Phi Equiangular Spiral-3 and 4



a

b

Formula Phi Equiangular Spiral-3 (a) is Formula Phi Equiangular Spiral-1 doubled and mirrored. Formula Phi Equiangular Spiral-4 (b) is essentially the equiangular spiral of Phi Equiangular Spiral-2 created on both sides of the original phi triangle and shows the intrinsic relationship of this formula to the pentagram.

CHAPTER 6

GEOMETRY IN MAYA VERNACULAR ARCHITECTURE

The basic designs of vernacular Maya structures, often called “Casas Mayas” in the Yucatán, are extremely conservative and have not changed significantly since at least the early 1800s. Archaeological and ethnohistorical evidence suggests that these designs can be traced back to Pre-Columbian times (Wauchope et al. 1940).

Two basic styles of vernacular houses exist in Mesoamerica today: the apsidal and the rectangular. Traditionally, the frames are constructed entirely of wood beams and poles lashed together with vines (*bejuco*) and the roofs are thatched with palm fronds or grasses. Walls are most often made of wattle and daub, though in some cases they are made of rough stone masonry, adobe bricks, or`wooden poles or planks. Sometimes, too, the walls are finished with a coating of stucco. The following diagram and photographs (Figures 36-39) show a few of the different styles of Maya vernacular houses:

Figure 36

Diagram A and Photograph 1 of Maya Vernacular Houses

a

b

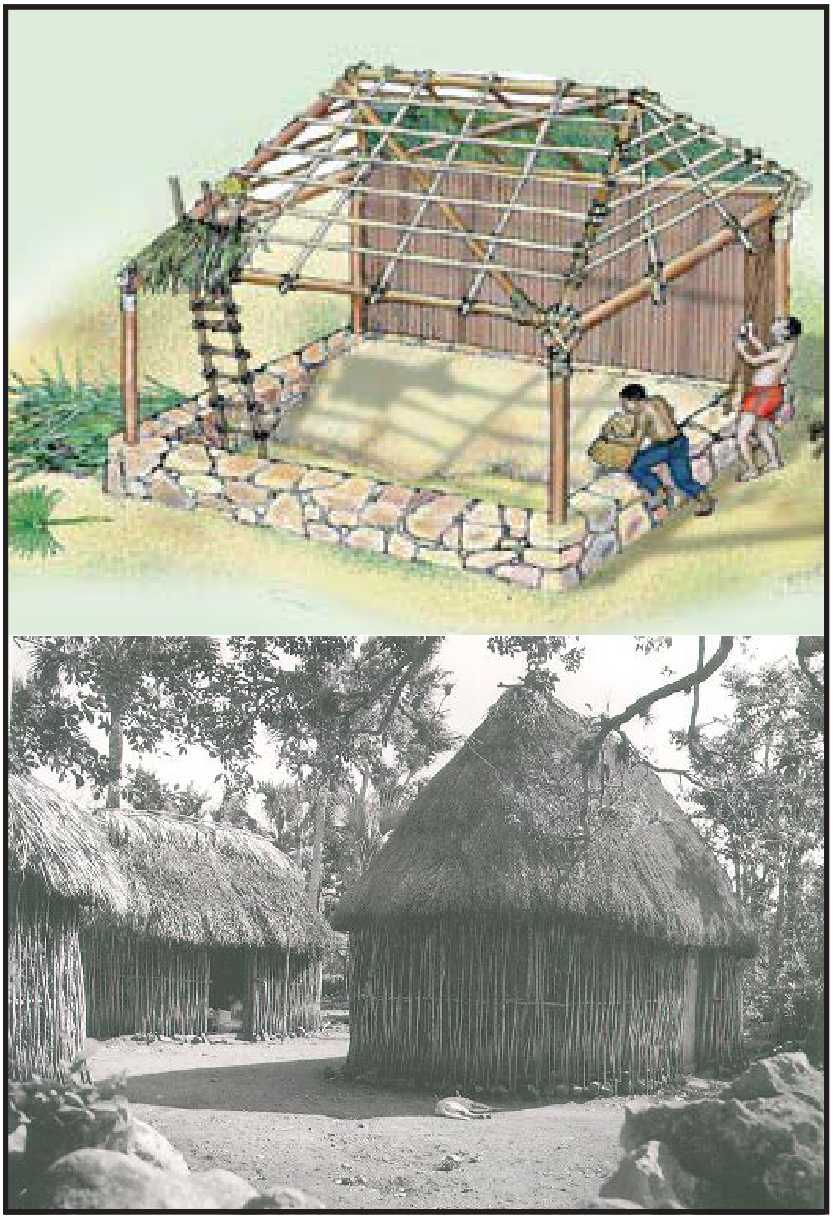


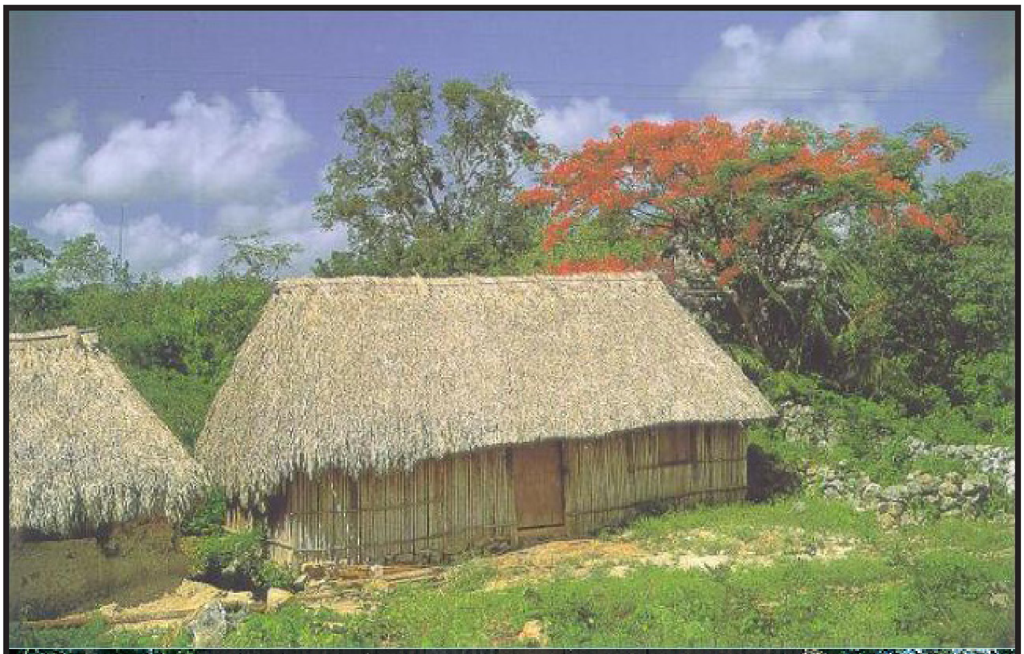
Diagram a, of a Maya house being built, shows some of the principal components of Maya houses: The low stone wall that is often built around the roof support posts to protect the walls from moisture and rot, the hard packed earth or *sascob* (deteriorated limestone) floor, the roof frame, and rafters, the *bijuco* lashings and thatch.

Photograph 1 (b) shows apsidal houses, with bare wattle and daub walls, from the Yucatan peninsula.

Figure 37

Photographs 2 and 3, Two Styles of Maya Vernacular Houses,
Yucatan Peninsula, Mexico

a



b

Photograph 2 (a) is of a rectangular Maya house with bare wattle and daub walls.
Photograph 3 (b) is of an apsidal Maya house with stuccoed and painted wattle and daub walls.

Figure 38

Photographs 4 and 5, Two Styles of Maya Vernacular Houses, Chiapas, Mexico

a

b

Photograph 4 (a) is of a rectangular Maya house with a porch and wood plank walls.

Photograph 5 (b) is of an apsidal Maya house with wattle and daub walls and a stuccoed door-way.

Figure 39

Photographs 6 and 7, Two Styles of Maya Vernacular Houses, Chiapas, Mexico

a

b

Photograph 6 (a) is of an apsidal Maya house with wattle and heavily daubed walls. Photograph 7 (b) is of an apsidal house with stuccoed masonry walls.

Sadly, the most complete source for descriptions of construction materials and techniques remains Wauchope's 1938 monograph on "modern Maya houses" (also see Wauchope 1940). A few more recent additions to Wauchope's detailed study include Anderson (1994), Breedlove and Laughlin (1993) and Laughlin (1996). Some localized ethnoarchaeological studies include Fauvet-Berthelot (1986: 235–263), Lee and Hayden (1988), and Smyth (1991).

Most of the following descriptions of how these vernacular houses are designed or laid out with measuring cords are the results of my own intermittent interviews with Maya shamans and house builders in the Yucatan peninsula, in the Highlands of Chiapas and Guatemala, and in Honduras between the years 1993 and 2001, and from similar interviews conducted by archaeologist Alfonso Morales in Honduras during 1995. Most of the measured drawings used to demonstrate the geometry incorporated into the designs of these houses were produced by myself. Others were produced by Wauchope (1938), Pivaral (1989), and Pierrebourg (2003).

Sometimes the ground plans of these vernacular houses are laid out with measuring cords as part of elaborate house-building ceremonies conducted by shamans. Often though, the same ground plans are laid out by house-building specialists after offering simple prayers or without any apparent ceremonies at all. The degree of ceremony involved is evidently determined by the religious proclivities of the house owners.

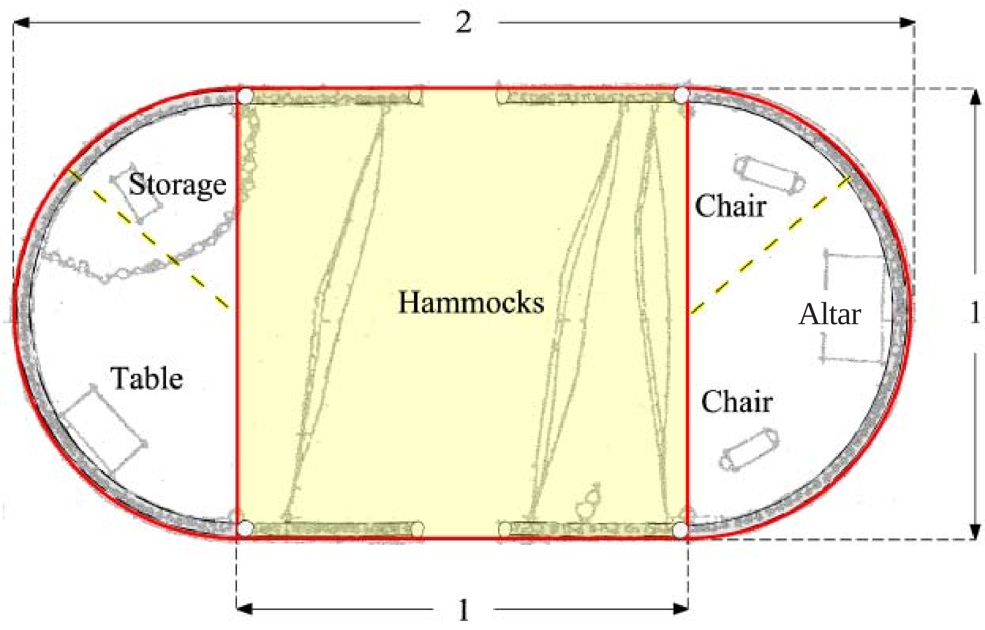
My first interview with a Maya house-building specialist was conducted in 1993 in the town of Izamal, in the state of Yucatan. Through an introduction by an acquaintance who lived in Izamal, I was received into the home of Don Pedro, a master house builder whose house was sparsely furnished, fastidiously clean and orderly, and beautifully built. The bejuco lashings of the principal support beams and rafters were snug and perfectly tied. The thatch roof was laid out in an unusual herringbone pattern that was pleasing to the eye.

Don Pedro used his own house as an example to respond to my questions about how he designed it. He began by pointing out that the principal vertical roof support posts formed an equal-sided square, and he told me that he used a cord to lay out this initial square. (I failed to ask him how he determined the right angles of the square, a mistake I would not repeat in subsequent interviews.) He then showed me how he stretched a cord from the corners of one side of the square, folded the cord in half to determine its center point, and re-stretched the cord to mark and stake a point midway between the corners. He then stretched his cord from this center point to both corners of the square, forming a half circle. He repeated this process on the opposite side of the square. The length of each side of this central square, which is also the width of the house, was measured as two "uinics" — twice the measure of the distance between his outstretched fingertips — in exactly the same manner as described in the house-building ceremony conducted by Don Aurelio. The height of the spring line of this particular house design is equal to one half of the width of the house, or one uinic. The height of the roof is also equal to one half the width of the house, so that the distance between the peak of the roof and the spring line is equal to 1.414… uinics (the square root of two). The slope of the roof forms a forty-five degree angle and the height of the house is equal to its width. This ratio of one to one, height to width, divided equally at the spring line, is common to most of the Maya houses I have examined. An interesting aspect of utilizing this particular formula of equal height to width is that whatever proportion is used to design the plan of the house will be identically reflected in the front elevation of the house (see Figures 40 and 41).

After kindly describing to me in detail how he designed his own house, Don Pedro suggested that I examine a recently abandoned traditional house of a different design, which was made by a friend whom Don Pedro described as a master house builder from the Caribbean coast. It was a well-constructed house of lashed wood beams and posts with wattle and daub walls and a palm-thatched roof. It was rectangular in plan and in good condition. With the help of a few graduate students, I produced a measured drawing. The exterior dimensions of the plan formed a nearly perfect phi rectangle and the interior posts used to support the roof were arranged as a Pythagorean 3,4,5 rectangle (see Figures 42 and 43).

Figure 40

Plan of Maya Vernacular House 1, Don Pedro's House,
Izamal, Yucatan, Mexico

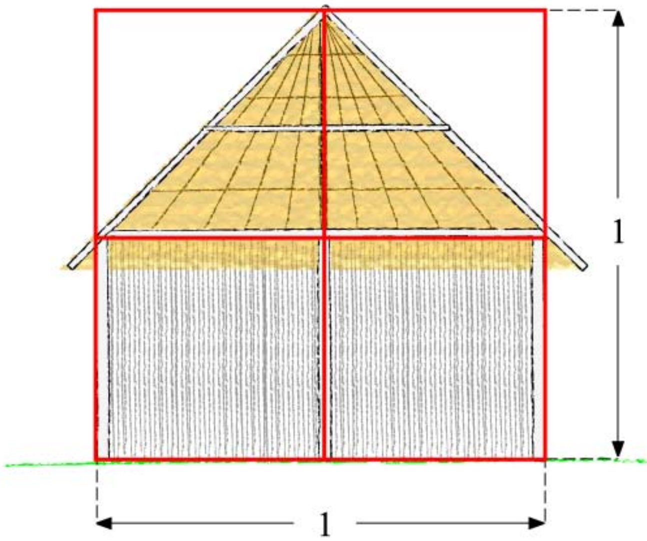


Maya Vernacular House 1 is probably the most common style of Maya vernacular house in the Yucatan Peninsula. The parallel sides of the apsidal plan and the four principle roof support posts form a square (shaded yellow). The center of the right and left-hand sides of this square, to the corners, are the radii for the half circle arcs that form the apsis. The width to length of the plan is one to two (square root of four).

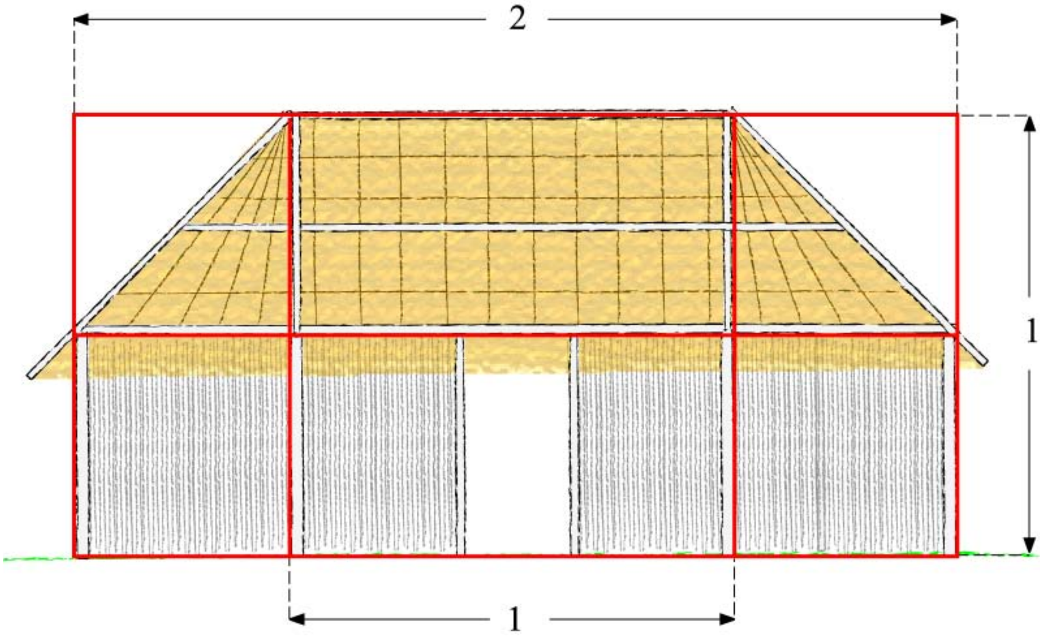
Figure 41

Elevations of Maya Vernacular House 1, Don Pedro's House,
Izamal, Yucatan, Mexico

a



b

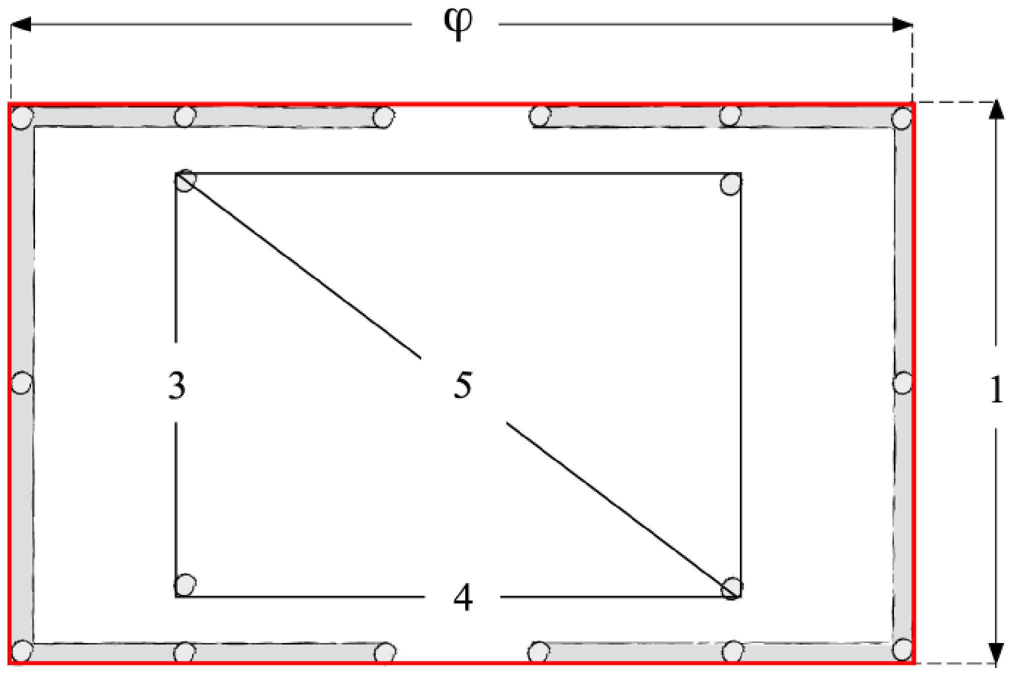


The side elevation (a) is of equal height and width and is inscribed by a quartered square. The spring line and roof are of equal height.
The front elevation (b) has a height of one to a length of two and the pitch of the roof in both elevations is forty-five degrees.

Figure 42

Plan of Maya Vernacular House 2, House of the “Master Builder from the Caribbean,”

Izamal, Yucatan, Mexico



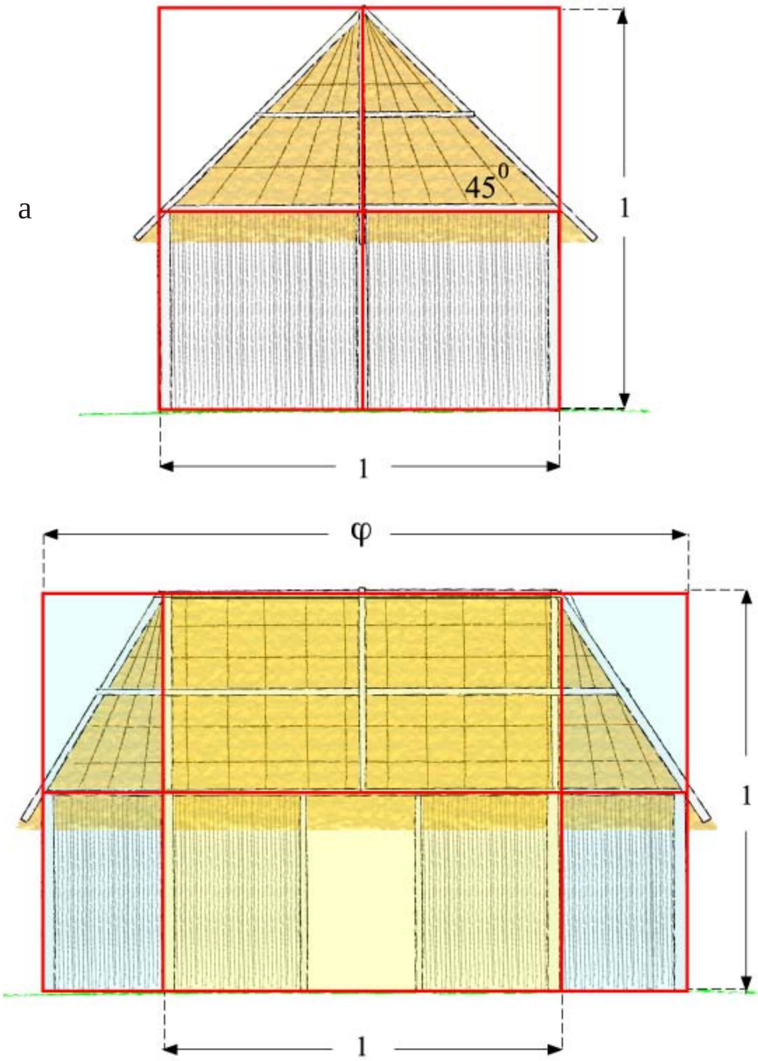
The plan of Maya Vernacular House 2 has a width to length ratio of one to phi. The interior support beams form a Pythagorean 3,4,5 rectangle whose length is equal to the width of the house.

Figure 43

Elevations of Maya Vernacular House 2, House of the "Master Builder from the Caribbean," Izamal, Yucatan, Mexico

a

b



The side elevation (a) is of equal height and width and is inscribed by a quartered square. The spring line and roof are of equal height. The pitch of the roof is forty-five degrees.

The front elevation (b) has a height of one to a length of phi. The width of the principal support beams to the height of the roof form a square bisected laterally at the spring line (shaded yellow), and the remaining space to either side of this square is inscribed by phi rectangles (shaded blue). The pitch of the roof is the diagonal of a phi rectangle (approximately 57.5 degrees).

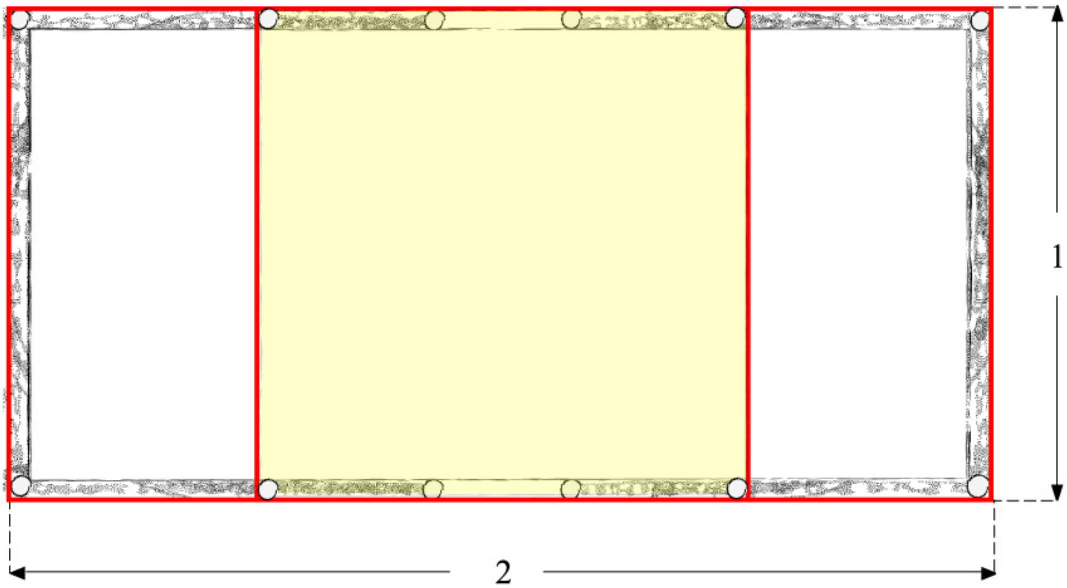
In 1995 I returned to Yucatan, sponsored by the Fundación Cultural Yucatan, to continue my ethnographical investigations of Maya vernacular houses. After documenting the house-building ceremony described in Chapter 3, I went on to help build three houses with Don Lucio Gonzales Kan Te and his apprentice, Jose Conteras. These houses were of identical design as Don Pedro's house, depicted above in Figures 40 and 41.

In response to questions I asked about house designs, Don Lucio told me that nearly all of the houses he built shared these same proportions. There were a couple of exceptions, however.` At times, he said, he preferred to construct the roof with beams that were the same length as the width of the house. With this method, the slope of the roof is sixty degrees and the ratio of the spring line to the height of the roof is one to the square root of three (see Figure 45). When I asked him why he sometimes chose this design over the forty-five degree roof, he told me that if he had "good wood," meaning very long, straight beams, he preferred this larger roof because there was more room to store food in the rafters. This was a practical answer to my question. Another example of Don Lucio's practical bent was that, unless a shaman was called for by the house owner, no ceremony at all was involved in his house-building activities. His particular method for laying out the initial square for the central supports of the house was also practical. He simply cut four straight beams, each measured with a cord to a length slightly longer than two uinics, and arranged them end-to-end to form a square. The inner corners of this square were marked with stakes, and postholes were dug at these locations for the principal vertical support beams of the house. The four beams that were used to lay out the square were then lashed to the tops of these forked, vertical supports to become the spring line and the principal support beams for the roof.

Don Lucio also recounted that he had built, per instructions of the house owners, houses that were identically proportioned to the houses we had built together but with rectangular plans instead of apsidal. He had also recently built a house, again per instruction of the house owner, whose principal vertical support beams were laid out in a ratio of three to four instead of the more common square. He was aware that the diagonal of this three-to four-sided rectangle was equal to five of the same units. He described the units as *metros*, or meters, but Don Lucio did not use a measuring tape, and "one metro" in his lexicon is equal to one-half a uinic. Figures 44-47 diagram the plans and elevations of these houses.

Figure 44

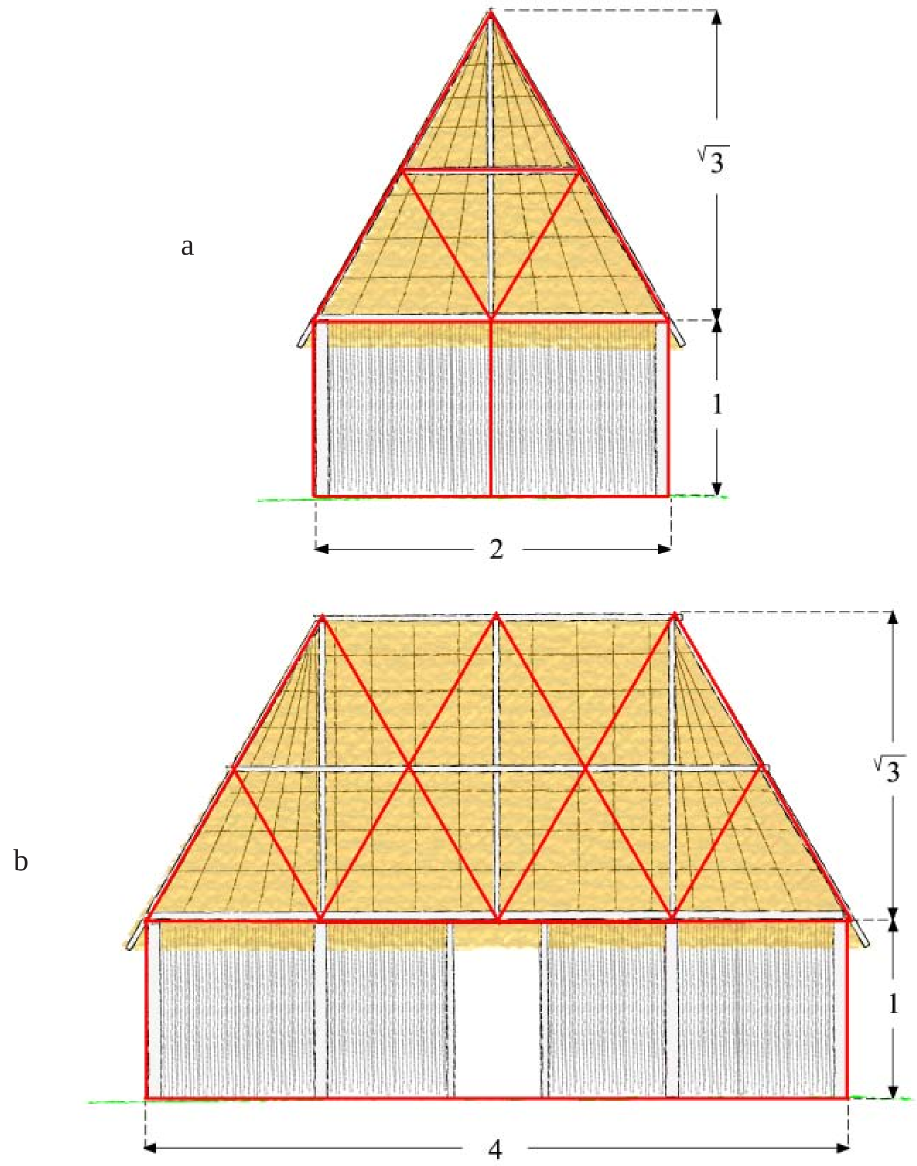
Plan of Maya Vernacular Houses 3, House Built by Don Lucio,
Yucatan, Mexico



The width to length of the plan of Maya Vernacular House 3 is one to two (root four). The principal roof support posts form a central square (shaded yellow).

Figure 45

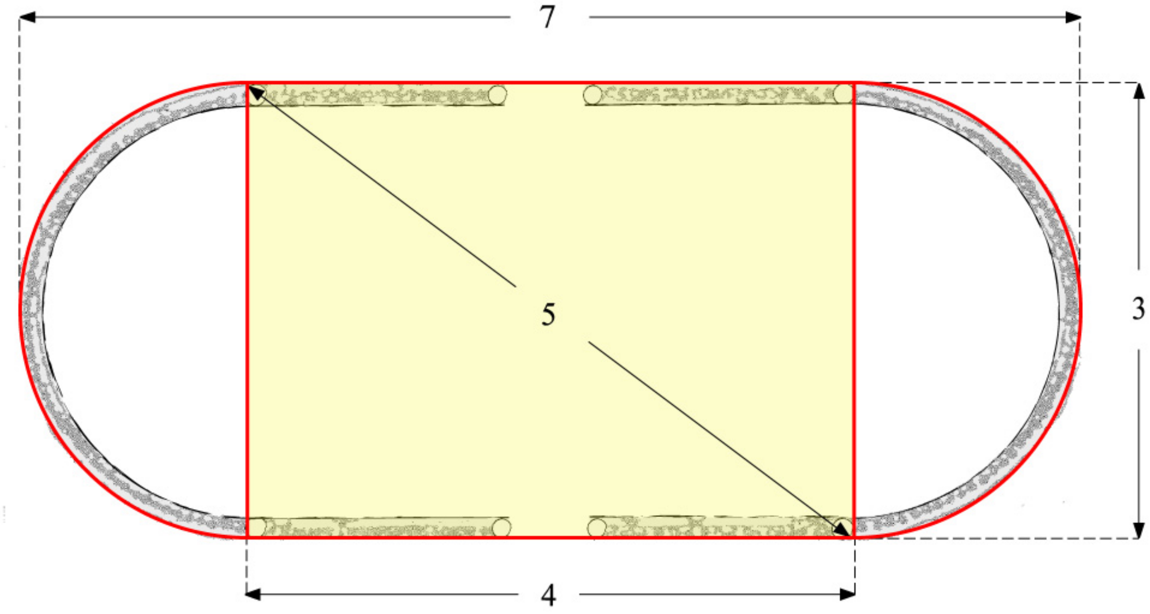
Alternate Elevations of Maya Vernacular Houses 1 and 3, Houses Built by Don Lucio, Yucatan, Mexico



These elevations (a and b) share the same plan designs as Maya Vernacular Houses 1 and 3.
The height of the spring line of the side elevation (a) to the width of the house is a ratio of one to two. The roof forms an equilateral triangle. The pitch of the roof in both elevations and all of the diagonal red lines are sixty degrees.
The height of the spring line of the front elevation (b) to the length of the house is a ratio of one to four. The height of the spring line to the height of the roof is one to root three.

Figure 46

Plan of Maya Vernacular House 4, House Built by Don Lucio,
Yucatan, Mexico

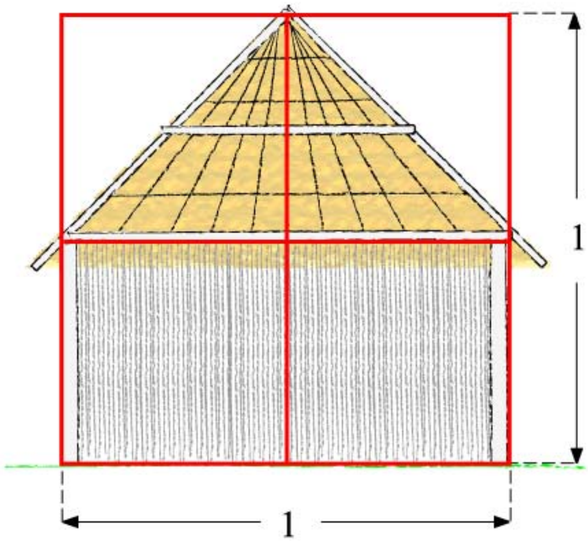


The plan of Maya Vernacular House 4 has a width to length ratio of three to seven (1 to 2.333...). The parallel sides of the apsidal plan and the principle support beams form a Pythagorean 3,4,5 rectangle (shaded yellow). The apsis are inscribed by half circles added to either side of the Pythagorean rectangle.

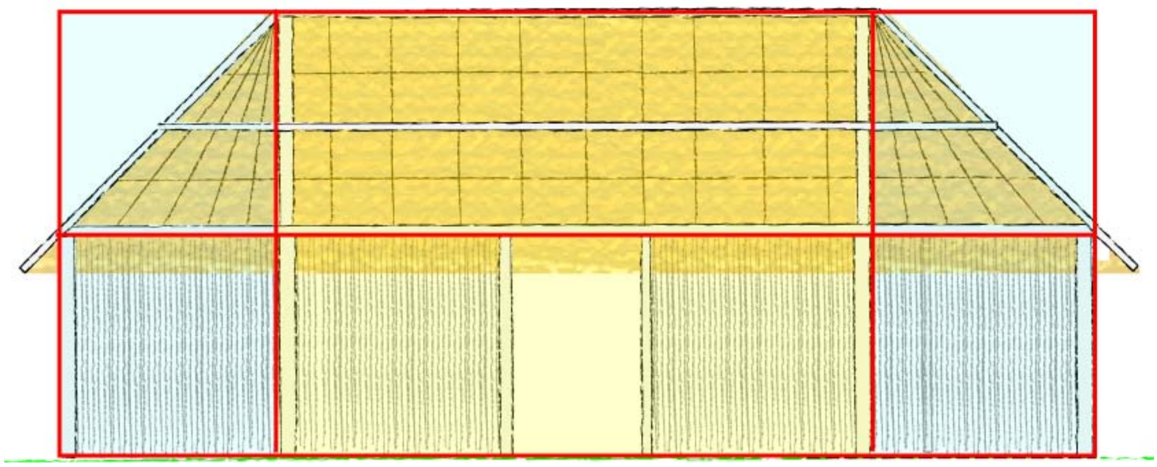
Figure 47

Elevations of Maya Vernacular House 4, House Built by Don Lucio,
Yucatan, Mexico

a



b



The side elevation (a) is of equal height and width and is inscribed by a quartered square. The spring line and roof are of equal height. The pitch of the roof in both elevations is forty-five degrees.

The front elevation (b) has a height of three to a length of seven. The width of the principal support beams to the height of the roof forms a Pythagorean 3,4,5 rectangle bisected laterally at the spring line (shaded yellow). The remaining space to either side of this Pythagorean 3,4,5 rectangle is inscribed by root four rectangles bisected into paired squares by the spring line (shaded blue).

In 1995, Liticia Rocha, Director of the Fundación Cultural Yucatán, introduced me to Francis Faller, a German-born, retired engineer, who had spent most of his life working in the Yucatan Peninsula. Francis Faller was then living on a small hacienda several kilometers southwest of the city of Merida. Before and during his retirement, he had taken a great interest in traditional Maya construction methods and had helped to construct (and to note the designs of) six Maya houses built by the same master house builder, Alfonso Chi. The plans of these houses were identical to the apsidal plans preferred by Don Lucio and Don Pedro: an initial square laid out to form the parallel sides of the apsidal plan and half circle arcs laid out to either side of the square to form the apsis. The roof, too, in side section, had a height of one to a width of two and a pitch of forty-five degrees, like those designed by Don Lucio and Don Pedro. But the side section of the walls, the height of the spring line to the width of the structure, was a ratio of one to phi. Mr. Faller watched Alfonso Chi manipulate a measuring cord to determine the height and width of the walls of the side elevation of these houses and he told me Alfonso did so using ”proportions and diagonals — not units of measure.” Though Mr. Faller did not recognize or remember the exact formula that Alfonso used, Mr. Faller measured the end result and realized that Alfonso was producing a very accurate phi proportion. Mr. Faller did know that the first roof support post that forms one corner of the initial square of the plan is set into the ground at a predetermined height, invariably one uinic, or slightly higher that the height of the house builder. Then a cord was stretched from the base of this corner post in a predetermined direction that established the orientation of the house. Alfonso Chi's houses were all orientated with the front door facing the general direction of north. From the height of this corner post, the length of the side elevation was then determined with a measuring cord.

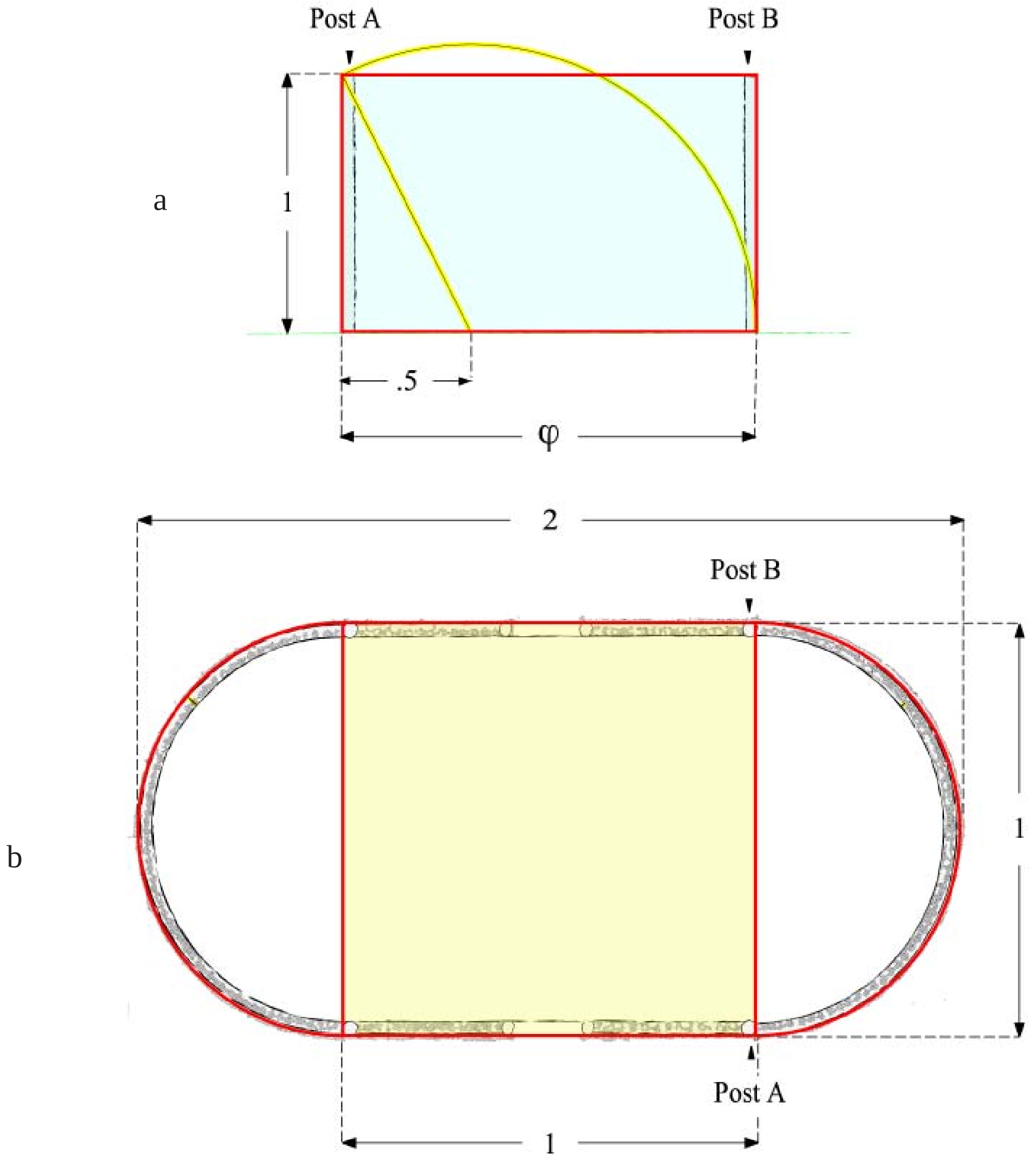
When measurng with a cord, the only simple way to lay out a phi rectangle using the height of the corner post as a starting point is as follows: The measuring cord would be stretched from the base of this corner post to its height, then folded in half. One end of this halved cord (one half uinic in length) is placed at the base of the post, the other end is laid out along the cord that determines the orientation of the house, and a wood stake is placed in the ground to mark this measure. Then the measuring cord is stretched from this point on the ground back to the top of the set corner post and this measure is added to the one-half uinic along the cord that determines the orientation of the house (see Figure 48). To determine the placement of the second corner of the initial square, the positions of the remaining two corners of the central square were dead-reckoned and adjusted until the diagonals of the square were of equal length.

Francis Faller also told me that he had once hired Don Alfonso to build an addition to his home. Mr. Faller had already staked out the corners of the extra room to be built, and the height of the room was predetermined by the height of the wall of the main house. Don Alfonso told him that the room was too short and asked if he could adjust it. Mr. Faller agreed to this, and when, out of curiosity, he measured the finished addition he found that the ratio of the height to width of the section was a very precise approximation of one to phi.

Though not related to the geometry of Maya houses, Mr Faller once witnessed Don Alfonso direct the preparation of a hard floor surface for a vernacular house in a manner that I had not heard of before nor since. The floor was made of a pre-measured quantity of ordinary dirt that was mixed with water and a small quantity (less than a liter) of sap from an unidentified species of bush or tree. (Mr. Faller had forgotten the names for the species of bush or tree.) When this mixture was of the consistency of wet concrete, five or so liters of honey were added that served as a catalytic agent. Once the honey was added, the mixture became warm to the touch and began to bubble. Using shovels and trowel, the men apply the mixture as quickly as possible before it sets into a hard, baked, clay-like floor.

Figure 48

Plan and Side Section of the Walls of Maya Vernacular House 5,
Houses Built by Don Alfonso Chi, Yucatan, Mexico

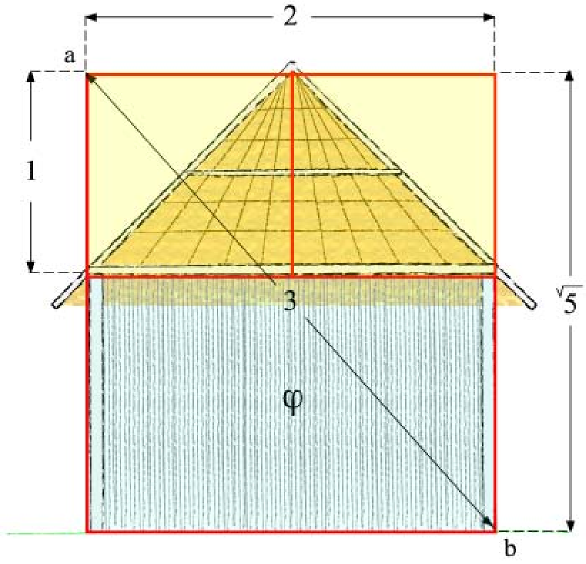


Section (a) diagrams a simple method for creating the walls of Alfonso Chi's houses with a height of one to a length of phi, beginning with the height of the initial corner post (Post A).
The apsidal plan of these houses (b) begins with an initial square that forms the parallel sides of the house, with half-circle arcs added to either side to form the apsis.

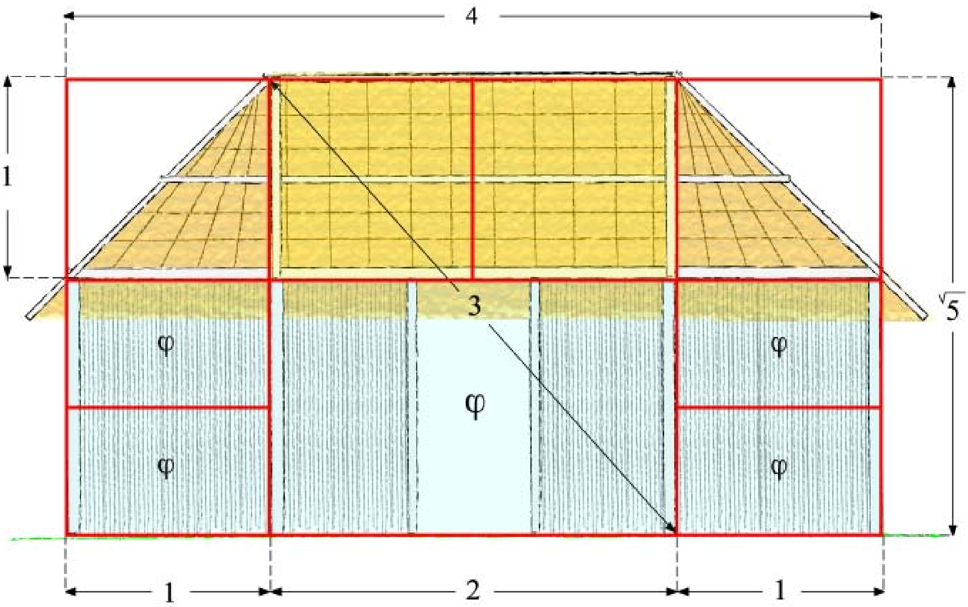
Figure 49

Elevations of Maya Vernacular House 5, Houses Built by Don Alfonso Chi, Yucatan, Mexico

a



b



The height of the spring line of the side elevation to the width of the house (a) is a ratio of one to phi. The height of the roof is one half the width of the house. The pitch of the roof in both elevations is forty-five degrees. Note that if the width of the side section is two, then its height is root five and its diagonal is three.

The length of the front elevation (b) is double that of the side elevation, with a length of four and a height of root five.

Unfortunately, I met Francis Faller just before I was to leave the Yucatan to conduct archaeological excavations at the site of Copan, in Honduras, and later, at Palenque, in Chiapas, Mexico. More than ten years passed before I was to return to the Yucatan Peninsula, and I was never able to interview Don Alfonso Chi.

However, earlier that year (1995), while helping a group of Mexican college students document and restore Hacienda Tabi, under the direction of architect and art historian Dr. Logan Wagner, I had the good fortune to conduct several interviews with Don Guadalupe. As a prominent shaman in the area of Oxkintok, Don Guadalupe was often called upon to perform the Cha Chac rain ceremony and other rituals. Although he did not perform house-building ceremonies, he had witnessed them many times during his sixty odd years. He also knew how to design three types of houses with a measuring cord. We spoke together in Spanish, and he used a string to show me how a measuring cord was used to design these houses.

The first design that he showed me was the same as the houses I had helped to build with Don Lucio (Figures 40 and 41). The ground plan of the second style of house was also apsidal and was accomplished by first laying out a square in the same way that Don Aurelio and Don Alfonso had, by dead reckoning the four equal sides of the square and then adjusting the corners until both diagonals of the square were of equal length. The center was determined by marking the intersection of the diagonals. From this center point of the square, the measuring cord was then stretched to one of the corners of the square, and an arc was drawn or etched in the soil, terminating at an adjacent corner. An identical arc was then drawn on the opposite side of the square. The result of this formula produces an apsidal ground plan whose width to length ratio is one to the square root of two (see Figure 50).

The third house design that Don Guadalupe described was one his grandfather taught him and was used exclusively for building *chosas*, a simple field house used seasonally by farmers when their field work takes them too far from home to return at night. After drawing a square in the dirt floor we were sitting on, Don Guadalupe demonstrated the formula for the plan of this house by stretching his piece of string along the base of the square, then folding the string in half to determine the center point of the base of the square. From this center point he stretched the string to the upper right hand corner of the square and then swung an arc from the upper corner to a point parallel with the base of the square, thus determining the length of the rectangle. This is a concise formula for creating a phi rectangle with a ratio of 1 to 1.618…. The side elevation of all three house designs utilized the same common formula described earlier: the spring line equal to one half the width of the structure and the height of the roof from the spring line also equal to one half the width, producing a pitch of forty-five degrees. When I asked Don Guadalupe why this particular plan was used specifically for chosas, he told me that he did not know (see Figures 52 and 53).

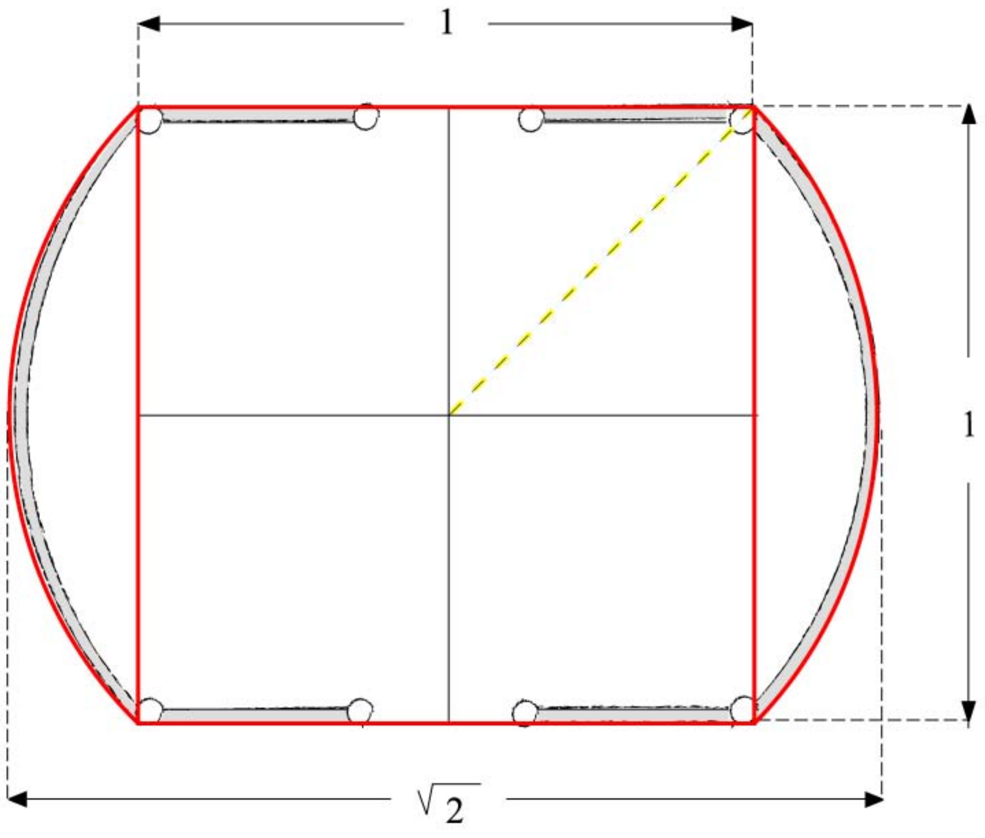
At this point in my investigations, I knew that the square root and phi proportions that were still being used to design Maya architecture were rectangular expressions of the proportions inherent in the equiangular triangle, the square, and the pentagon. I also knew that most flowers had three, four, or five petals or multiples of three, four, and five petals, and that apparently, there were no seven- or nine-petaled flowers. Thus, the set of proportions inherent in most flowers is the same set of proportions used to design Maya architecture. To be clear about my bias here, I admit that I suspected early on that the Maya would have been aware that the set of proportions they chose to design their houses was the very same set of proportions that the "Maker and Modeler" used to create the heavens and the earth. Nowhere in nature are these proportions more clearly and abundantly expressed than in the shapes of flowers.

So, being very careful not to lead Don Guadalupe in any way, I asked him, "*Que puede decir me sobre las flores*? ("What can you tell me about flowers?") He began by explaining that flowers were very important to the Maya and that most Maya did not cultivate flowers but instead harvested wild flowers for use in specific ceremonies. He told me that the word "flowery" was used by Yucatec Maya as a synonym for “sacred.” He said that the human soul was a white flower and that one’s ancestors and offspring were called flowers.

He finished by recounting that his grandfather, who was also a shaman and who had taught him how to use the measuring cord, had explained to him that, “*Las formas de las flores son dentro nuestros casas*.” (“The shapes of the flowers are in our houses.”) When I asked him what his grandfather meant by that, he told me that he did not know.

Figure 50

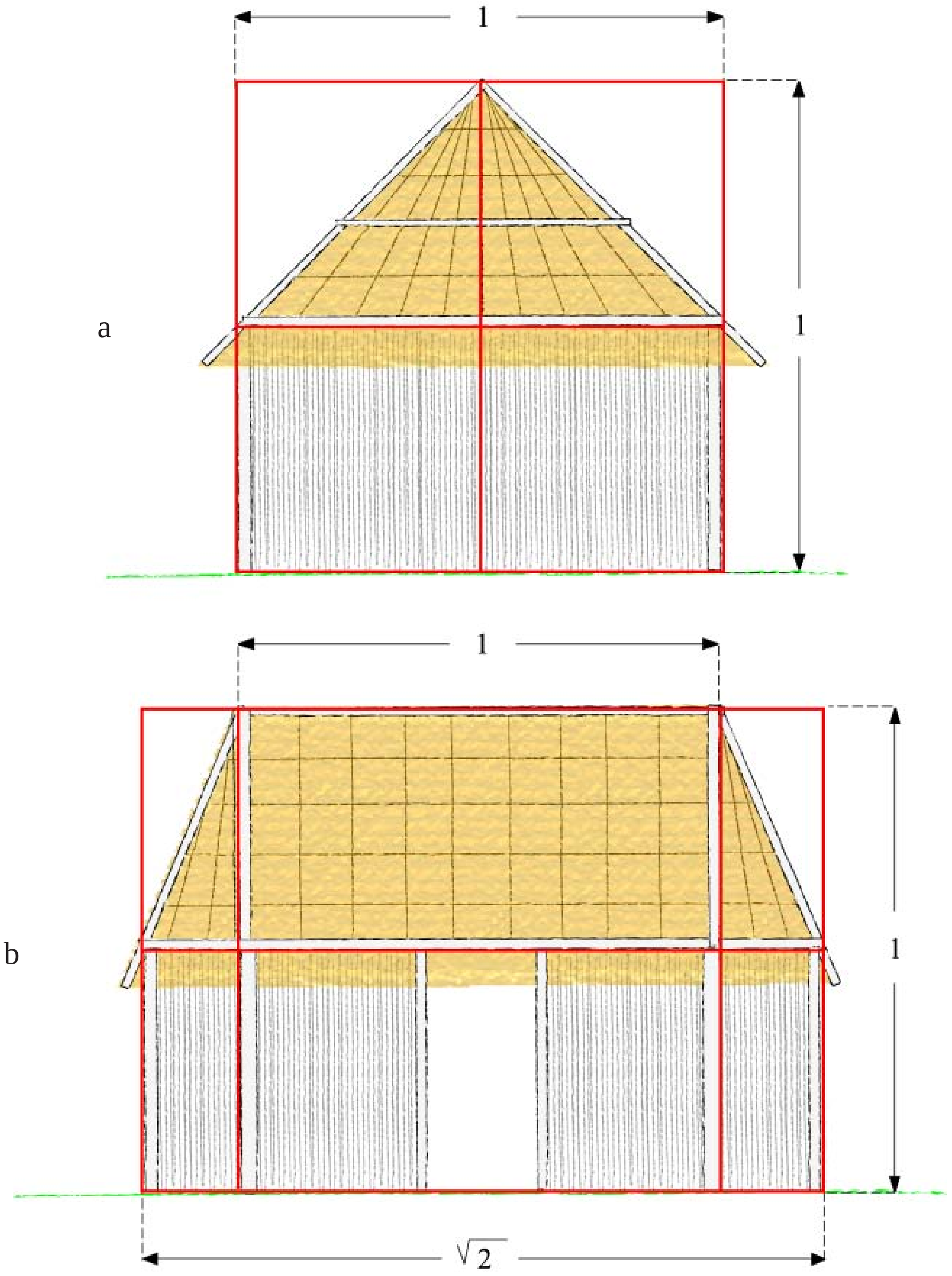
Plan of Maya Vernacular House 6, A House Described by Don Guadalupe, Oxkintok, Yucatan, Mexico



The plan of Maya Vernacular House 6 has a width to length ratio of one to root two. The principal support beams and the parallel walls of the apsidal plan form a square, and the radii of the apsis run from the center of the square to its corners (yellow highlighted dashed line).

Figure 51

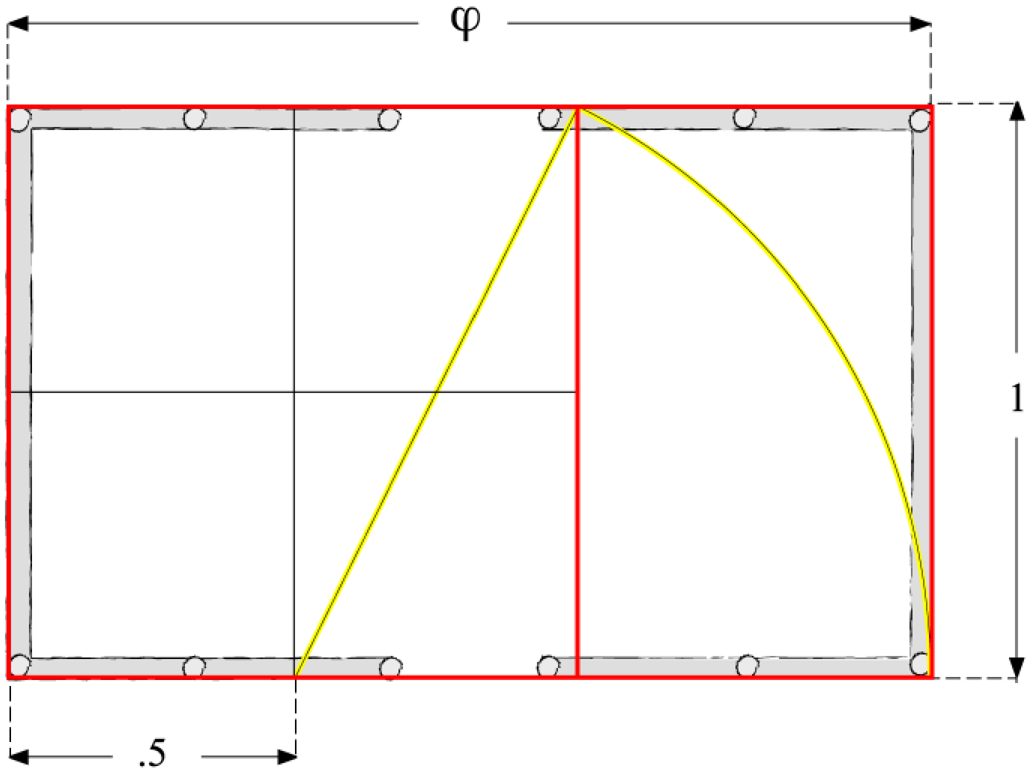
Elevations of Maya Vernacular House 6, A House Described by Don Guadalupe, Oxkintok, Yucatan, Mexico



The side section (a) is of equal height to width and is inscribed by a square. The height of the spring line and the roof are also equal. The pitch of the roof is forty-five degrees.
The height to width of the front elevation (b) is a ratio of one to root two. The pitch of the roof is approximately 67.5 degrees

Figure 52

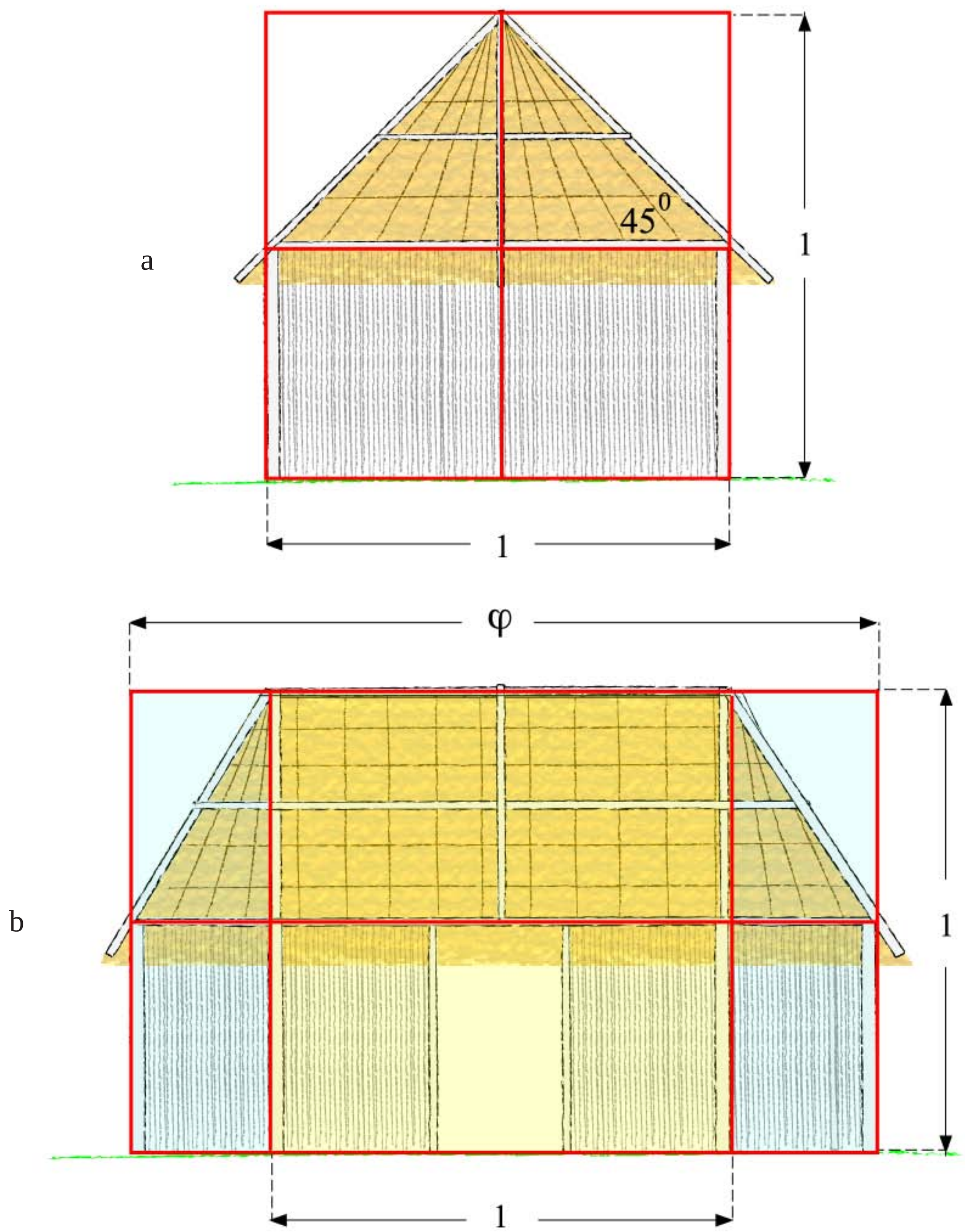
Plan of Maya Vernacular House 7, A House Described by Don Guadalupe,
Oxkintok, Yucatan, Mexico



The plan of Maya Vernacular House 7 has a width to length ratio of one to phi. The plan is laid out by first laying out a square and from the center of the base of this square, stretching a diagonal to its upper right corner. This diagonal is the radius for the arc that determines the length of the plan (highlighted yellow lines).

Figure 53

Elevations of Maya Vernacular House 7, A House Described by Don Guadalupe, Oxkintok, Yucatan, Mexico



The side elevation (a) is of equal height to width and is inscribed by a square. The height of the spring line and the roof are equal. The pitch of the roof is forty-five degrees.

The height to width of the front elevation (b) is also a ratio of one to phi. The pitch of the roof is the diagonal of a phi rectangle (approximately fifty-six degrees and shaded blue).

I learned about the layout, design, and ceremonies related to Maya traditional houses in and around Chichicastenango, Guatemala, from my friend Fernando Lopez. We met in the tunnel excavations deep beneath the surface structures at Copan, Honduras, on my first day of work, in 1994. Fernando was the assistant to David Sedat, Field Director of the Early Acropolis Project.

As a young man, Fernando was conscripted into the Guatemalan army, but he soon escaped and took refuge in a small village near Chichicastenango where he became an apprentice to a local shaman. Among the many things he learned there was how to build a traditional Maya house and the ceremonies associated with the design and layout of the house.

One weekend, during my third field season at Copan, Fernando invited me to help him clear his milpa for burning. Though he and his family lived in a modern house in town, between excavations and on weekends he worked with his uncle on a milpa in the hills north of town. On the milpa was a beautifully constructed traditional house of a design I had not seen before. When we took a noon break to eat some *papusas* under the shade of the wide veranda, I asked him about his house.

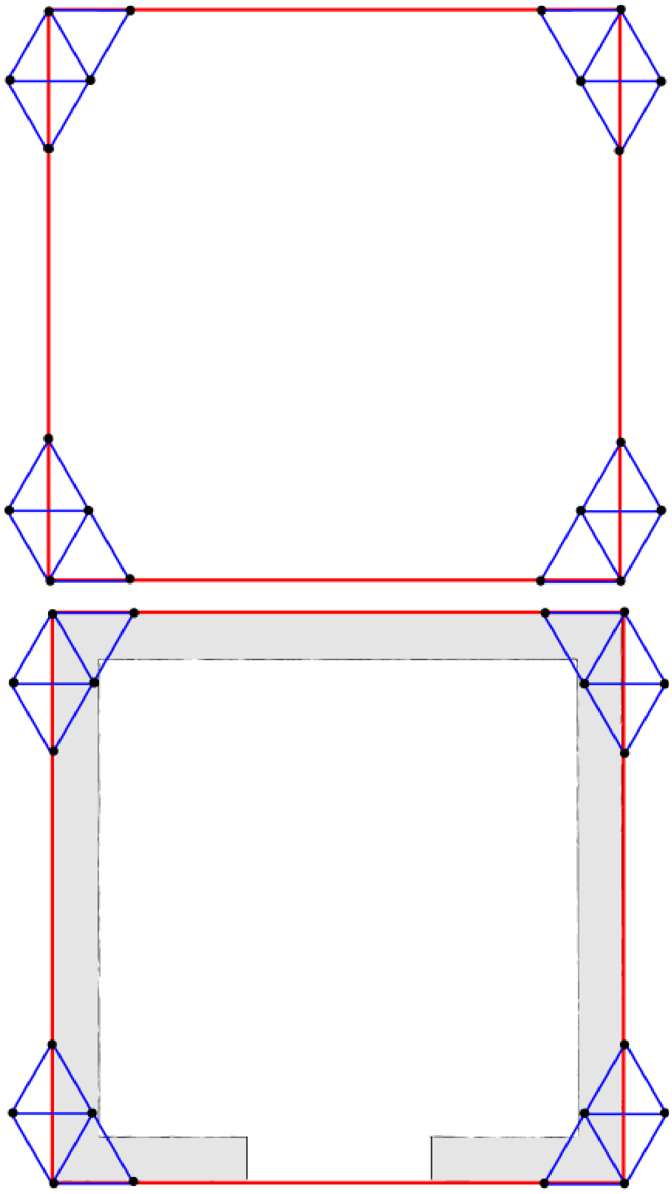
He told me the story of how he became a shaman's apprentice and how he built this particular house to the specifications that he learned during his apprenticeship. He also described the ceremony performed as part of the activities involved in laying out the ground plan of the house. All the shamans living in that area of the Highlands, including the shaman who taught Fernando, used a special knotted cord to lay out the right angles of a house. They made these cords once and used them for the rest of their lives. Fernando described the ceremony for making this knotted cord. He said that first a straight pole of wood was cut to a length exactly three times the height of the shaman and set into the earth and plumbed vertically with a plumb bob. This was done before sunrise on a certain day of the year. Fernando could not remember which day of the year that this ceremony was performed. (I would guess that it may have been the day of the zenith passage when some shamans in the Highlands use a plumbed vertical stick to determine the day of the zenith passage by observing that the stick casts no shadow at noon). At the moment of sunrise, the shaman counted off six and a half steps along the shadow of the pole and marked this measure with a wooden stake. Fernando described these as *pasos elegantes, no demaciado largos* ("elegant steps, not too long"). At sunset the shaman would repeat this process, following the shadow of the pole in the opposite direction. This procedure was used to determine the overall length of the cord, which of course would be thirteen steps long. The cord was then carefully divided by knots into seven equal portions. There were eight knots in all, including the knots at the ends of the cord. He said that the important numbers here were thirteen and eight.

The ground plan of this particular type of Maya house is a square, and the knotted cord is used to form the right angles at each corner of the square. Beginning with the first three sections of the cord, an equilateral triangle is stretched and staked into the ground. The remaining four sections of the cord are stretched and staked to form two more equilateral triangles joined to the first one to form a half hexagon divided by three equilateral triangles (see Figure 54). Next, a length of cord that is four times the intended length of one side of the square plan of the house is measured and cut. Generally, the length of one side of the square plan of the house is twice the height of the house builder. Fernando called this length a *brasada*, which is measured in the same manner as a uinic; that is, by stretching a cord horizontally from fingertip to fingertip. To determine the perimeter of the house, two brasadas are measured from one end of the cord, and this measure is stretched across the two knots that form the base of the first equilateral triangle that was staked out to form the half hexagon. The opposite end of the two-brasada measure is then tied to a wooden stake pounded into the ground, thus delineating one side of the square house. Using the long end of the same cord, two more brasadas are measured and stretched across the vertices of the two remaining triangles of the half hexagon and pulled tight between them. Again, the opposite end of the cord is staked into the ground to form the second side of the house plan. Using a half hexagon divided into three equilateral triangles in this manner forms a true right angle. The seven-sectioned measuring cord is then removed, but a stake at the base of the triangle that marks the corner of the right angle is replaced with a third stake that is pounded into the ground. Thus, the shaman has laid out a true right angle, two brasadas to a side. The shaman then repeats these steps, laying out half hexagons made of three equilateral triangles at the ends of this right angle to complete the square plan. At the final corner of the square, he also lays out this half hexagon. After the first right angle is determined, laying out the half hexagon at each corner of the square is unnecessary and redundant. When I pointed this out to Fernando, he told me that these triangles laid out at the four corners of the house were considered *las raices de la casa* ("the roots of the house") and that there were prayers for laying out of each of twelve triangles. He could not remember all of the prayers to each of the triangles but mentioned that one was for the corn god and the milpa and that the last three were prayers for the husband, wife, and children who were to live in the house (see Figure 54).

Forked wooden vertical roof support posts are set into the ground at the corners of the square plan, and adobe bricks are used to build the walls inside of the staked out square plan. Like many Maya vernacular houses, the height of the spring line is equal to one half of the length of the walls. But the roof of this particular house is supported by vertical posts set into the ground that extend one brasada beyond the corners of the house, thus creating a roof that has an area four times that of the house proper (see Figure 55). The pitch of the roof of Fernando's house was sixty degrees, though sometimes a pitch of forty-five degrees is used (see Figures 56 and 57). Fernando preferred the larger roof and used the extra space for hammocks and food storage.

Figure 54

Plan of Maya Vernacular House 8, Don Fernando's House,
Copan, Honduras

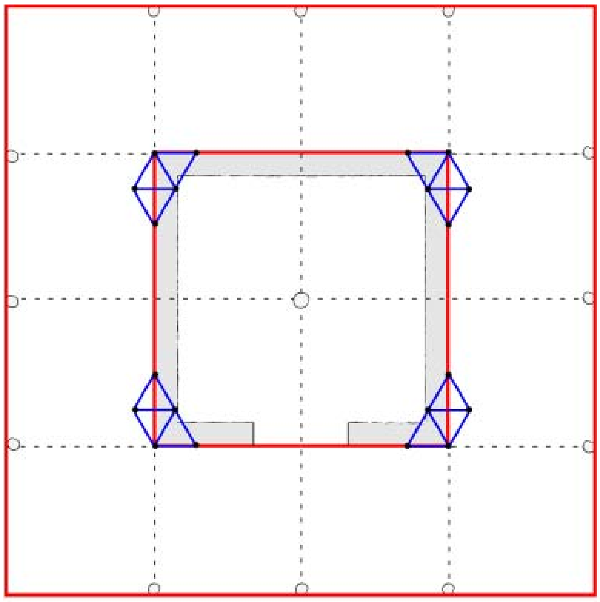


The plan of Maya Vernacular House 8 is a square. The walls are made of adobe bricks. The half hexagons (blue lines) in the corners demonstrate how the Maya shaman staked out the four true right angled corners of the square plan with his special cord that is thirteen footsteps in length and divided with eight knots (including the knots at the ends of the cord) into seven equal sections.

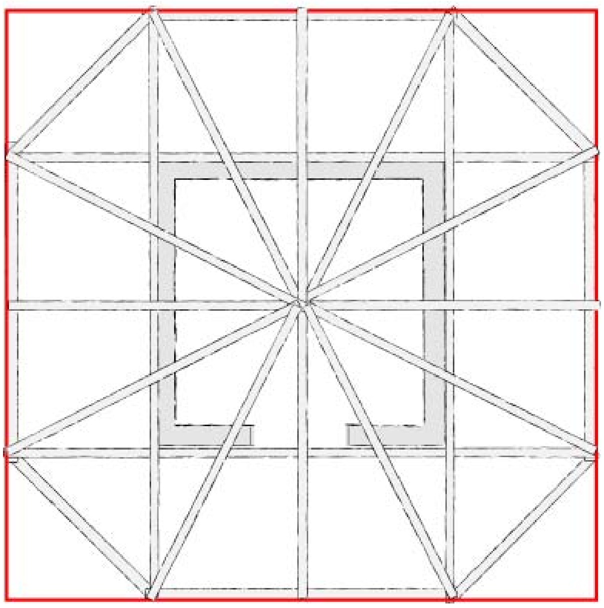
Figure 55

Plan of Maya Vernacular House 8 With Roof, Don Fernando's House, Copan, Honduras

a



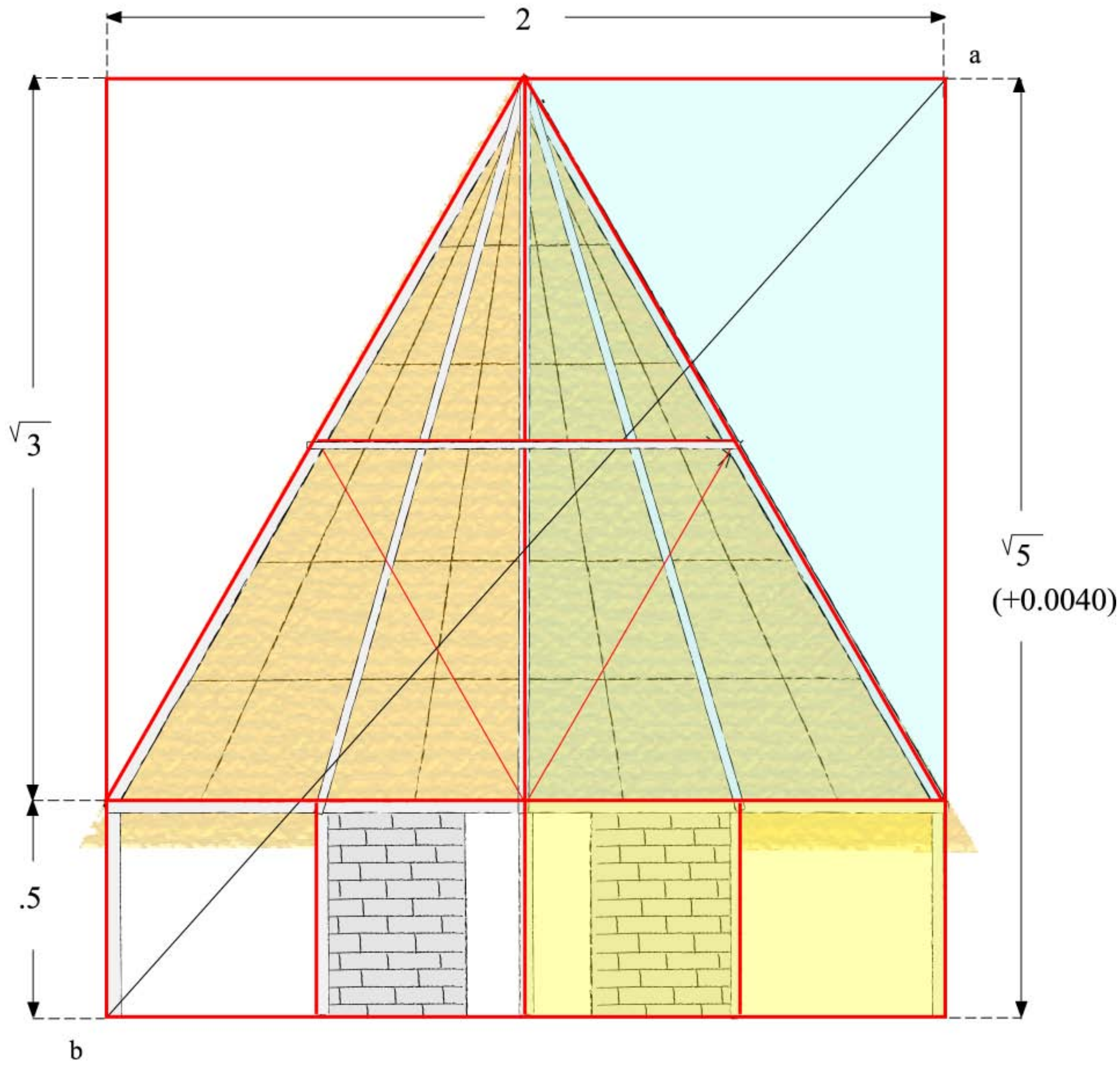
b



The plan of Maya Vernacular House 8 (a), with roof support posts and (b), with rafters, demonstrates that the square that inscribes its roof has an area exactly four times larger than the area of the exterior dimensions of the adobe brick house (shaded gray). Note the unusual central roof support post that extends to the height of the roof.

Figure 56

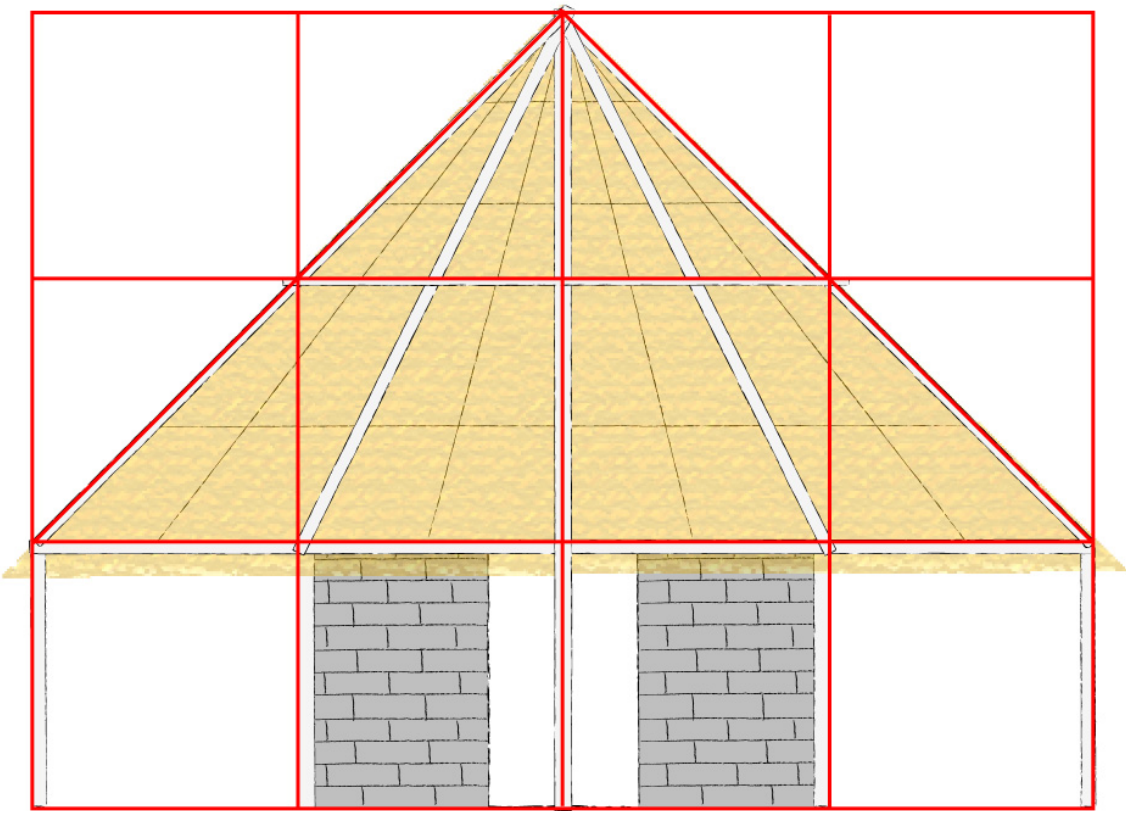
Elevation of Maya Vernacular House 8, Don Fernando's House,
Copan, Honduras



The elevation of Maya Vernacular House 8 has a roof that is inscribed by an equilateral triangle (red lines). The pitch of the roof is sixty degrees. If the width of the house (including the roof) is two units, then the height of the roof is root three and the height of the spring line is .5. The height of the entire structure would be root five, as per the virtually perfect Formula Square Root of Five-3. The diagonal a,b is virtually three (3.00006) units.

Figure 57

Alternate Elevation of Maya Vernacular House 8,
Don Fernando's House, Copan, Honduras

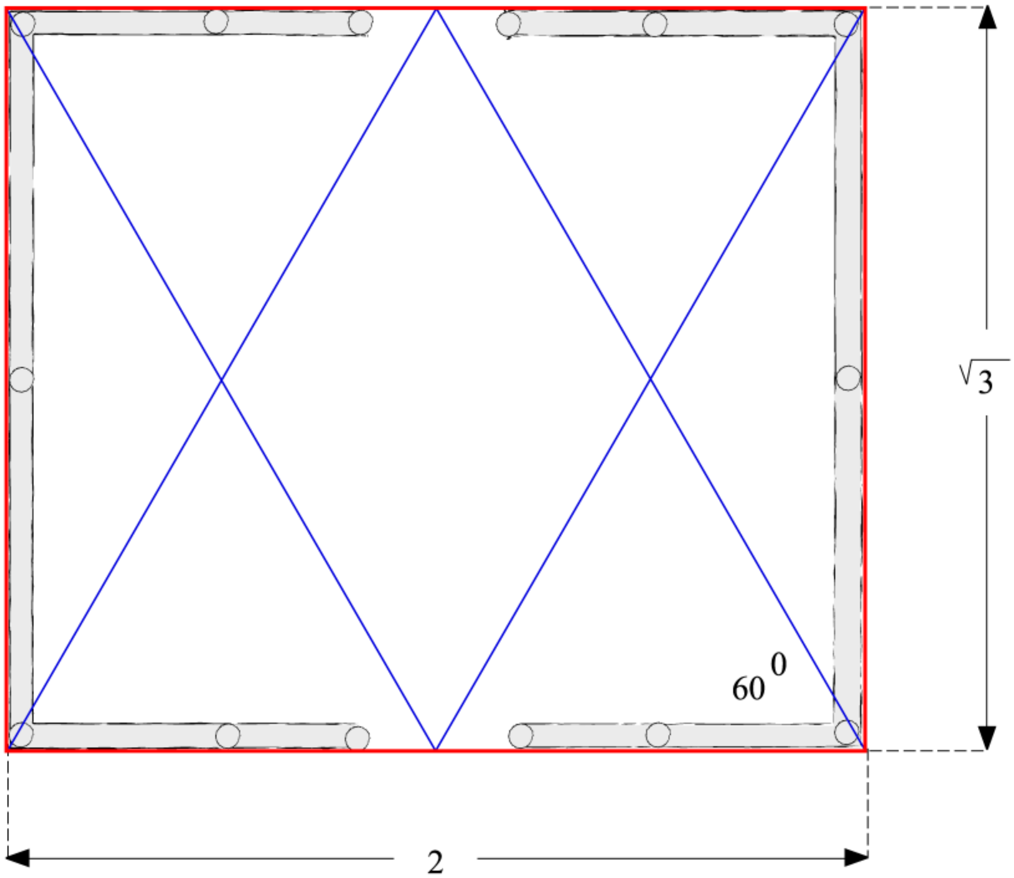


This Alternate elevation of Maya Vernacular House 8 has a roof that is twice the height of the spring line of the house and twice the width of the house. The pitch of the roof is forty-five degrees. The bold red lines demonstrate how a Pythagorean 3,4,5 rectangle divided into twelve equal squares both inscribe and neatly subdivide the elevation of the house.

Years ago, my friend and colleague Alfonso Morales Cleveland told me about a Maya vernacular house style he saw in northern Honduras, as well as a ceremony for creating a measuring cord. His general description was similar in many ways to the more detailed information provided by Fernando Lopez. The shaman also produced his measuring cord by measuring its length from the shadow of a straight stick placed in the ground at sunrise and sunset on a particular day of the year. Whether this cord was subdivided with knots was not ascertained. In any case, this cord was divided into three equal sections, each of which was equal to the intended length of the future house, and opposing equilateral triangles were laid out. The vertices of each equilateral triangle was centered at the base of the opposing triangle and then staked. A longer cord was then tied to and stretched from these corner stakes. The walls of the house were built within this rectangle, which had a proportion of two over root three (see Figure 58). Alfonso did not ask his informant about the elevations of this house style.

Figure 58

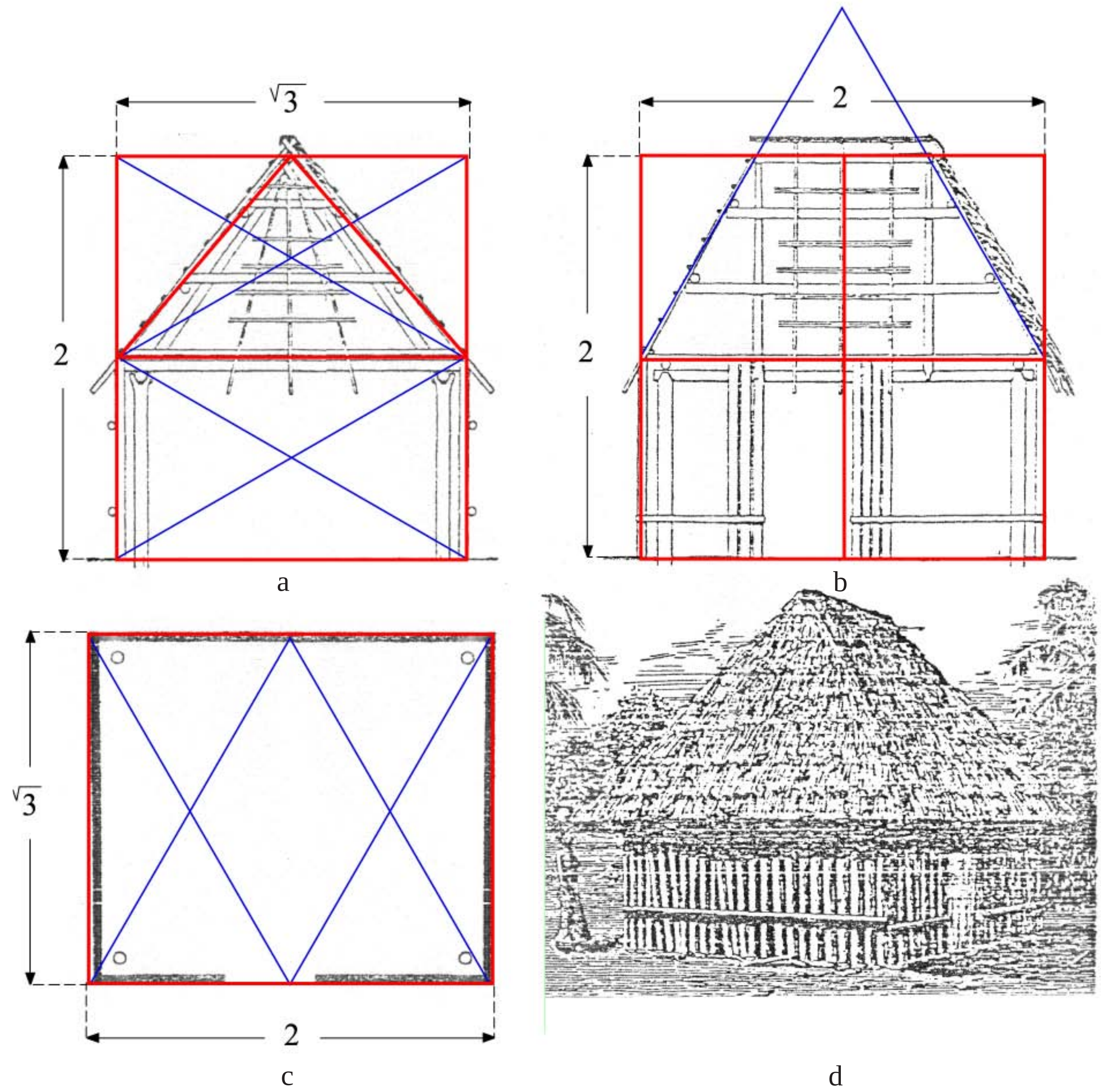
Plan of Maya Vernacular House 9,
Copan, Honduras



The plan of Maya Vernacular House 9 has a width to length ratio of one to two over root three and is formed by laying out opposing equilateral triangles (in blue lines).

Figure 59

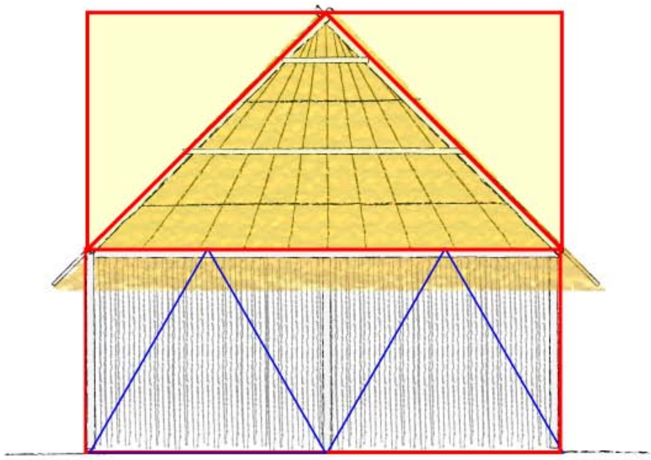
Plan, Elevations, and Perspective of Maya Vernacular House 10,
Highland Guatemala
Measured Drawing by Wauchope (1940)



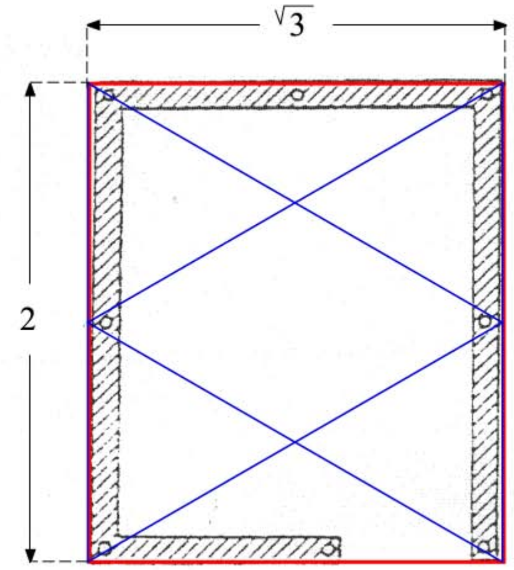
The height to width of the side elevation (a) is a ratio of two over root three. The pitch of the roof is approximately forty-nine degrees.
The front elevation (b) is of equal height to width and is inscribed by a square. The height of the spring line and the roof are equal. The pitch of the roof is sixty degrees.
The plan (c) has a width to length ratio of two over root three. All blue lines form equilateral triangles.

Figure 60

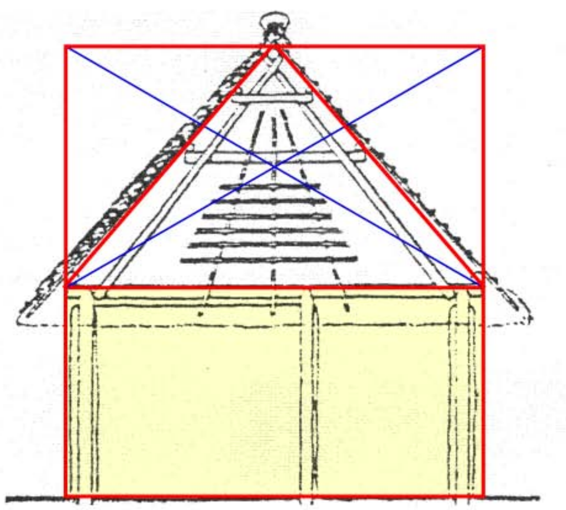
Plan and Elevations of Maya Vernacular House 11,
Santiago Atitlan, Guatemala
Measured Drawing by Wauchope (1940)



a



b

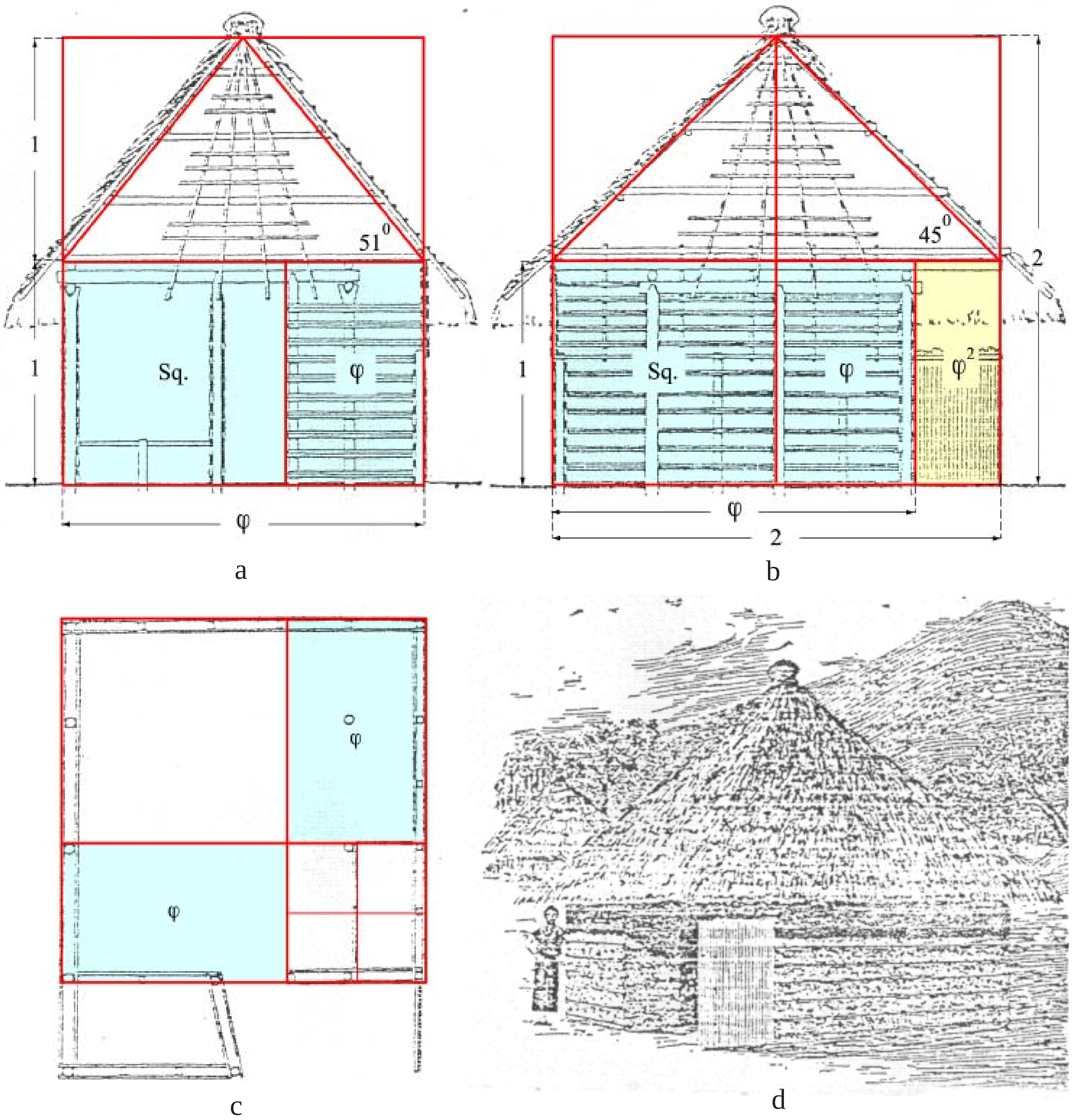


c

The length to width of the plan (b) is a ratio of two over root three.
The height of the walls of the front elevation (c) is equal to one half the width of the plan and is inscribed by a root four rectangle (shaded yellow). The roof is inscribed by a root three rectangle with a pitch of approximately forty-nine degrees.
The height of the roof of the side elevation (a) is equal to one half the length of the plan, has a pitch of forty-five degrees and is inscribed by a root four rectangle (shaded yellow). The walls are inscribed by two, two over root three rectangles. All blue lines form equilateral triangles.

Figure 61

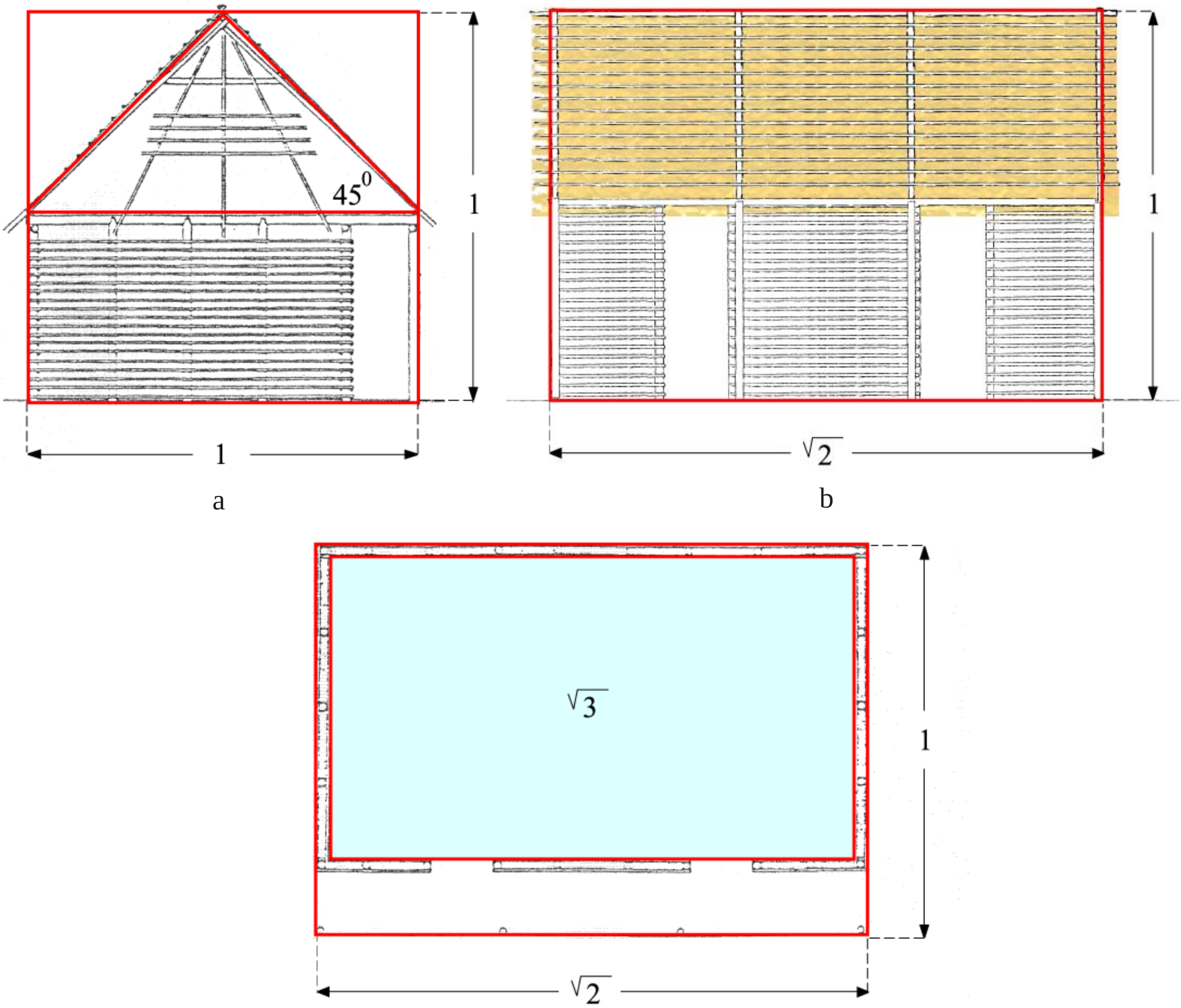
Plan, Elevations and Perspective of Maya Vernacular House 12,
San Lucas Toliman, Guatemala
Measured Drawing by Wauchope (1940)



The plan (c) is inscribed by a square that is subdivided at the doorway and the length of the small interior room into squares (white) and phi rectangles (shaded blue) as per Formula Square-2.
The front elevation (a) has a height of two to a width of phi, divided equally at the spring line. The wall is inscribed by a phi rectangle (shaded blue) that is subdivided by a square and a smaller phi rectangle at the right edge of the doorway. The pitch of the roof is fifty-one degrees.
The side elevation (b) is inscribed by a square that is quartered. The wall is inscribed by a phi rectangle (shaded blue) that is subdivided by a square and a smaller phi rectangle by the center line. The porch is inscribed by a phi squared rectangle. The pitch of the roof is forty-five degrees.

Figure 62

Plan and Elevations of Maya Vernacular House 13,
San Cristobal, Guatemala
Measured Drawing by Wauchope (1940)



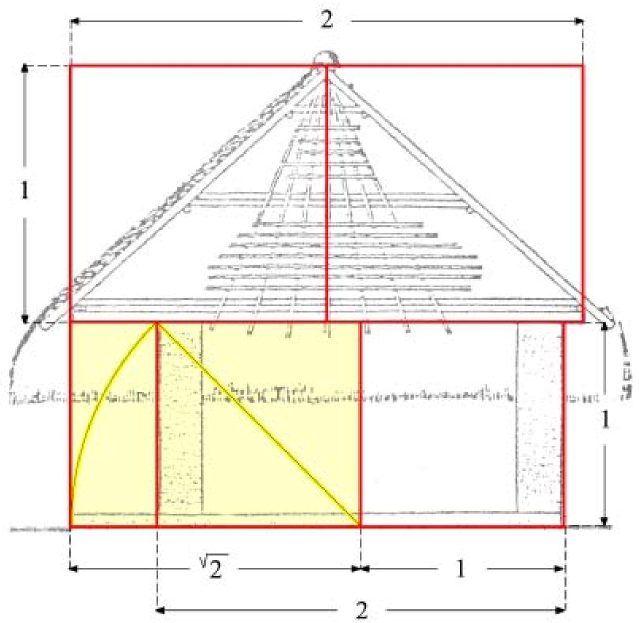
c

The plan of Maya Vernacular House 13 (c), with porch, is inscribed by a root two rectangle. The interior room is inscribed by a root three rectangle (shaded blue).
The side elevation (a) is inscribed by a square that is halved at the spring line. The pitch of the roof is forty-five degrees.
The front elevation (b) is inscribed by a root two rectangle the same size as the root two rectangle that inscribes the plan. Note the unusual overhanging gable roof.

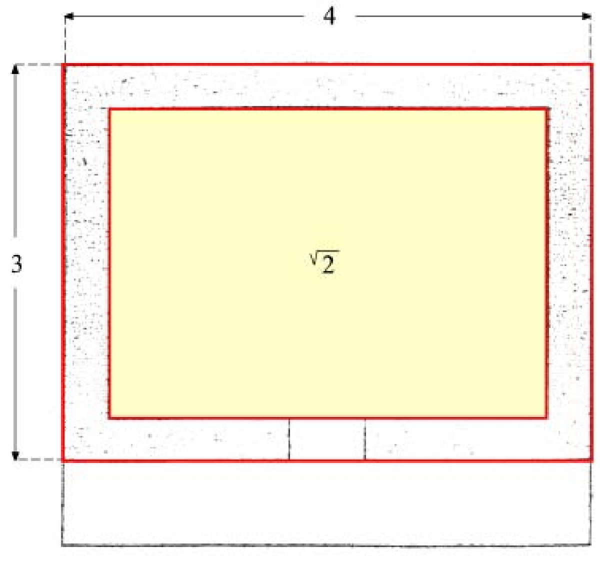
Figure 63

Plan and Elevation of Maya Vernacular House 14,
San Pedro de Laguna, Guatemala
Measured Drawing by Wauchope (1940)

a



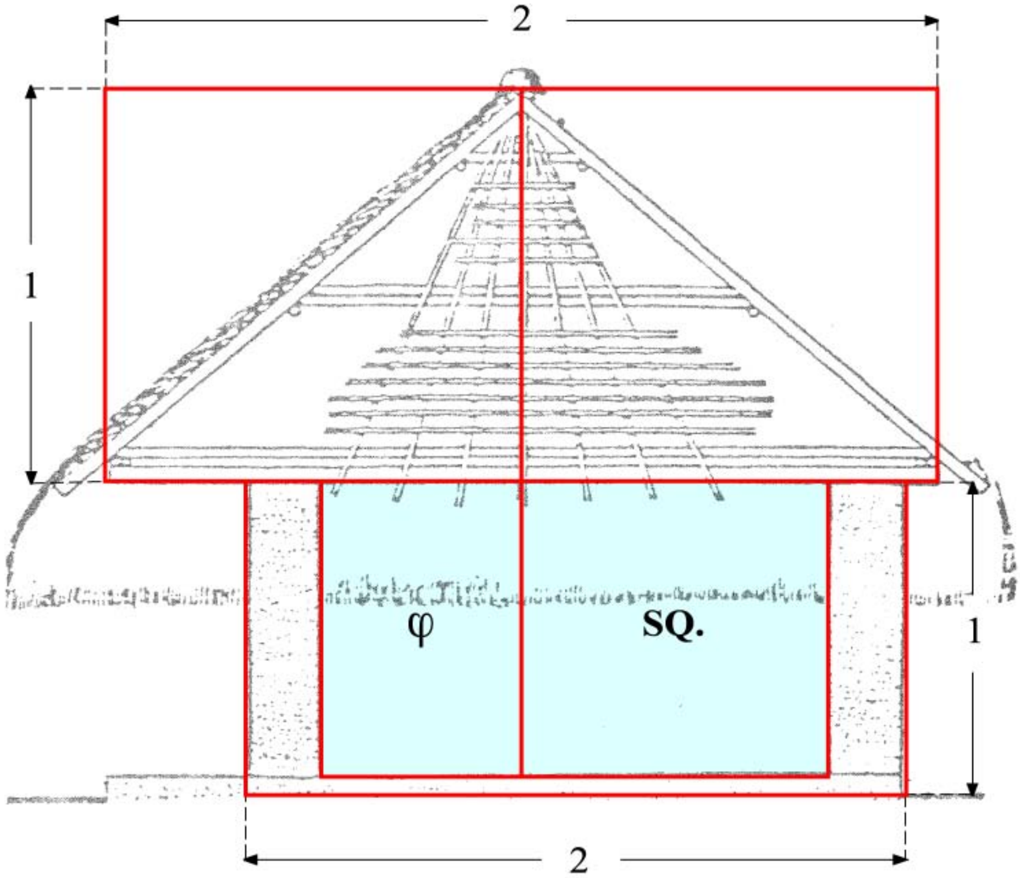
b



The plan of the adobe brick walls of Maya Vernacular House 14 (b) is inscribed by a Pythagorean 3,4,5 rectangle. The interior room is inscribed by a root two rectangle (shaded yellow).
The wall of the side elevation (a) is inscribed by a root four rectangle. One half of this wall and the porch are inscribed by a root two rectangle (shaded yellow).

Figure 64

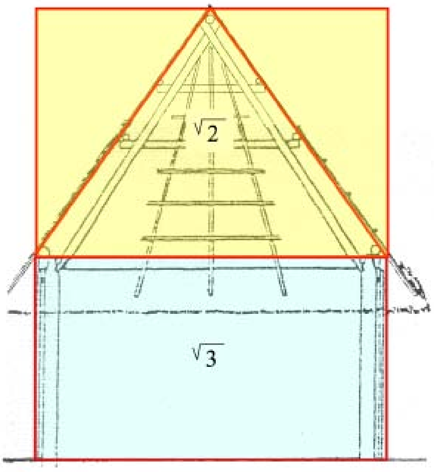
Section of Maya Vernacular House 14,
San Pedro de Laguna, Guatemala
Measured Drawing by Wauchope (1940)



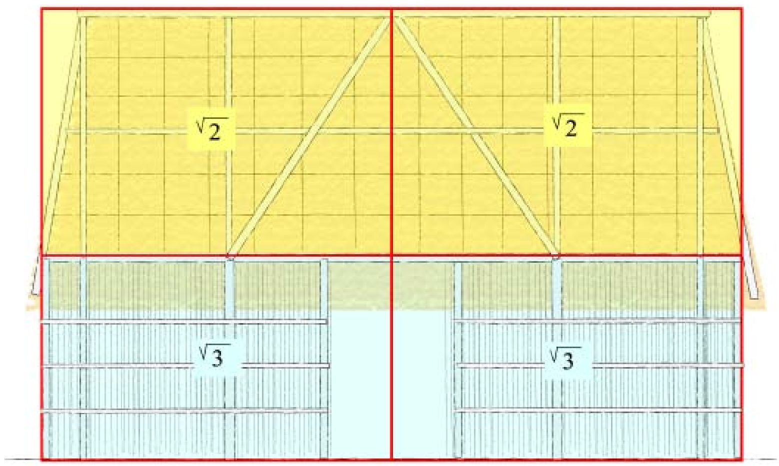
The side section of the interior room of Maya Vernacular House 14 is inscribed by a phi rectangle (shaded blue) that is subdivided into a square and a smaller phi rectangle by the center line of the roof.

Figure 65

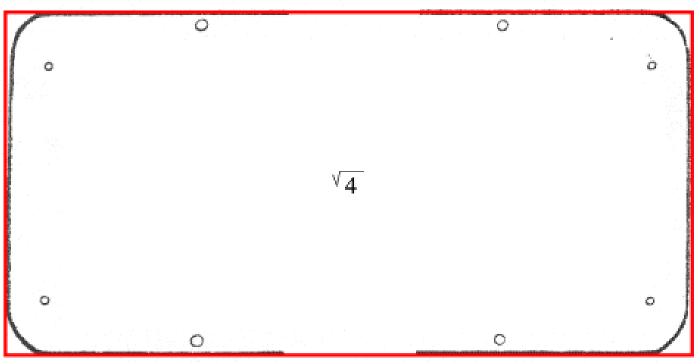
Plan and Elevations of Maya Vernacular House 15,
Tzimin, Yucatan, Mexico
Measured Drawing by Wauchope (1940)



a



b

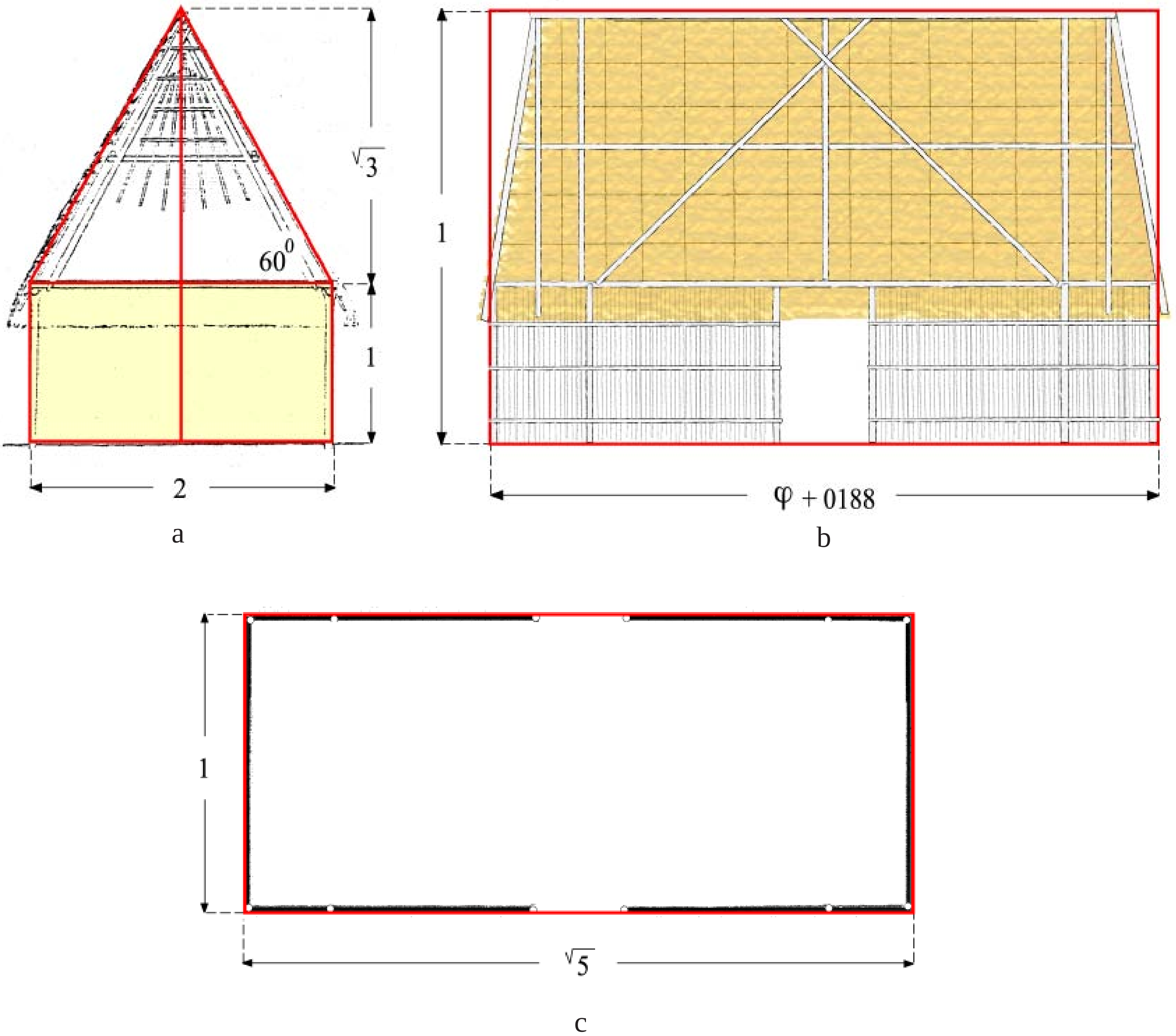


c

The plan of Maya Vernacular House 15 (c) is inscribed by a root four rectangle.
The wall of the side elevation (a) is inscribed by a root three rectangle (shaded blue). The roof is inscribed by a root two rectangle (shaded yellow). The pitch of the roof is approximately fifty-five degrees (the diagonal of a root two rectangle).
The front elevation (b) is double that of the side elevation. Note the very steep pitch of the roof (approximately eighty degrees).

Figure 66

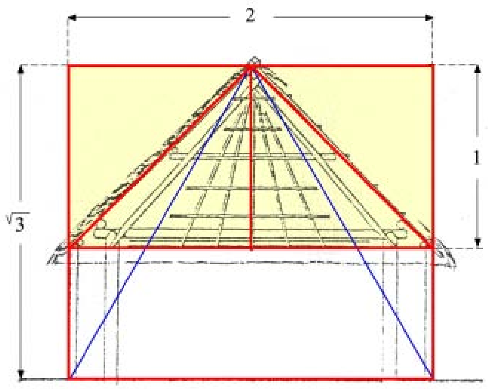
Plan and Elevations of Maya Vernacular House 16,
Lerma, Yucatan, Mexico
Measured Drawing by Wauchope (1940)



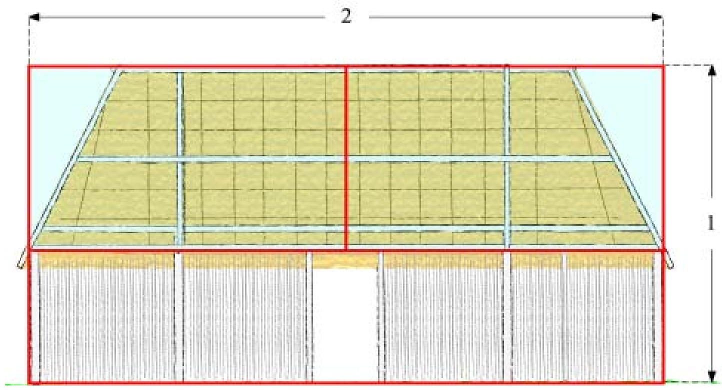
The plan of Maya Vernacular House 16 (c) is inscribed by a root five rectangle.
The wall of the side elevation is inscribed by a root four rectangle (shaded yellow). The roof is inscribed by an equilateral triangle.
The front elevation is inscribed by a rectangle with a ratio of one to phi + 0.188 (root three plus one divided by two over root five). This may have been considered a perfect formula.

Figure 67

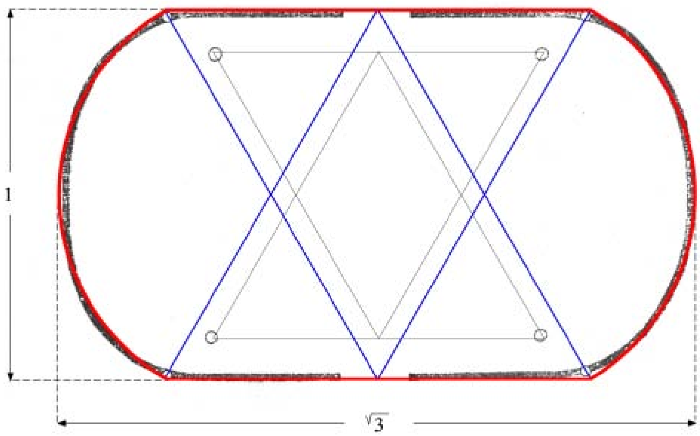
Plan and Elevations of Maya Vernacular House 17,
Chan Kom, Yucatan, Mexico
Measured Drawing by Wauchope (1940)



a



b



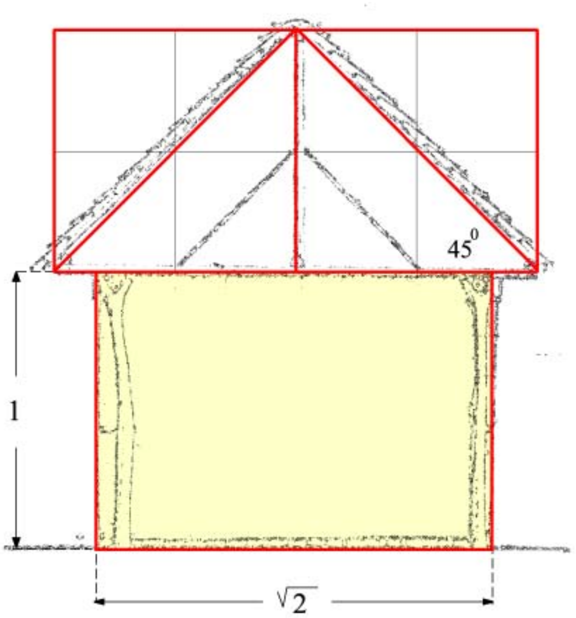
c

The parallel sides of the plan (c) are formed by the bases of two opposing equilateral triangles (blue lines). The intersections of theses equilateral triangles are the radii for the apsis of the plan. This formula creates a plan with a width to length ratio of one to root three. The interior roof support posts are also laid out using opposing equilateral triangles (fine black lines).
The height of the side elevation is equal to one half the length of the plan. The height of the roof is one half the width of the elevation, is inscribed by a root four rectangle (shaded yellow), and has a pitch of forty-five degrees.
The front elevation (b) is inscribed by a root four rectangle, the roof is inscribed by two root three rectangles (shaded blue), and its pitch is approximately seventy-four degrees (the diagonal of a root four rectangle).

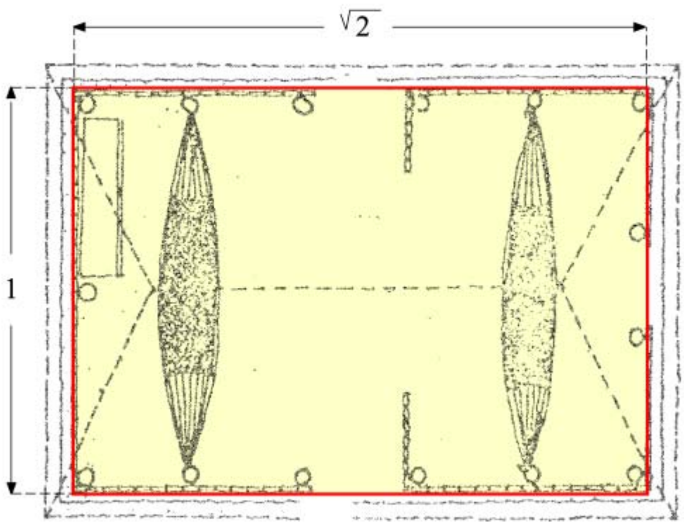
Figure 68

Plan and Elevation of Maya Vernacular House 18,
Yucatan, Mexico
Measured Drawing by Wauchope (1940)

a



b

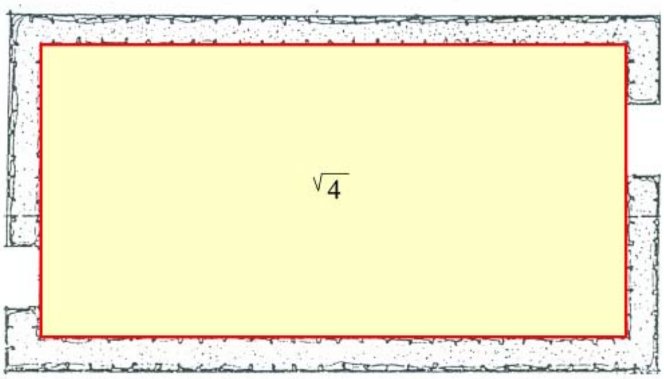


The plan of Maya Vernacular House 18 (b) is inscribed by a root two rectangle (shaded yellow). The wall of the side elevation (a) is also inscribed by a root two rectangle (shaded yellow). The roof is inscribed by a root four rectangle and has a pitch of forty-five degrees.
The wall of the front elevation (not shown) would be inscribed by a root four rectangle.

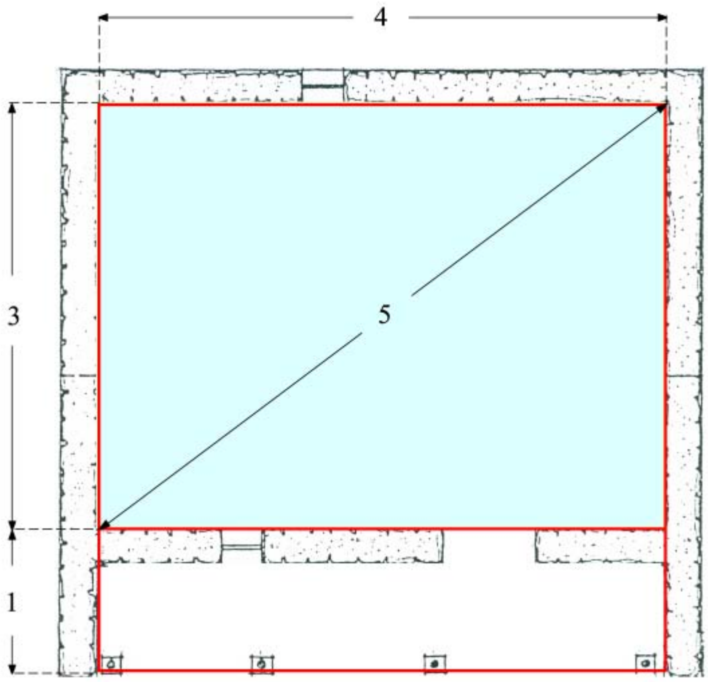
Figure 69

Plans of Maya Vernacular Houses 19 and 20,
San Andres Xicul, Guatemala
Measured Drawing by Bonilla Privaral (1989)

a



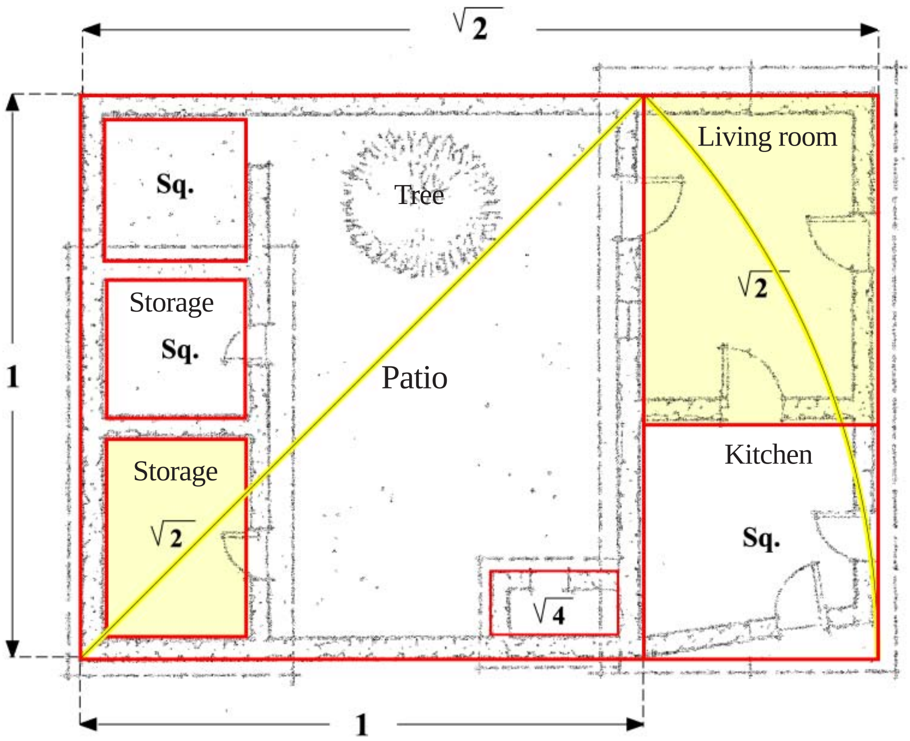
b



The interior room of Maya Vernacular House 19 (a) is inscribed by a root four rectangle (shaded yellow).
The interior of the "U" shaped wall and the four porch support posts of Maya Vernacular House 20 are inscribed by a square. The interior room is inscribed by a Pythagorean 3,4,5 rectangle (shaded blue). The elevations of these houses were not recorded.

Figure 70

Plan of Maya Vernacular Houses 21,
Santa Maria Chiquimula, Guatemala
Measured Drawing by Bonilla Privaral (1989)



The plan of Maya Vernacular House 21 is of the adobe brick walls that enclose the patio, and the form the walls of the storage sheds, living room, and kitchen. The storage sheds, living room, and kitchen are framed by wood posts (embedded in the adobe brick walls) and beams, the roofs are framed with wood beams and poles and thatched.

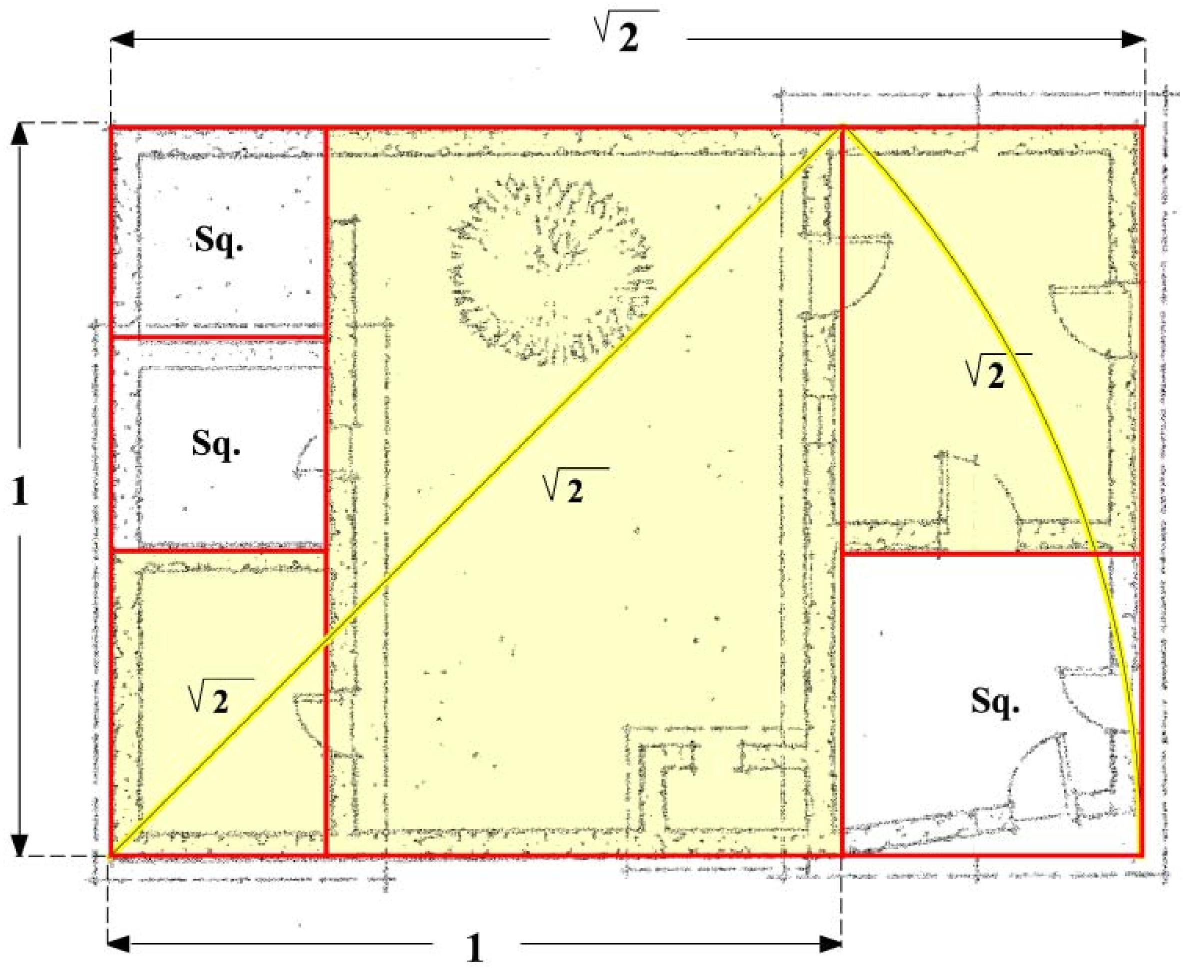
The entire structure is inscribed by a root two rectangle. The adobe brick walls that enclose the patio and storage sheds are inscribed by a square, the diagonal of which (highlighted yellow) is equal to the length of the plan and is the radius for the arc that terminates at the lower right corner of the diagram. The kitchen is inscribed by a square (in white), and the living room is inscribed by a root two rectangle (shaded yellow). The square patio and the square and root two subdivisions of the kitchen and living room are a clear example of Formula Square Root of Two-1.

The interior of the smaller of the two storage sheds is inscribed by a square whose diagonal is equal to the length of the interior of the larger storage shed, which is inscribed by a root two rectangle (shaded yellow). The remaining, semi-enclosed space in the upper left corner of the patio is also inscribed by a square of equal size to the square inscribed in the interior of the smaller storage shed.

The front wall of the kitchen (lower right corner) is also the main entrance to the house and is intentionally skewed or angled inward to leave a roofed dry entry to the house, protected from the rains.

Figure 71

Plan of Maya Vernacular Houses 21b,
Santa Maria Chiquimula, Guatemala
Measured Drawing by Bonilla Privaral (1989)

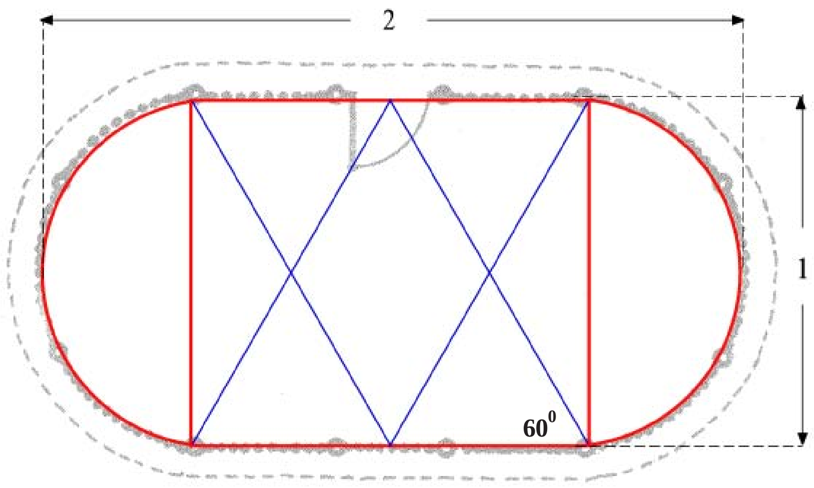


The plan of Maya Vernacular House 21b subdivides the square patio into two root two rectangles (shaded yellow) and two squares (in white) and implies that the architect of this structure may also have been familiar with the root two and square subdivisions of a square diagrammed in Formula Square-1.

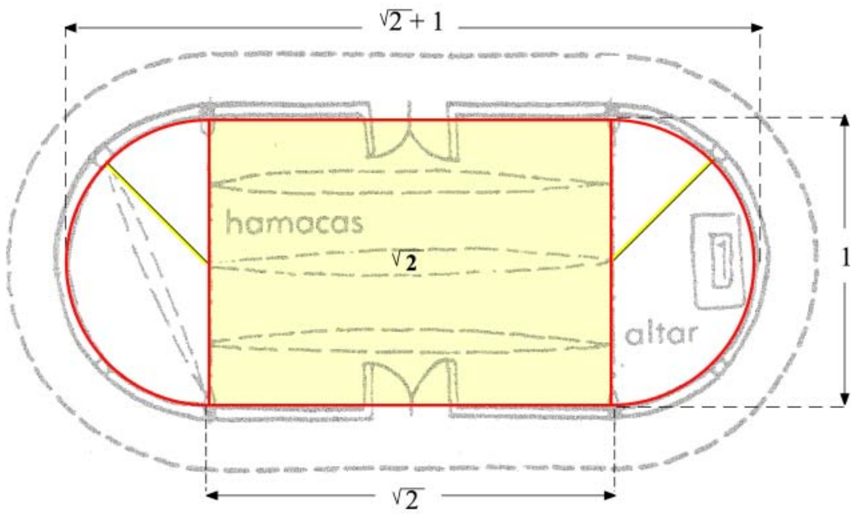
Figure 72

Plan of Maya Vernacular Houses 22 and 23,
Yucatan, Mexico
Measured Drawing by Pierrebourg (2003)

a



b



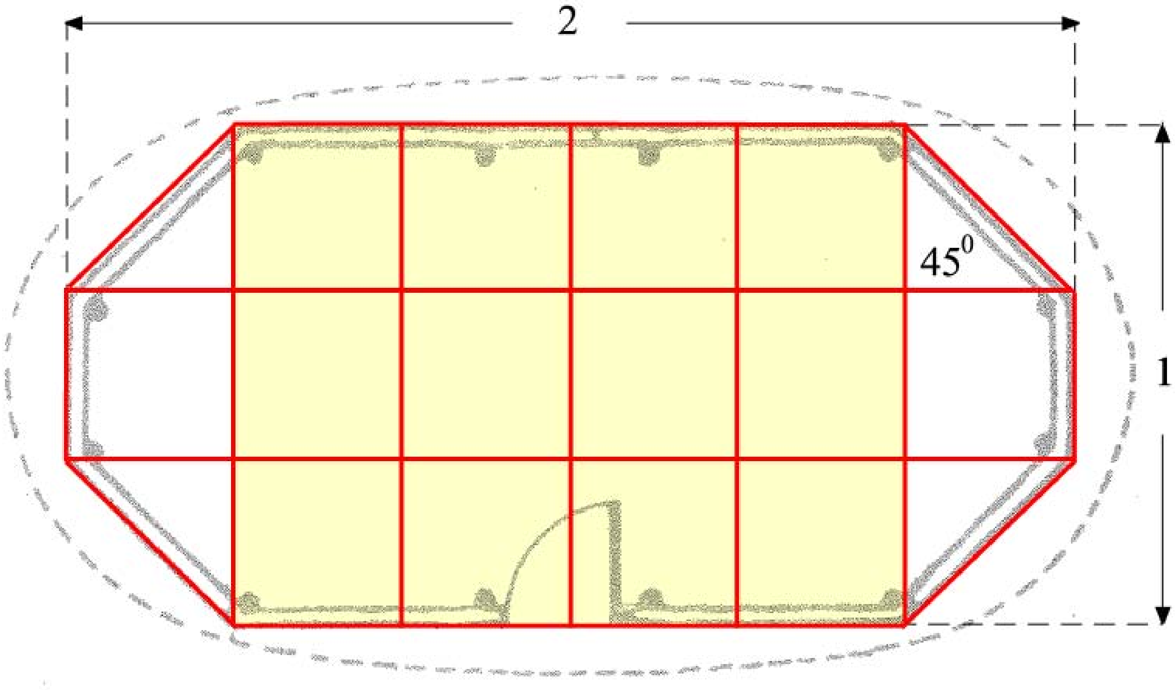
The plan of Maya Vernacular House 22 (a) has a width to length ratio of one to two. The principle roof support beams, and the parallel walls of the apsidal plan, are formed by laying out opposing equilateral triangles.

The plan of Maya Vernacular House 23 (b) has a width to length ratio of one to root two plus one. The principle roof support beams and the parallel walls of the apsidal plan form a root two rectangle (shaded yellow). The radii for the apses of the plan are half circles drawn from the centers of the sides of the root two rectangle to its corners.

The elevations of these structures were not documented.

Figure 73

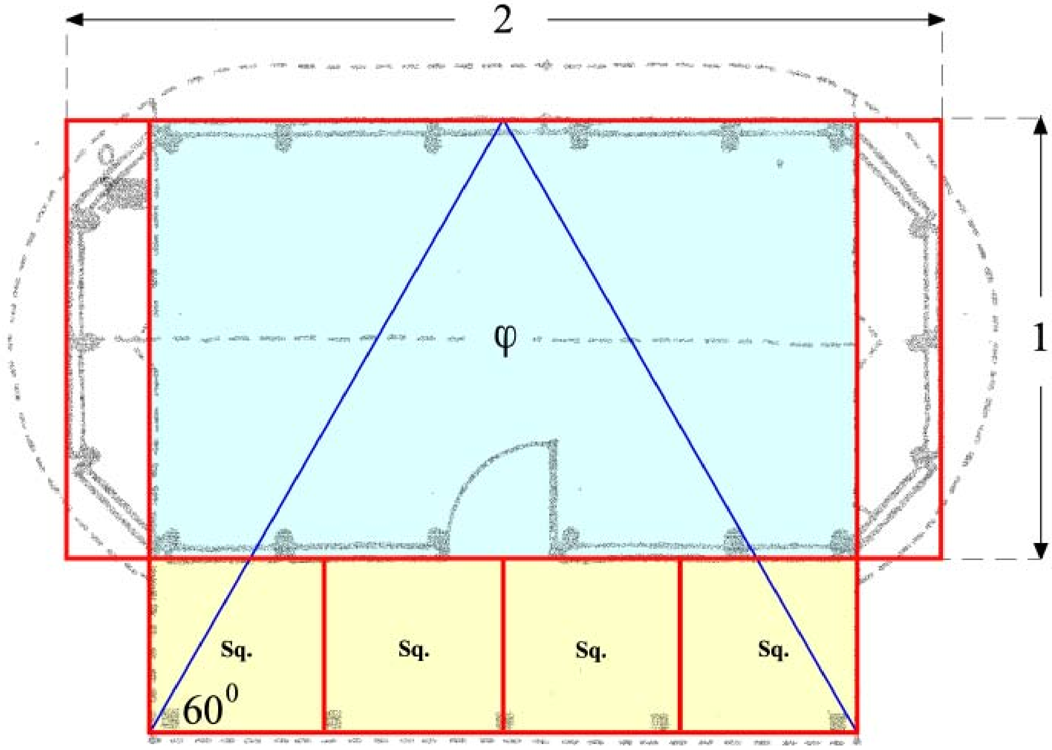
Plan of Maya Vernacular House 24,
Yucatan, Mexico
Measured Drawing by Pierrebourg (2003)



The width to length ratio of the polygonal plan of Maya Vernacular House 24 is one to two. The parallel walls and principle roof support posts of the plan are inscribed by a Pythagorean 3,4,5 rectangle (shaded yellow). The diagonal sides of the polygonal plan are forty-five degrees.

Figure 74

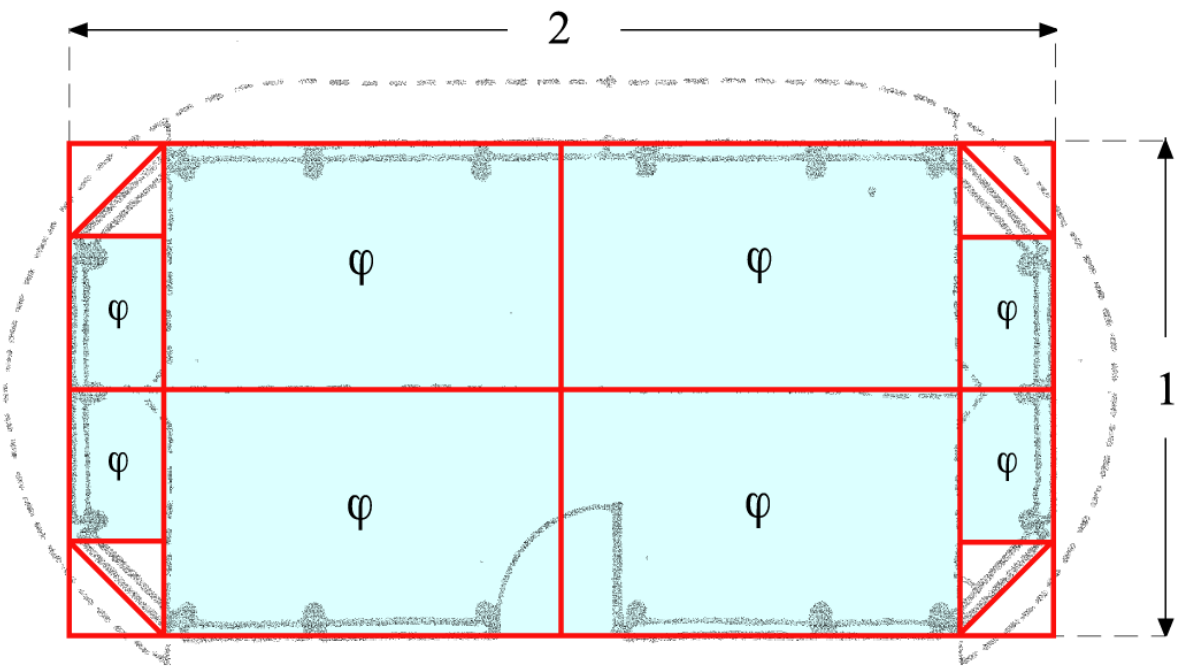
Plan of Maya Vernacular House 25,
Yucatan, Mexico
Original Measured Drawing by Pierrebourg (2003)



The polygonal plan of Maya Vernacular House 25 has a width to length ratio of one to two. The principle roof support beams and the parallel walls of the polygonal plan form a phi rectangle (shaded blue). The roof extends beyond the front wall of the house to form a porch whose width is one to a length of four (shaded yellow). The phi rectangle formed by the principal roof support beams and the one to four proportion of the porch forms a rectangle that inscribes an equilateral triangle (blue lines), as per the virtually perfect Formula Two Over Square Root of Three-3.

Figure 75

Plan of Maya Vernacular House 25b,
Yucatan, Mexico
Measured Drawing by Pierrebourg (2003)



The root four rectangle that inscribes the polygonal plan of Maya Vernacular House 25 is subdivided into a phi rectangle (shaded blue) and squares (white). The diagonals of the four squares at the corners of the plan define the angled ends (forty-five degrees) of the polygonal plan.
Each quarter of the plan is a root four rectangle subdivided into horizontal phi rectangle and a smaller phi rectangle and square (a phi squared rectangle), as per Formula Square Root of Four-3.

CHAPTER 7

GEOMETRY IN CLASSIC AND POST-CLASSIC MAYA ARCHITECTURE

The following analyses of the geometry of Classic and Post-Classic period architecture begins with some simple plans of Classic Period structures from Palenque and proceeds to include the plans and elevations of more complex Classic Period structures from Palenque, Tikal, and Altar de los Sacrificios. These Classic examples are followed by analyses of simple plans from the Post-Classic site of Chichen Itza and more complex examples of plans and elevations from Chichen Itza, Acanceh, Dzibilchaltun, Xpuhil, Zempoala, and El Tajin.

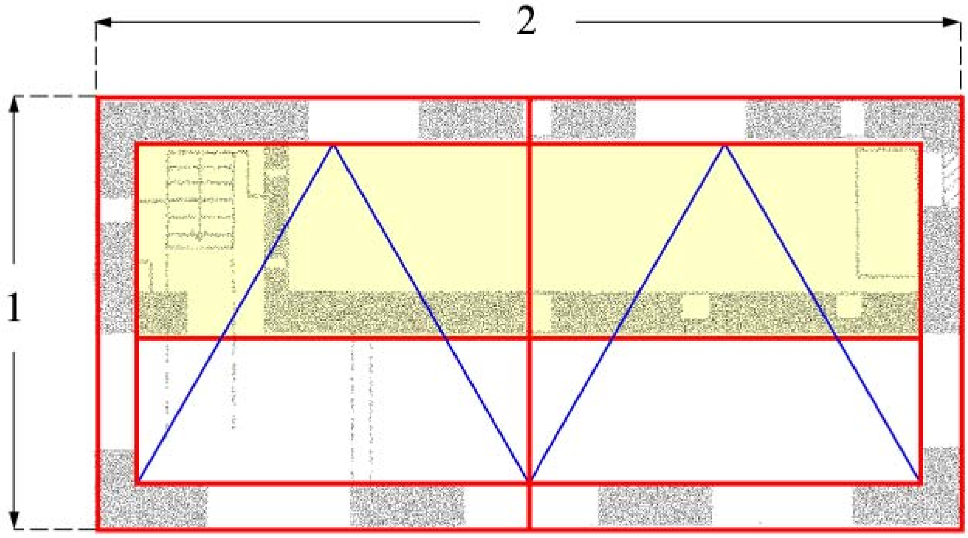
These examples were selected to represent a wide temporal and spacial range of Pre-Columbian Maya ceremonial architecture, but this selection is also biased by the availability of published measured drawings.

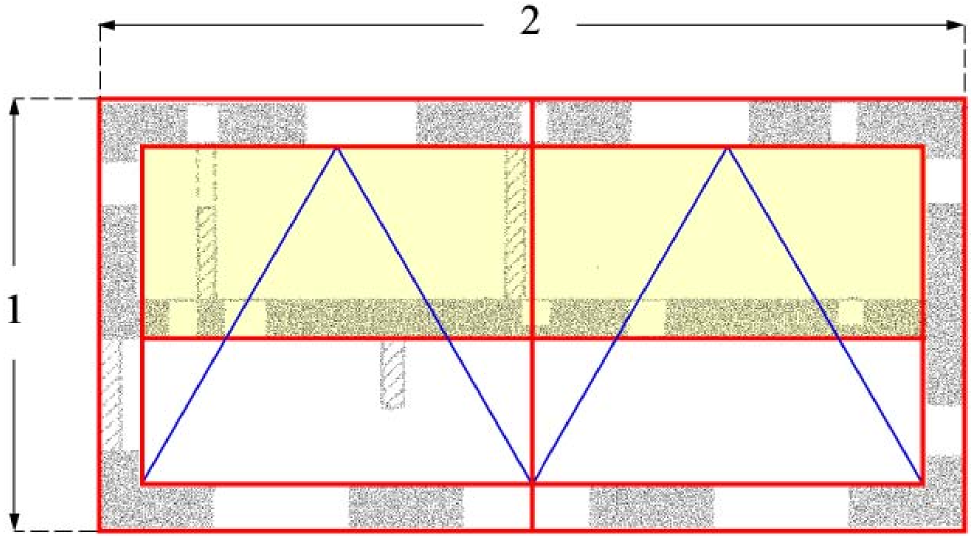
The set of square root, phi, and Pythagorean 3,4,5 rectangles and their subdivisions that are manifested in Pre-Columbian architecture are the same as those expressed in the examples of vernacular architecture. But, as might be expected, some of the suggested geometrical formulae seen in Pre-Columbian ceremonial architecture are more complex.

Note that the square root, phi, and Pythagorean rectangles are always expressed relative to the interior and exterior walls of the structures (never the centers of walls) and that the suggested subdivisions of these rectangles generally focus on the front galleries and entrances of these structures.

Figure 76

Plan of Houses G and H, Palenque, Chiapas, Mexico
Measured Drawing by G. F. Andrews (1974)

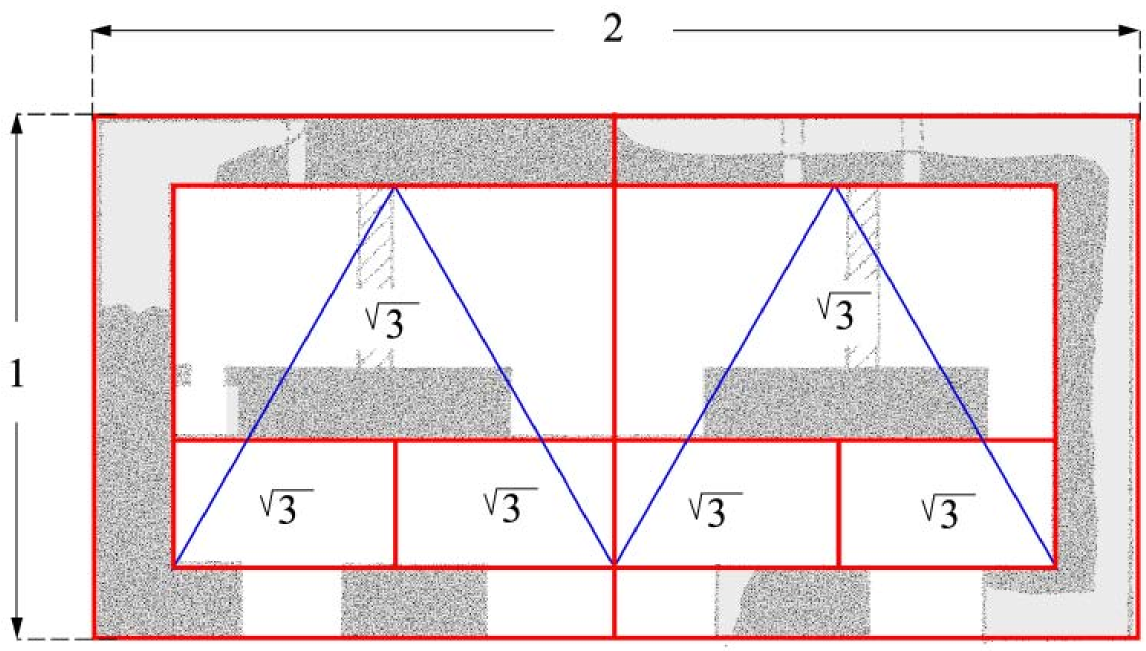




The exterior walls are inscribed by a square root of four rectangles (one to two), and the interior space is subdivided into two, two over root three rectangles, each of which are inscribed by equilateral triangles (blue lines).
The interior vault support walls and the rear galleries are inscribed by root four rectangles (shaded yellow).

Figure 77

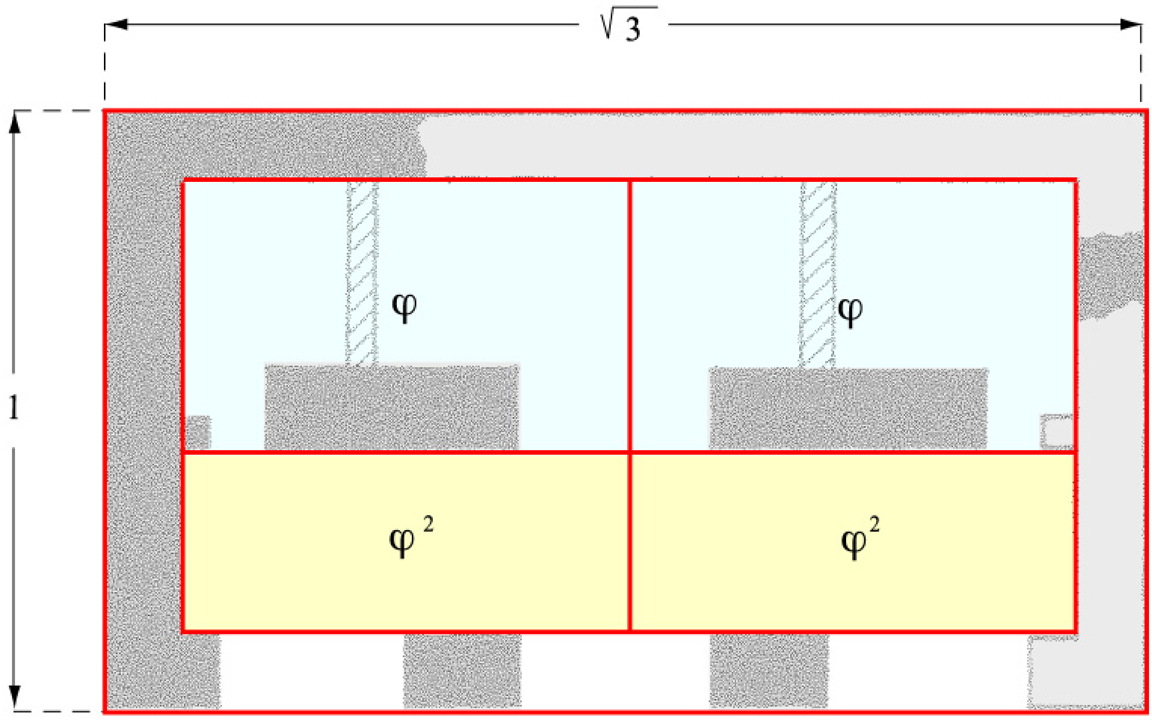
Plan of Temple XVIIIa, Palenque, Chiapas, Mexico
Measured Drawing by G. F. Andrews (1974)



Like Houses G and H, the exterior walls of Temple XIIIa are inscribed by a root four rectangle (one to two), and the interior space is subdivided into two, two over root three rectangles, each of which are inscribed by equilateral triangles (blue lines).
The interior vault support walls further subdivide the interior space into four square root of three rectangles (front gallery) and the remaining space into two square root of three rectangles, as per Formula Two Over Square Root of Three-2.

Figure 78

Plan of Temple XVIII, Palenque, Chiapas, Mexico
Measured Drawing by G. F. Andrews (1974)

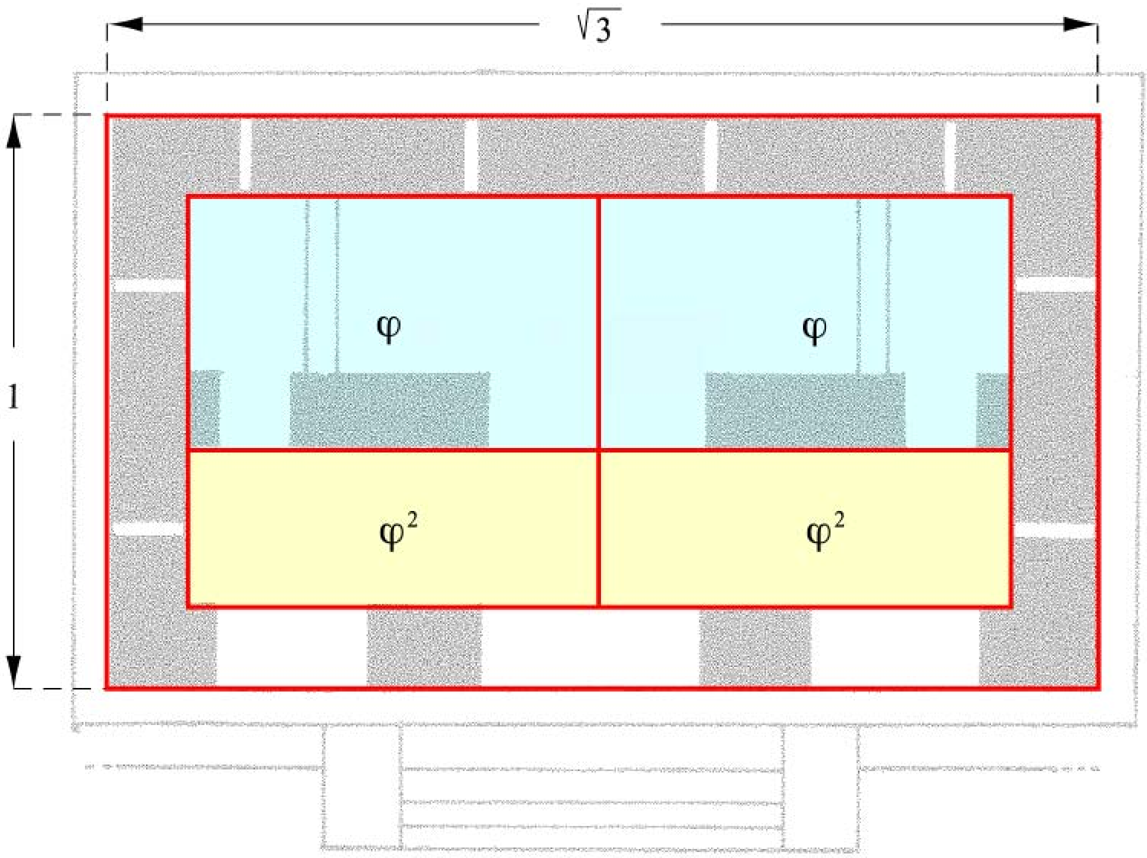


Temples XVIII and XVIIIa are a pair of temples that share a platform and are arranged parallel to to each other, about one meter apart. Note that the interior and exterior geometry of these temples are essentially reversed. The exterior walls of Temple XVIII are inscribed by a root three rectangle, and the interior space inscribes a root four rectangle.

The interior vault support walls further subdivide the double squares of the interior space into two phi-squared rectangles (front gallery, shaded yellow) and the remaining space into two phi rectangles (shaded blue), as per Formula Square-2.

Figure 79

Plan of Temple II, Palenque, Chiapas, Mexico
Measured Drawing by G. F. Andrews (1974)

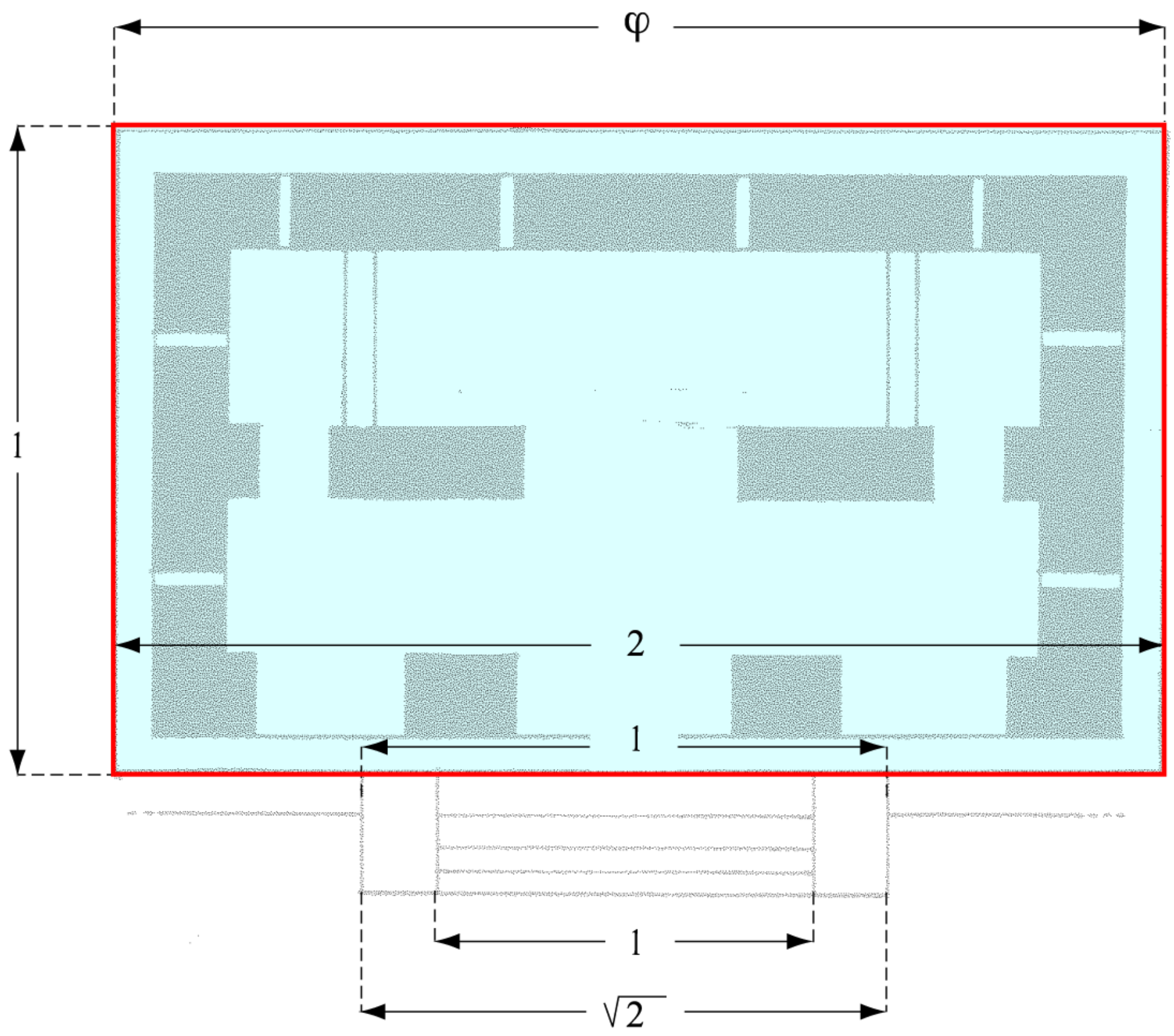


Temple II shares identical proportions as those described for Temple XVIII on the previous page. The exterior walls are inscribed by a root three rectangle, and the interior space inscribes a root four rectangle.

As in Temple XVIII, interior vault support walls further subdivide the double squares of the interior space into two phi-squared rectangles (front gallery, shaded yellow) and the remaining space into two phi rectangles (shaded blue), as per Formula Square-2.

Figure 80

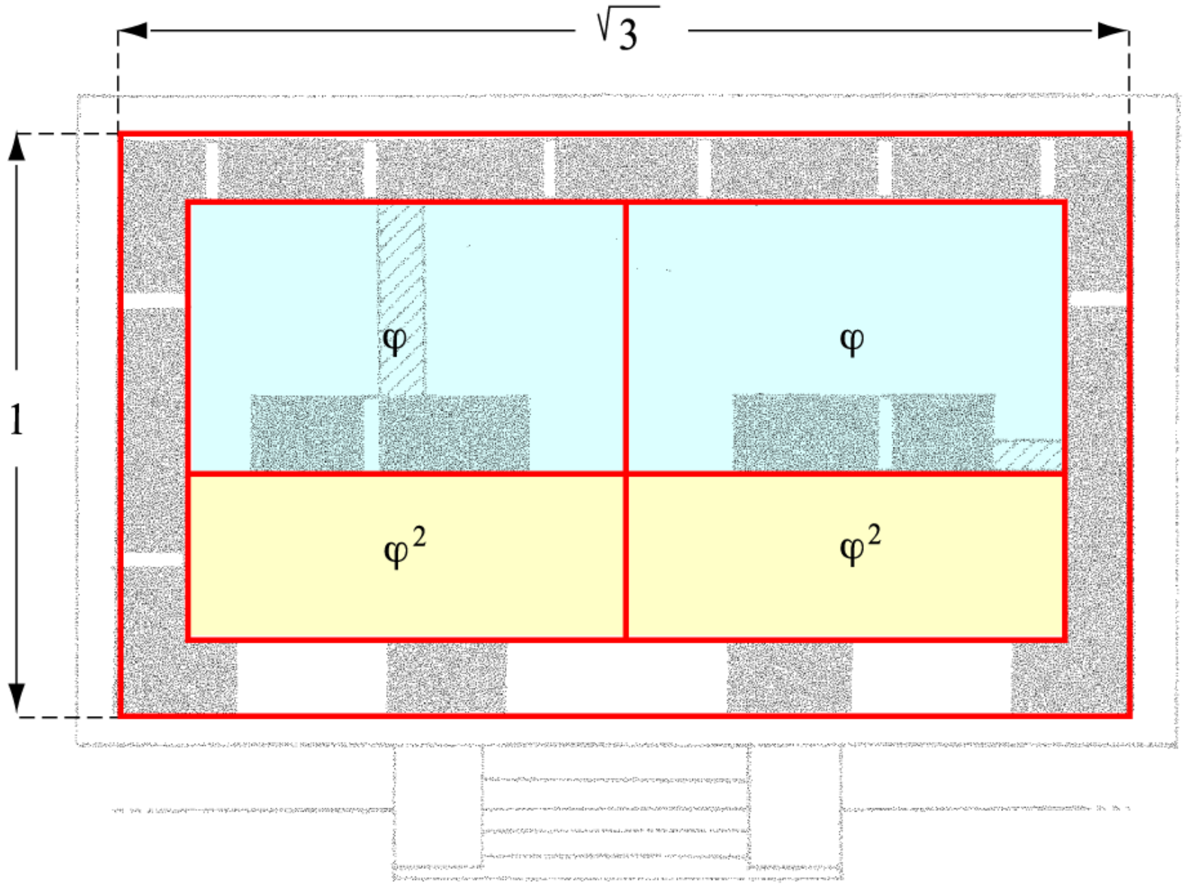
Plan of the Raised Platform and Staircase of Temple II,
Palenque, Chiapas, Mexico
Measured Drawing by G. F. Andrews (1974)



The raised platform of Temple II is inscribed by a phi rectangle (shaded blue). If the width of the staircase is one, than the width of the balustrade is square root of two. If the width of the balustrade is one, the width of the platform is two.

Figure 81

Plan of Temple IV, Palenque, Chiapas, Mexico
Measured Drawing by G. F. Andrews (1974)

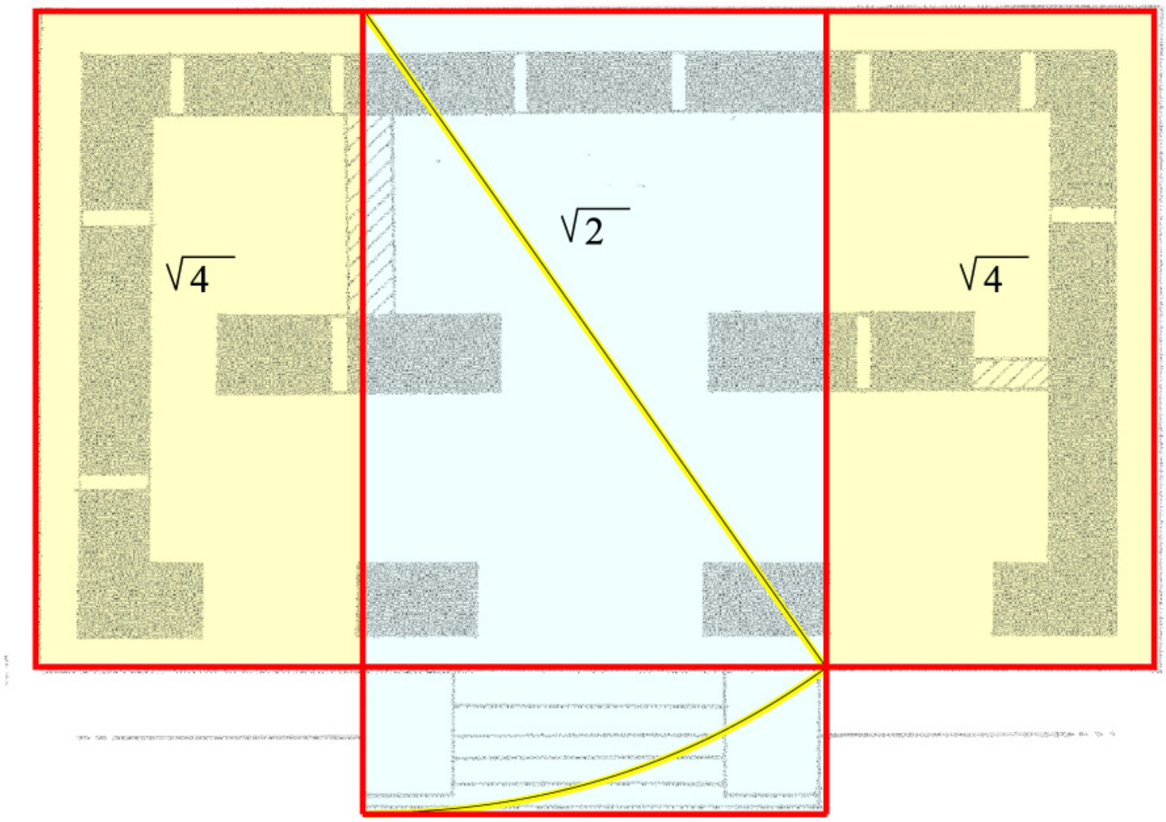


Temple IV also shares identical proportions with those described for Temples II and XVIII on the previous pages. The exterior walls are inscribed by a root three rectangle, and the interior space inscribes a root four rectangle.

As in Temples II and XVIII, interior vault support walls further subdivide the double squares of the interior space into two phi-squared rectangles (front gallery, shaded yellow) and the remaining space into two phi rectangles (shaded blue), as per Formula Square-2.

Figure 82

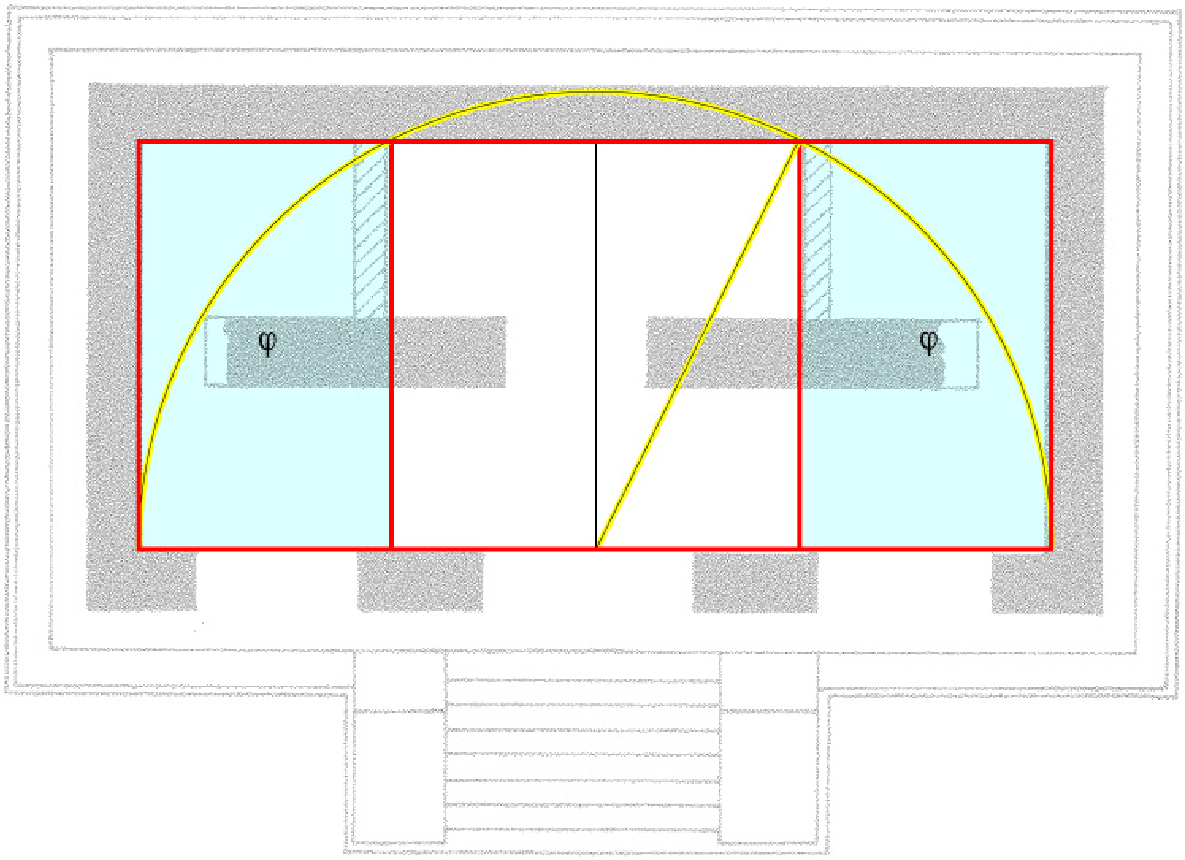
Plan of the Raised Platform and Staircase of Temple IV,
Palenque, Chiapas, Mexico
Measured Drawing by G. F. Andrews (1974)



Where the balustrade is attached to the raised platform of Temple IV, straight lines were probably drawn across the width of the platform, creating a root two rectangle, the diagonal of which is demarcated with a yellow highlighted line. This diagonal may have been used as the radius of an arc to determine the width of the balustrade and to produce a root three rectangle (shaded blue). The remaining rectangles to either side of the balustrade form root four rectangles (shaded yellow).

Figure 83

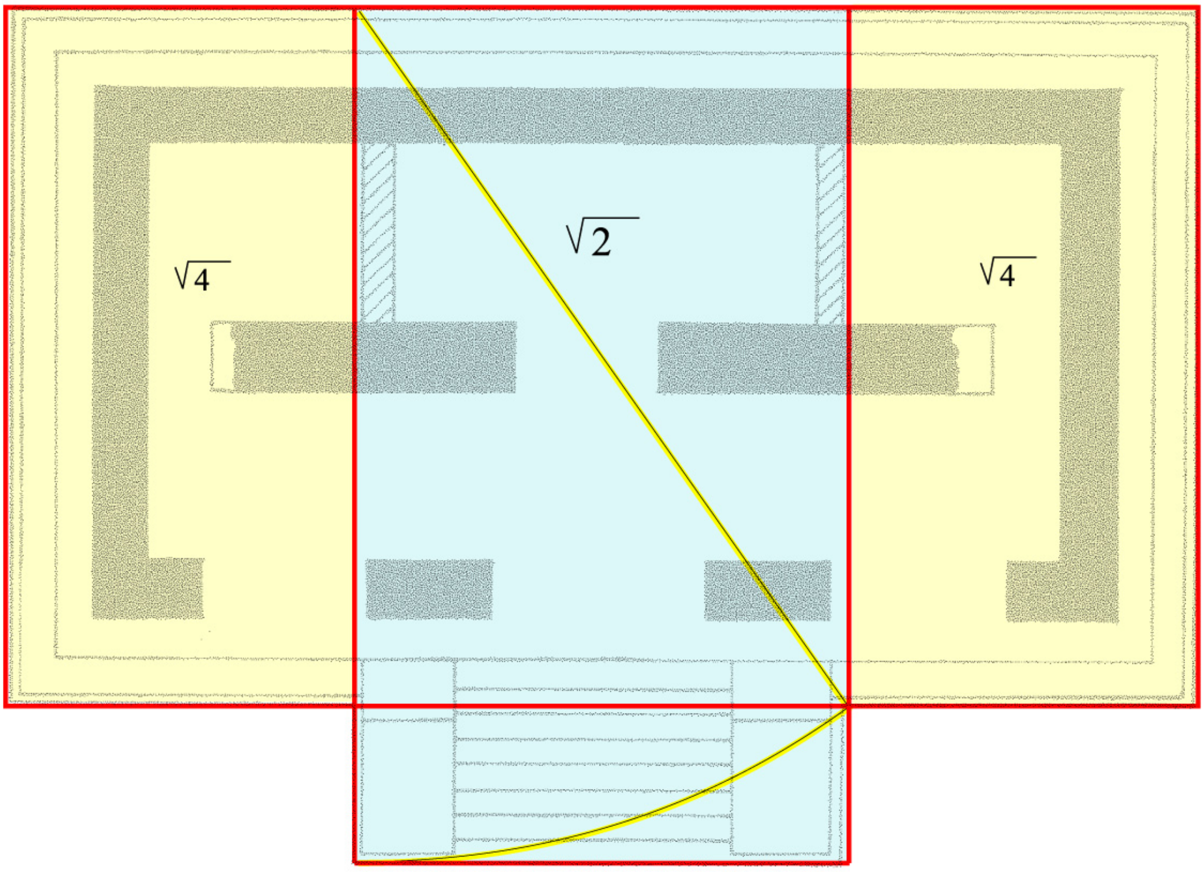
Plan of Temple VIII, Palenque, Chiapas, Mexico
Measured Drawing by G. F. Andrews (1974)



The interior space of Temple VIII is inscribed by a root five rectangle that is subdivided into a square (white) that is defined by the placement of the curtain walls of the rear gallery and two phi rectangles (shaded blue), as per Formula Square Root of Five-3.

Figure 84

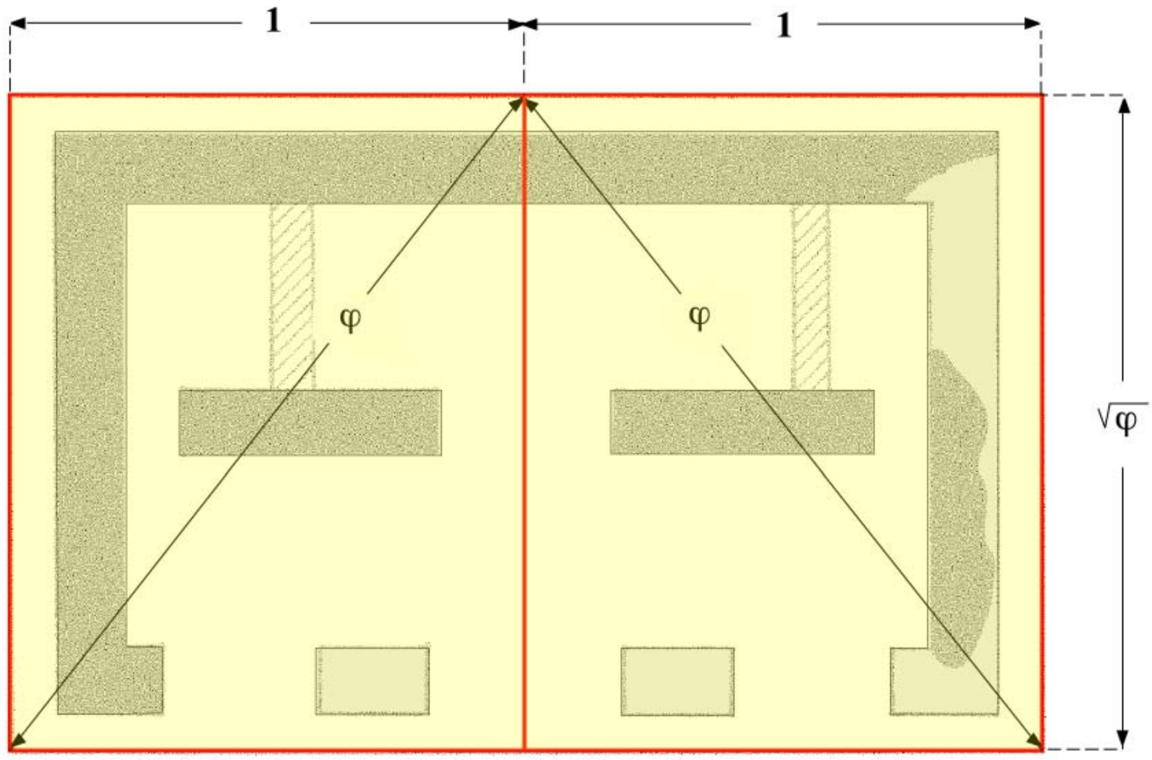
Plan of the Raised Platform and Staircase of Temple VIII,
Palenque, Chiapas, Mexico
Measured Drawing by G. F. Andrews (1974)



The geometry of the platform of Temple VIII is identical to that of the platform of Temple IV. Where the balustrade is attached to the raised platform of Temple IV, straight lines were probably drawn across the width of the platform, creating a square root of two rectangle, the diagonal of which is demarcated with a yellow highlighted line. This diagonal may have been used as the radius of an arc to determine the width of the balustrade and to produce a square root of three rectangle (shaded blue). The remaining rectangles to either side of the balustrade form square root of four rectangles (shaded yellow).

Figure 85

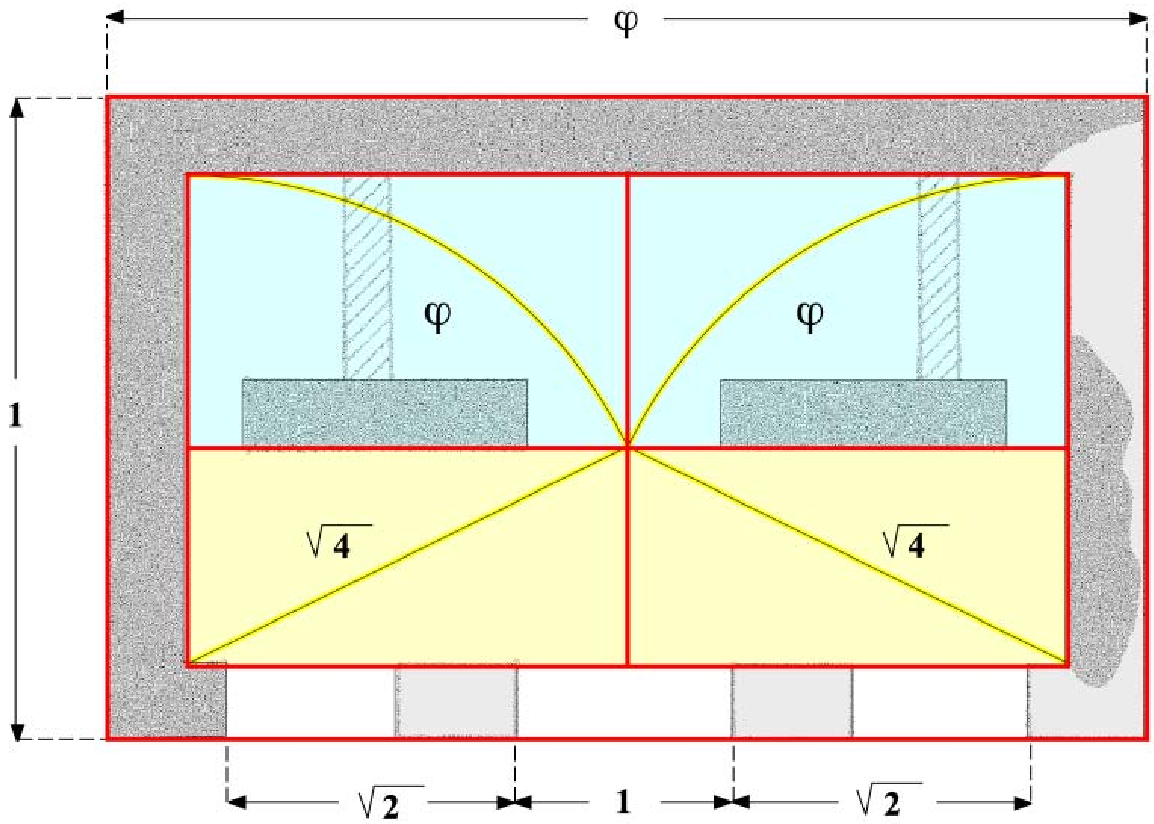
Plan of the Raised Platform of Temple XII,
Palenque, Chiapas, Mexico
Measured Drawing by G. F. Andrews (1974)



The raised platform of Temple XII is inscribed by a rectangle with a length of two to a width of root phi.

Figure 86

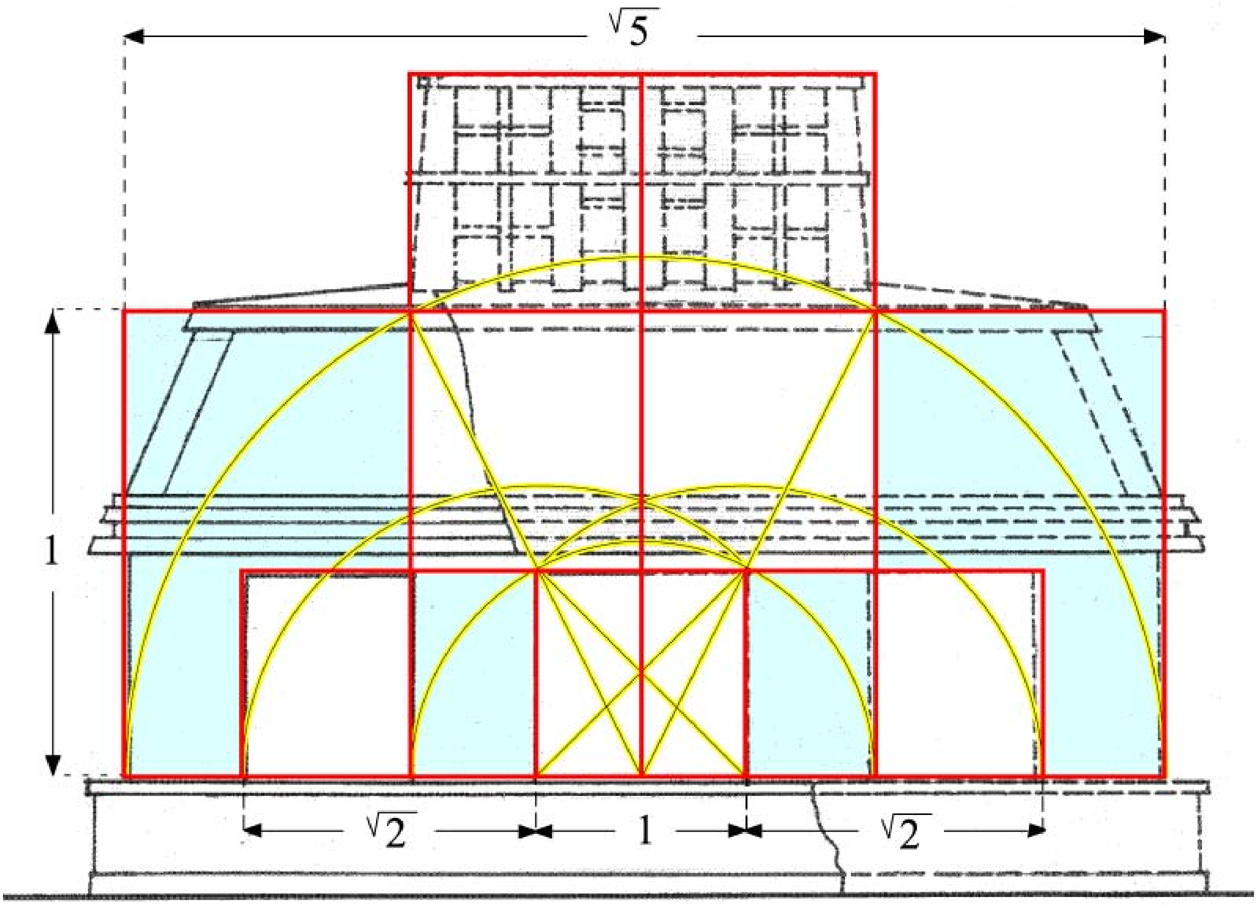
Plan of Temple XII,
Palenque, Chiapas, Mexico
Measured Drawing by G. F. Andrews (1974)



The exterior walls of Temple XII are inscribed by a phi rectangle. The front gallery is inscribed by two root four rectangles, the diagonals of which are the radii for the arcs that define the two phi rectangles that inscribe the rear gallery and principle roof support piers.
If the width of the central entrance is one, then from the central entrance to the outer edges of the lateral entrances is root two.

Figure 87

Elevation of Temple XII,
Palenque, Chiapas, Mexico
Measured Drawing by G. F. Andrews (1974)



The central entrance is inscribed by a square (white), and the piers that separate the three doorways are inscribed by phi rectangles (shaded blue). Together, they form a root five rectangle, as per Formula Square Root of Five-3. The width of the central entrance and piers is equal to the height of the temple to the top of the roof moulding, forming a larger square. To either side of this larger square are phi rectangles (also shaded blue). Together, these also form a root five rectangle, as per Formula Square Root of Five-3.

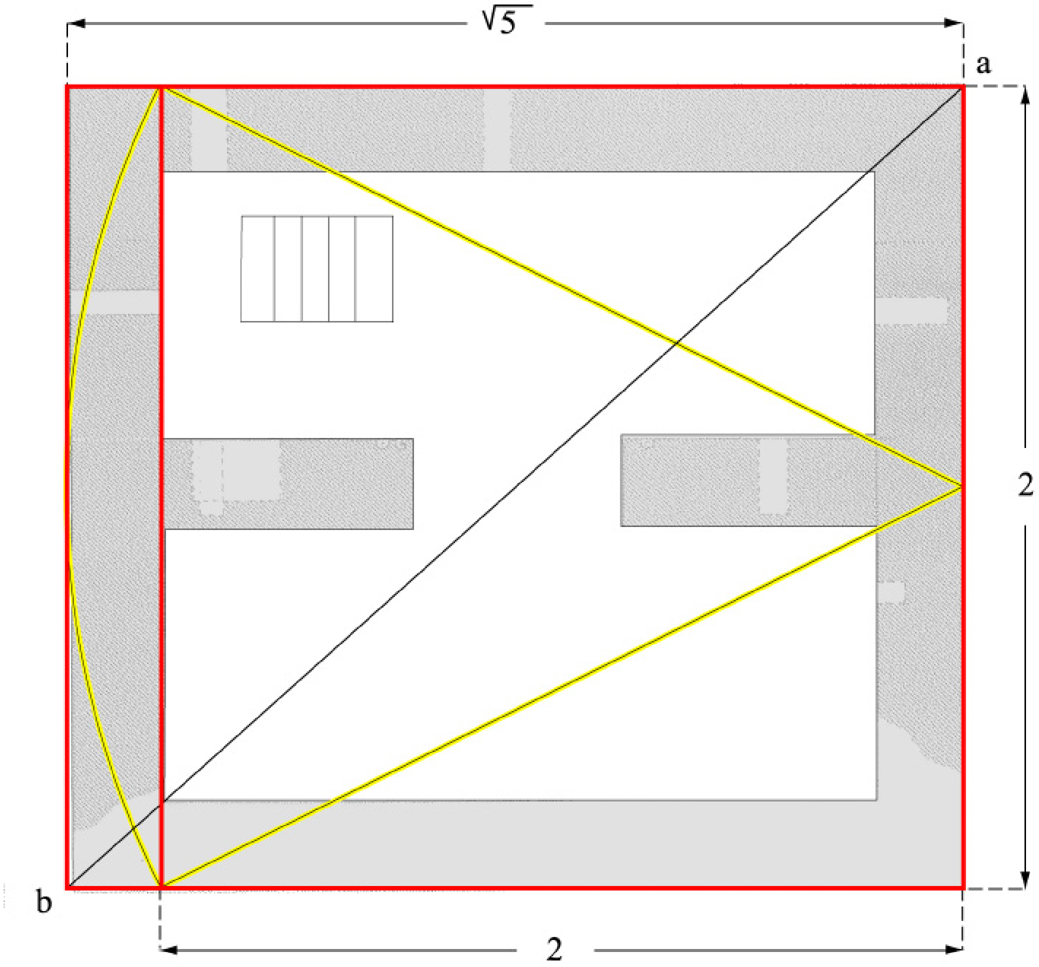
The radii for the half-circle arcs that define the phi rectangles are the diagonal lines that extend from the center base of the squares to their upper corners. The forty-five degree diagonals of the square central entrance are the radii for the arcs that determine the outer lower corners of the lateral entrances.

Note that the height of the roof to the length of the temple is inscribed by a square root of five rectangle that subdivides identically to the square root of five rectangle created by the central doorway and its piers.

Though the base of the roof comb is intact, its height is estimated here.

Figure 88

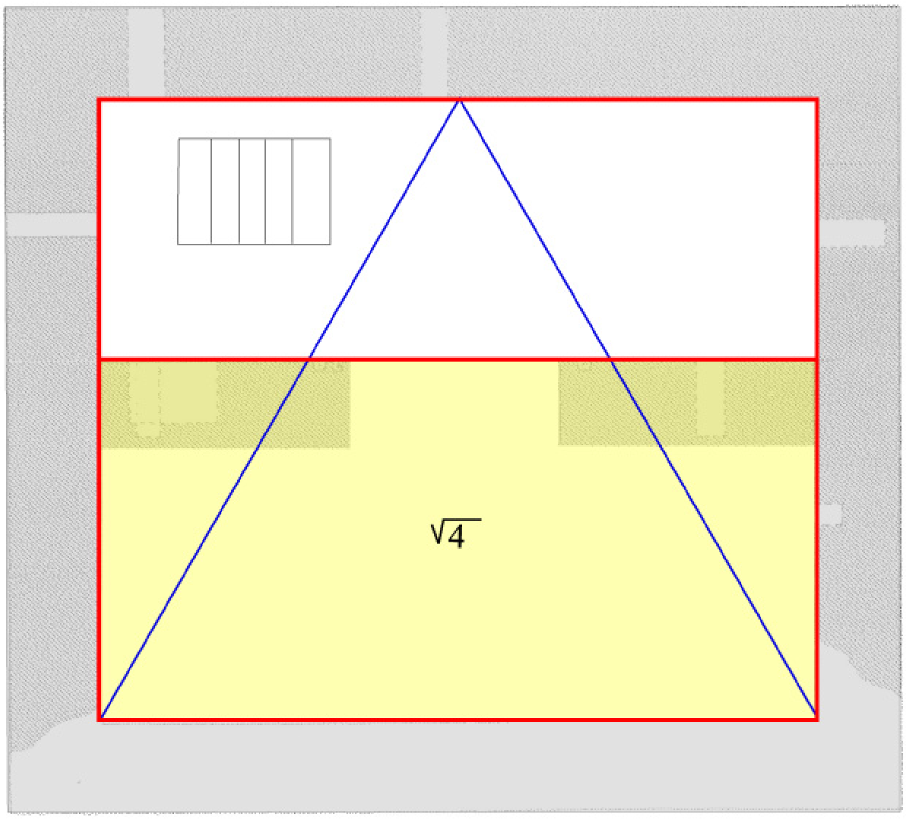
Plan of the Upper Floor of the Temple of the Jaguars,
Palenque, Chiapas, Mexico
Measured Drawing by Merle G. Robertson (1991)



The plan of the upper floor of the Temple of the Jaguars is inscribed by a rectangle with a width of two and a length of root five. The diagonal of this rectangle (a,b) is three. From the outer wall to the opposite inner wall is two, and forms a square two units to a side. The diagonals drawn from the horizontal center of this square to the opposite corners are the radii for the arc that determines the width of the walls.

Figure 89

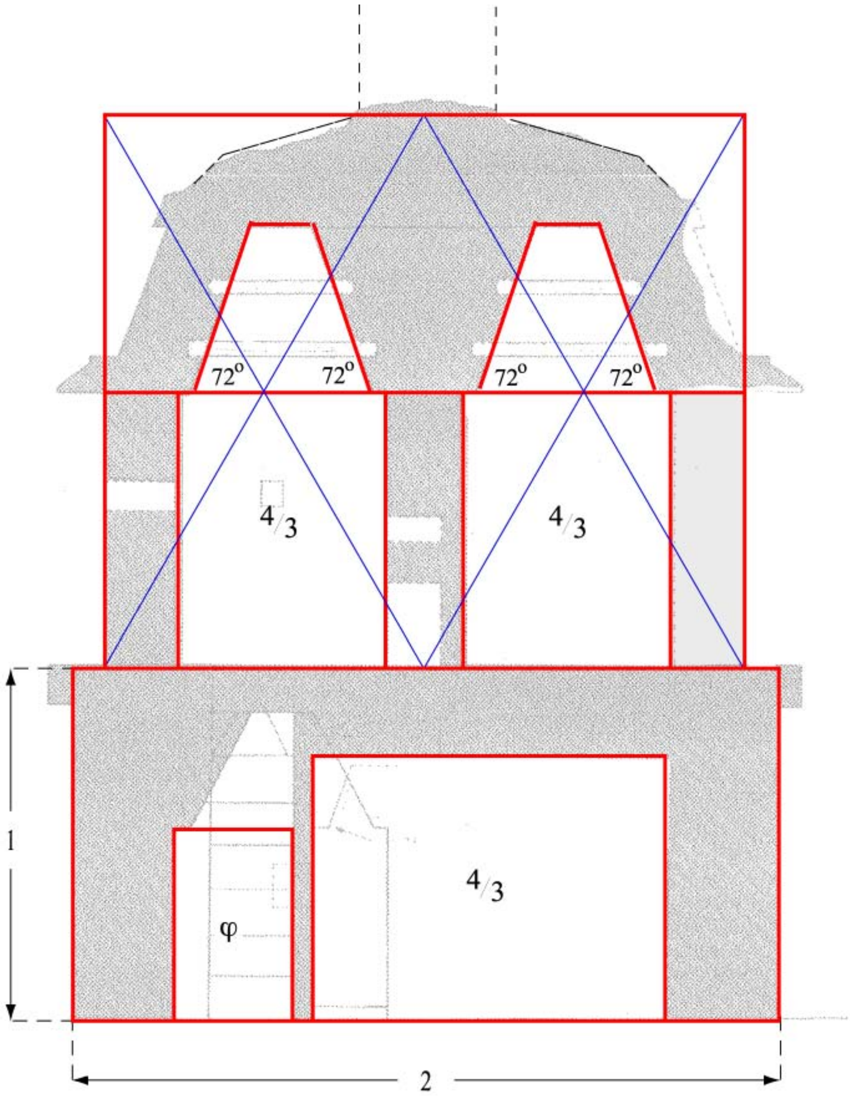
Plan of the Upper Floor of the Temple of the Jaguars-b,
Palenque, Chiapas, Mexico
Measured Drawing by Merle G. Robertson (1991)



The interior walls of the plan of the upper floor of the Temple of the Jaguars are inscribed by a two over root three rectangle that inscribes an equilateral triangle (blue lines). The central support piers and the front gallery are inscribed by a root four rectangle (shaded yellow). Because the front and rear galleries are of equal size, a root four rectangle would also inscribe the rear gallery and central support piers.

Figure 90

Section of the Temple of the Jaguars,
Palenque, Chiapas, Mexico
Measured Drawing by Merle G. Robertson (1991)

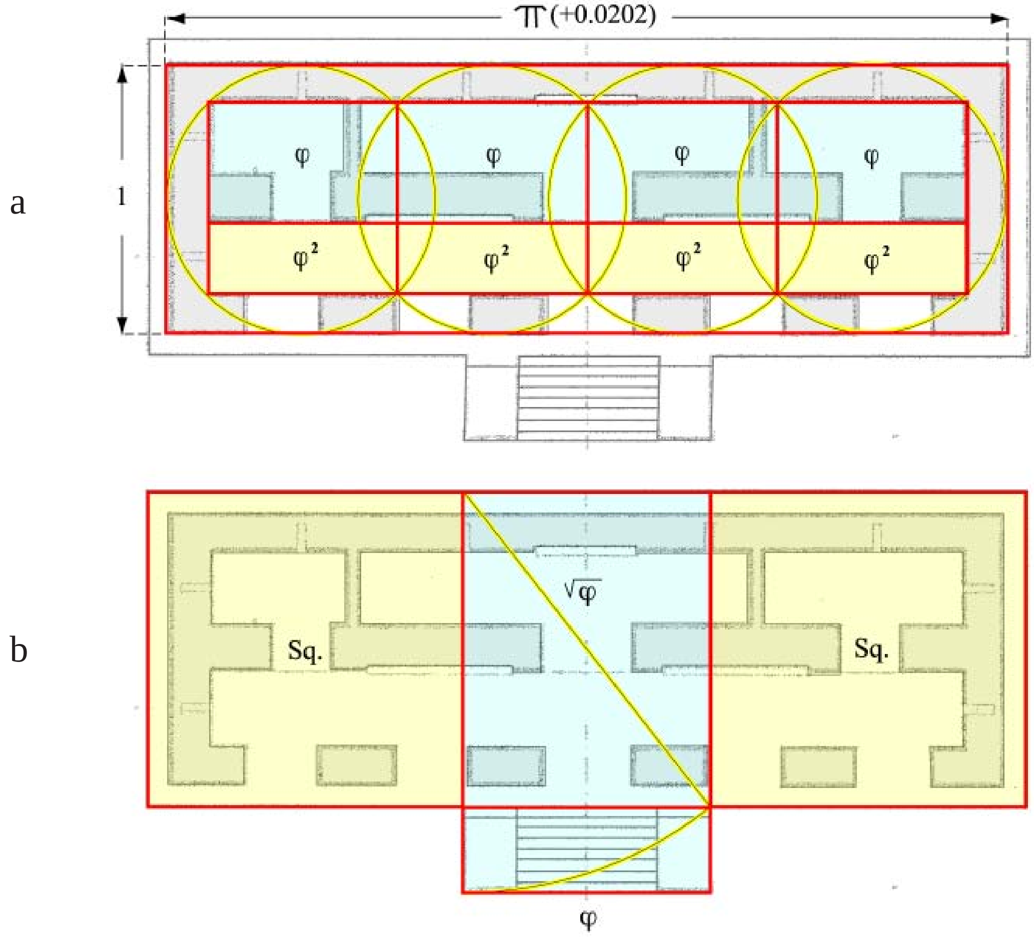


The section of the lower story of the Temple of the Jaguars is inscribed by a root four rectangle. The section of the interior room is inscribed by a Pythagorean 3,4,5 rectangle, and the doorway is inscribed by a phi rectangle.

The upper story, including the roof, is inscribed by a two over root three rectangle that is divided in half at the spring line. The interior rooms are inscribed by Pythagorean rectangles, and the angles of the vaults are seventy-two degrees (the angle of a phi triangle).

Figure 91

Plan of the Temple of the Inscriptions,
Palenque, Chiapas, Mexico
Measured Drawing by G. F. Andrews (1974)

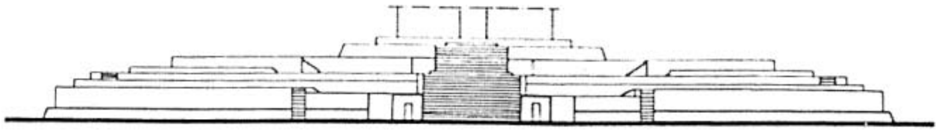


The interior walls of the Temple of the Inscriptions are inscribed by a rectangle with a ratio of one to four that is divided into four squares. The front gallery subdivides these squares into phi squared rectangles (shaded yellow) and phi rectangles (shaded blue), as per Formula Square-2. Four circles inscribe the four squares and define the width of the walls. Whether intentional or not, this formula produces a width to length ratio of one to pi minus 0.0202.

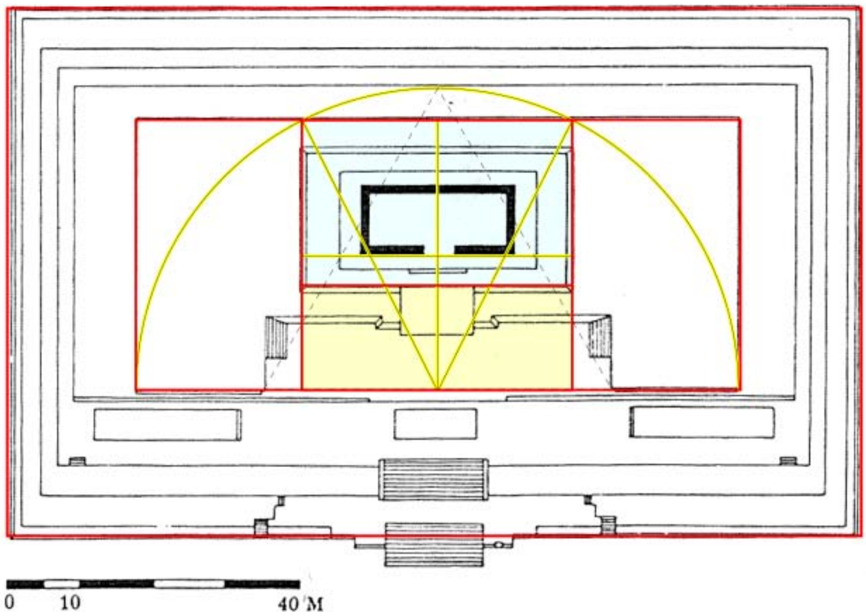
Where the balustrade is attached to the raised platform (Figure b), straight lines are drawn across the width of the platform, creating a root phi rectangle, the diagonal of which is demarcated with a yellow highlighted line. This diagonal is the radius of an arc that determines the width of the balustrade and creates a phi rectangle (shaded blue). The remaining rectangles to either side of the balustrade are squares (shaded yellow).

Figure 92

Plan of of Structure A-1, Altar de los Sacrificios, Peten, Guatemala
Measured Drawing by A.L.Smith (Marquina 1951)



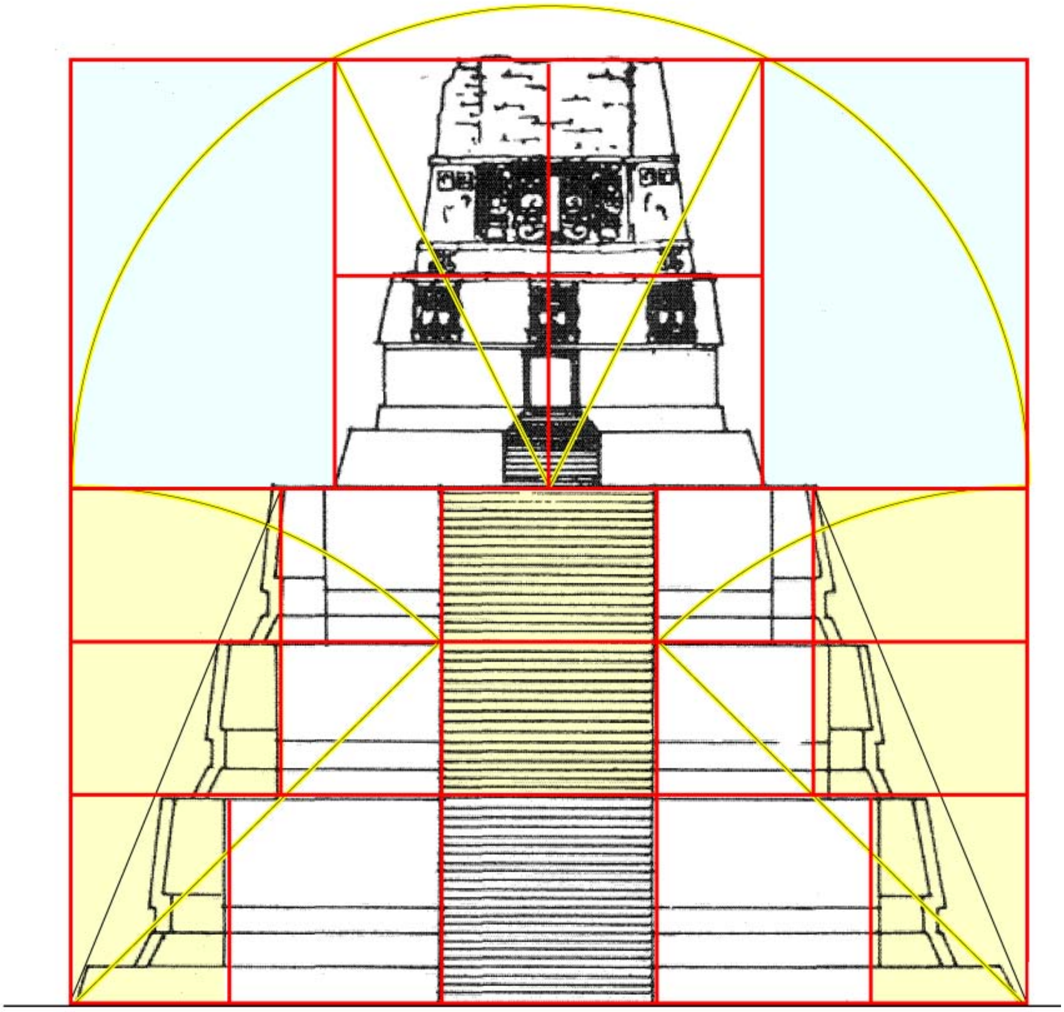
A



The lower platform of this enormous structure is inscribed by a phi rectangle. The uppermost level of the principle platform is inscribed by a square root of five rectangle whose width is determined by the smaller platform and whose length is used to define the centered square. This square is further subdivided into a phi rectangle (shaded blue) and a phi-squared rectangle (shaded yellow). This square is also quartered (yellow highlighted lines), and the front edge of the temple rests on the horizontal center of the square. The yellow highlighted diagonals from the center base of the square are the radii used to determine the arc which then determines the length of the square root of five rectangle.

Figure 93

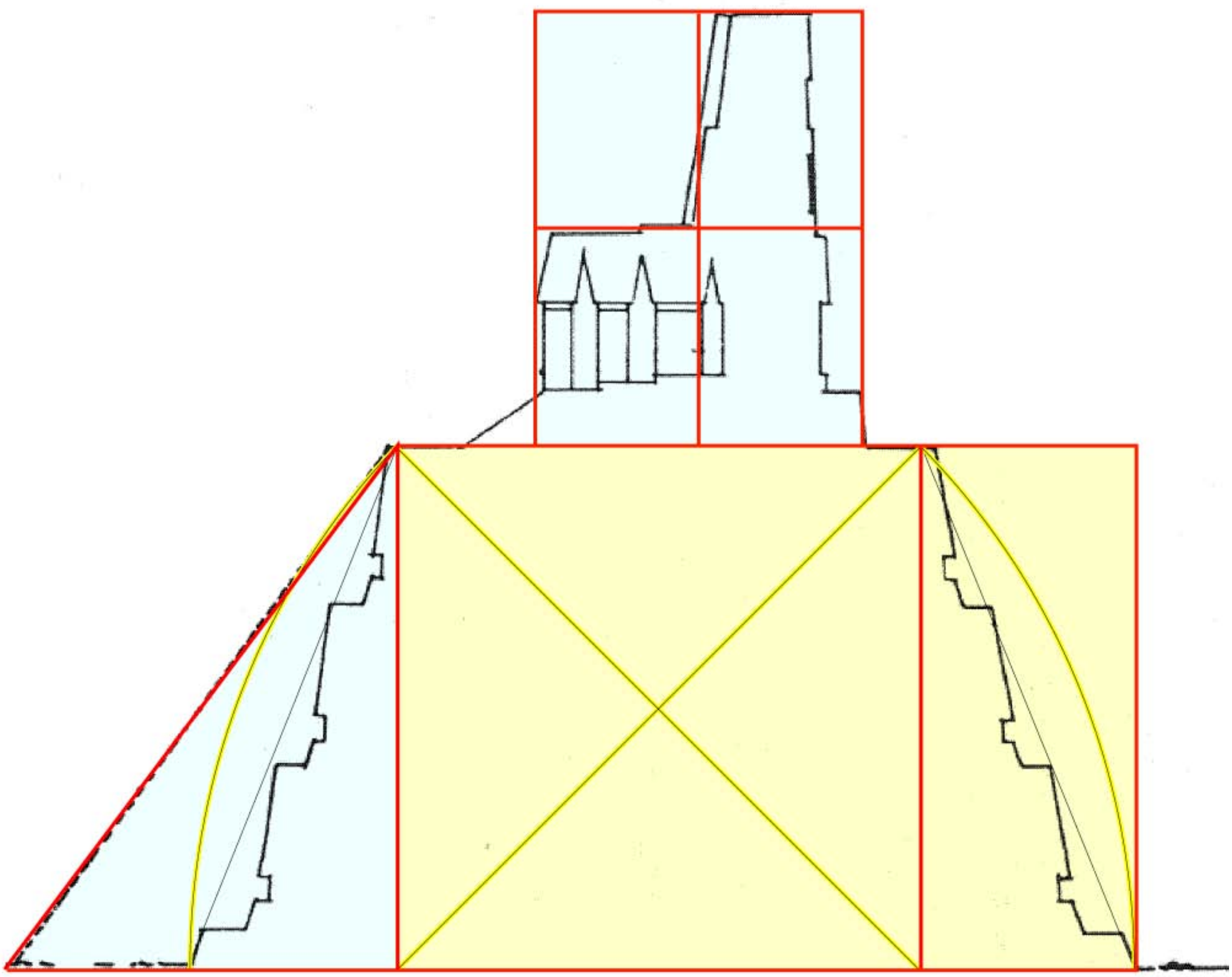
Elevation of Temple I, Tikal, Guatemala
Measured Drawing by J.A. Gomez R. (Marquina 1951)



Inscribed on either side of the staircase are square root of two rectangles that are subdivided into squares (the vertical risers of the three terraces in white). The remaining spaces and inset corners are inscribed by square root of two rectangles (shaded yellow). The staircase, too, is subdivided into a square root of two rectangle and a square. The yellow highlighted diagonals are forty-five degrees and are the radii for the arcs that define the height of the upper terrace.
The temple proper is inscribed by a square (in white). The horizontal center of the square divides the roof from the roof comb. The diagonals from the center base of this square to its upper corners are the radii for the arc (shaded blue) that defines the phi rectangles to either side of the temple. The phi rectangles, along with the central square, form a square root of five rectangle subdivided via Formula Square Root of Five-3.

Figure 94

Section of Temple I, Tikal, Guatemala
Measured Drawing by J.A. Gomez R. (Marquina 1951)

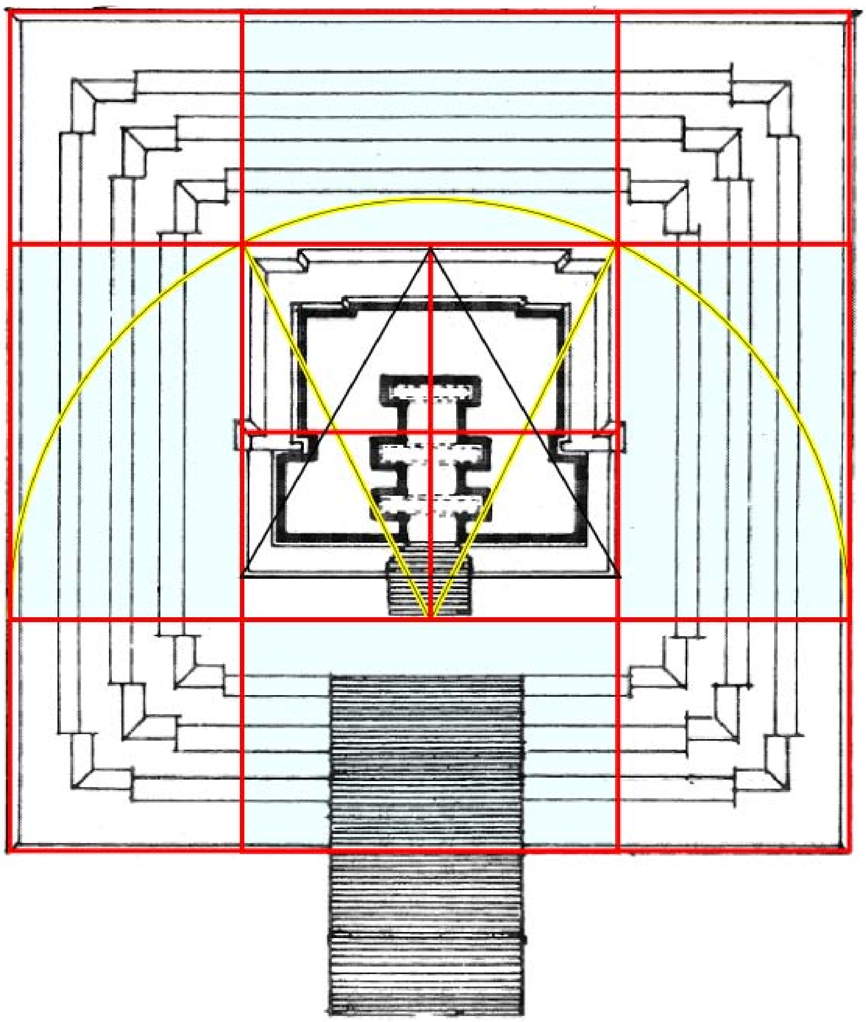


The width of the upper platform of the three-tiered base of Temple I is equal to its height, forming a square whose diagonals are shown in yellow highlighted lines. These are the radii for the arcs that form square root of two rectangles to either side of the square (shaded yellow on the right side).

The section of the temple is inscribed by a Pythagorean 3,4,5 rectangle (shaded blue) that is quartered. The triangle formed from where the top of the staircase meets the upper platform to the base of the pyramid is a Pythagorean 3,4,5 triangle (also shaded blue). If the base of this triangle is three, its height is four and the slope of the stairs is five.

Figure 95

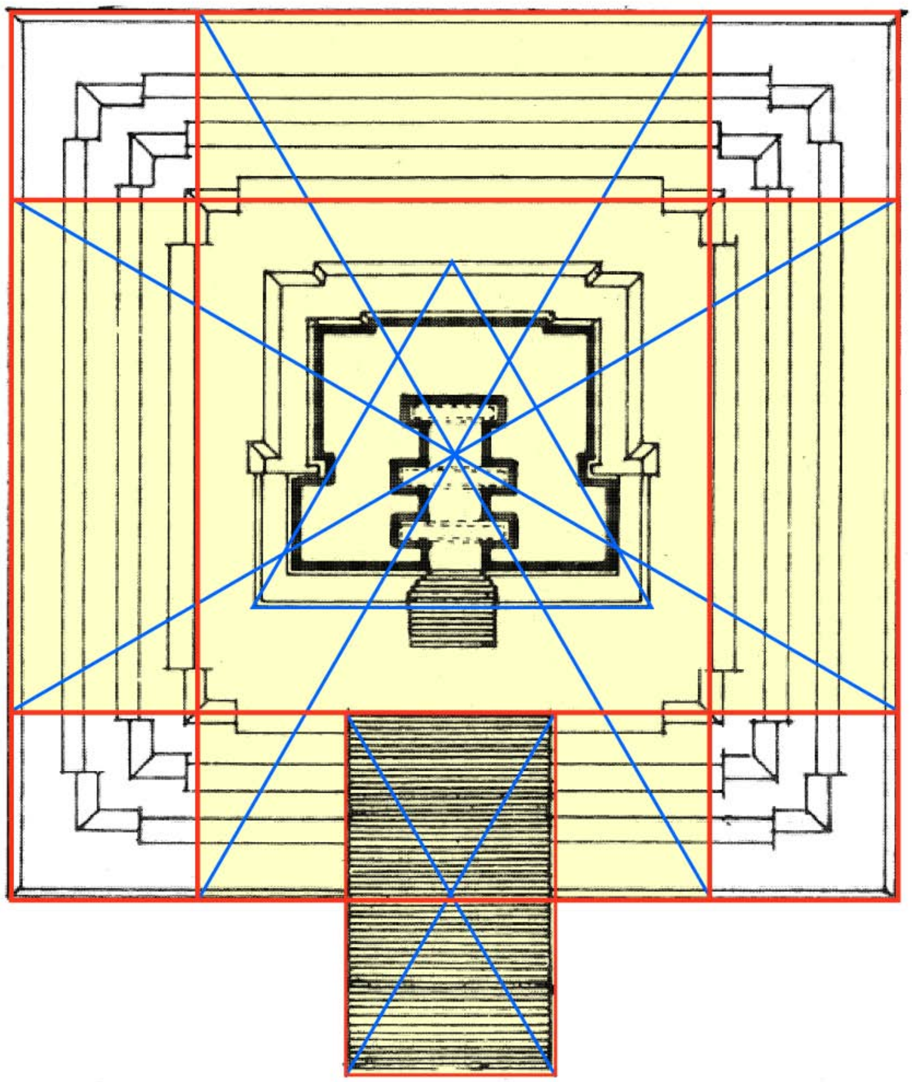
Plan of Temple I (a), Tikal, Guatemala
Measured Drawing by J.A. Gomez R (Marquina 1951)



The plan of the pyramidal base of Temple I and the plan of the temple proper, including its staircase, are inscribed by squares. The square that inscribes the temple is quartered and the diagonals drawn from the lower center of this square to its upper corners are radii foe the arc that defines the phi rectangles (shaded blue) to either side of it. This process would be repeated at a ninety-degree angle to produce the phi rectangles to the front and rear of the temple.

Figure 96

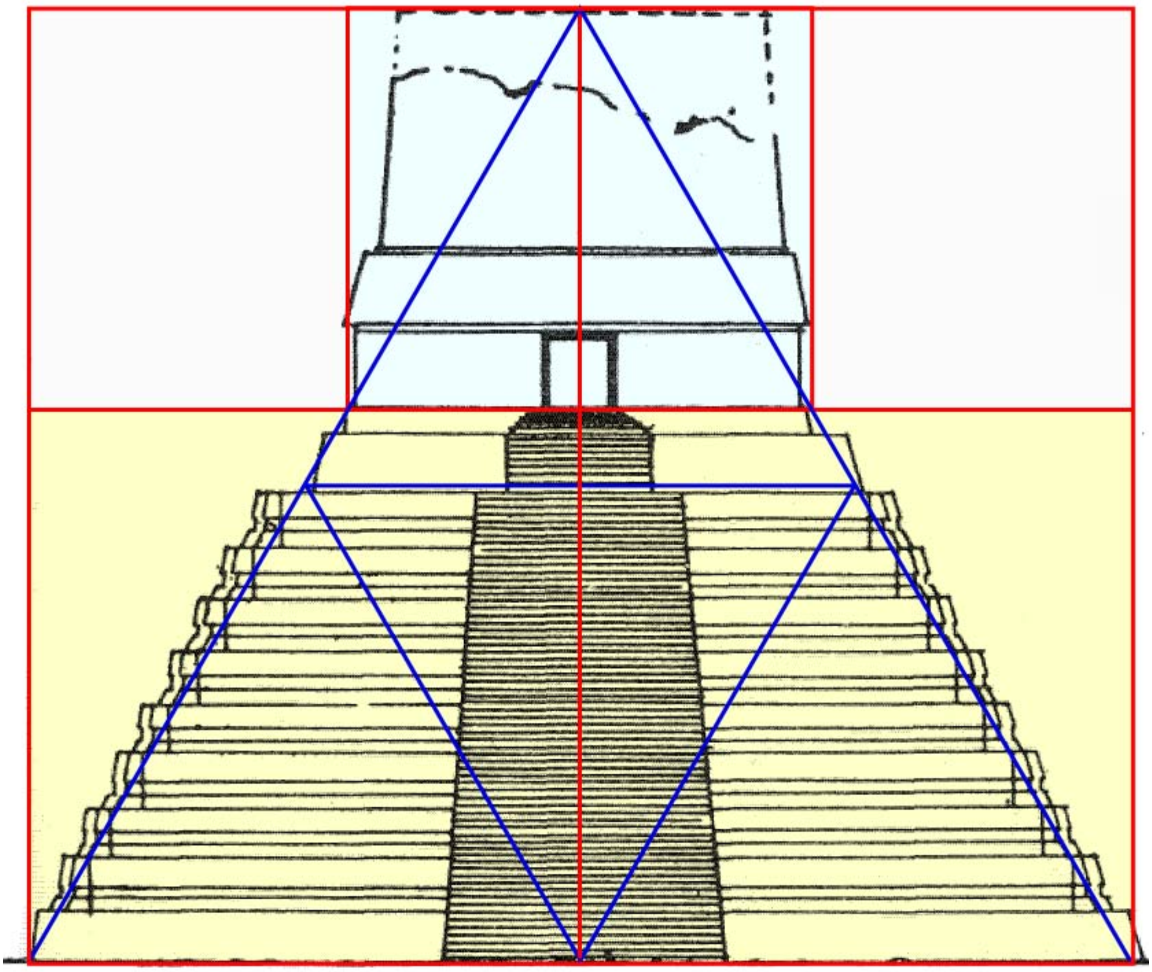
Plan of Temple I (b), Tikal, Guatemala
Measured Drawing by J.A. Gomez R. (Marquina 1951)



The plan of the temple proper, not including the staircase, inscribes an equilateral triangle (blue lines). Straight lines drawn from the edges of the square upper platform to the edges of the base of the pyramid form square root of three rectangles (shaded yellow, with their diagonals, drawn in blue lines, forming equilateral triangles). The plan of the principle staircase is also inscribed by a square root of three rectangle whose diagonals (also in blue lines) form equilateral triangles.

Figure 97

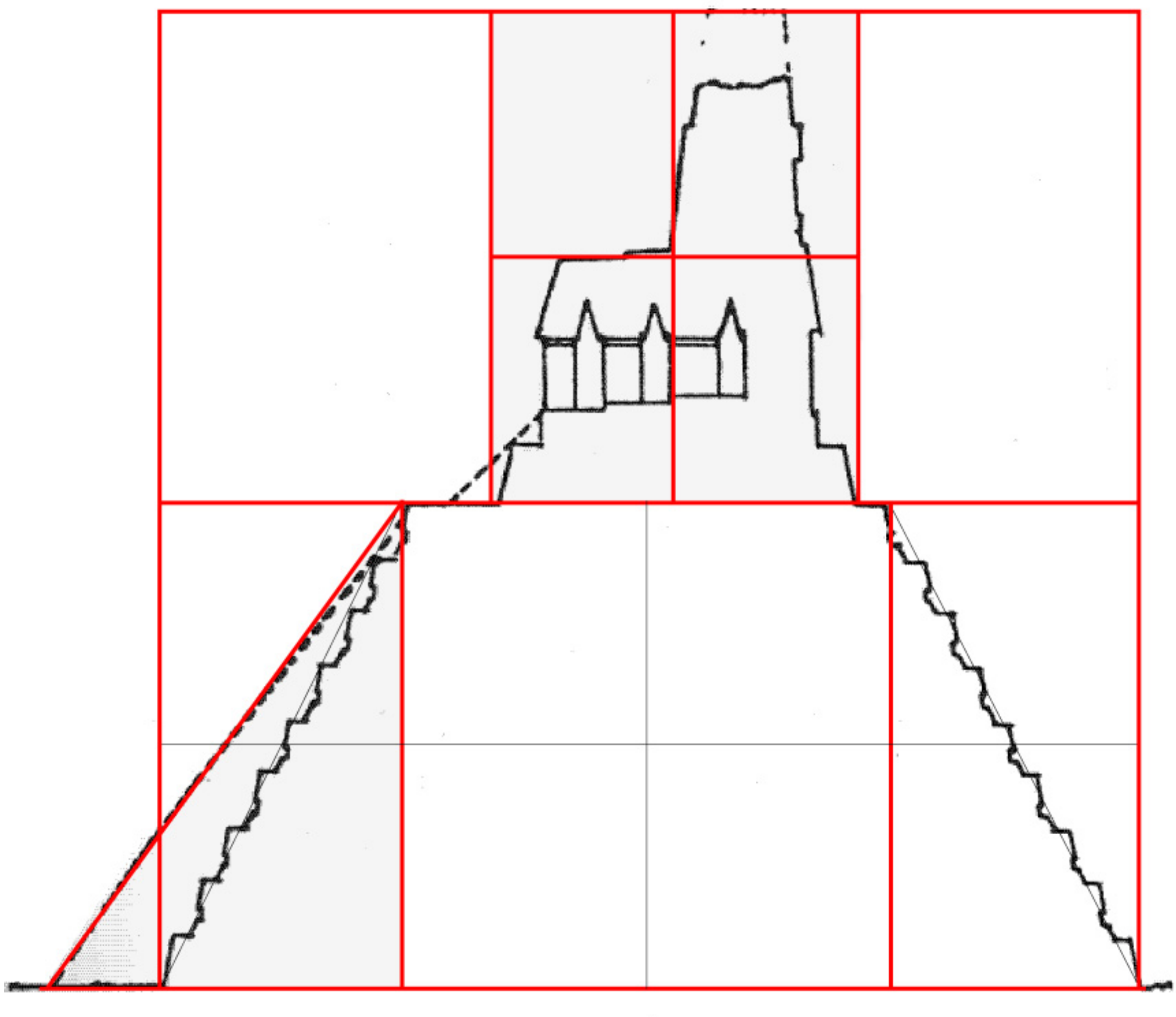
Front Elevation of the Temple II, Tikal, Guatemala
Measured Drawing by J.A. Gomez R. (Marquina 1951)



The front elevation of Temple II is inscribed by a virtually perfect two over square root of three rectangle that is subdivided via Formula Two Over Square Root of Three-e. The width of the pyramid, to the base of the temple, is inscribed by a square root of four rectangle (shaded yellow). The temple is inscribed by a two over square root of three rectangle (shaded blue), and the remaining space to either side of the temple is inscribed by square root of phi rectangles (shaded gray).
An equilateral triangle is inscribed in the virtually perfect two over square root of three rectangle that defines the slope of the pyramid, and this equilateral triangle is divided in half at the top of the pyramid.

Figure 98

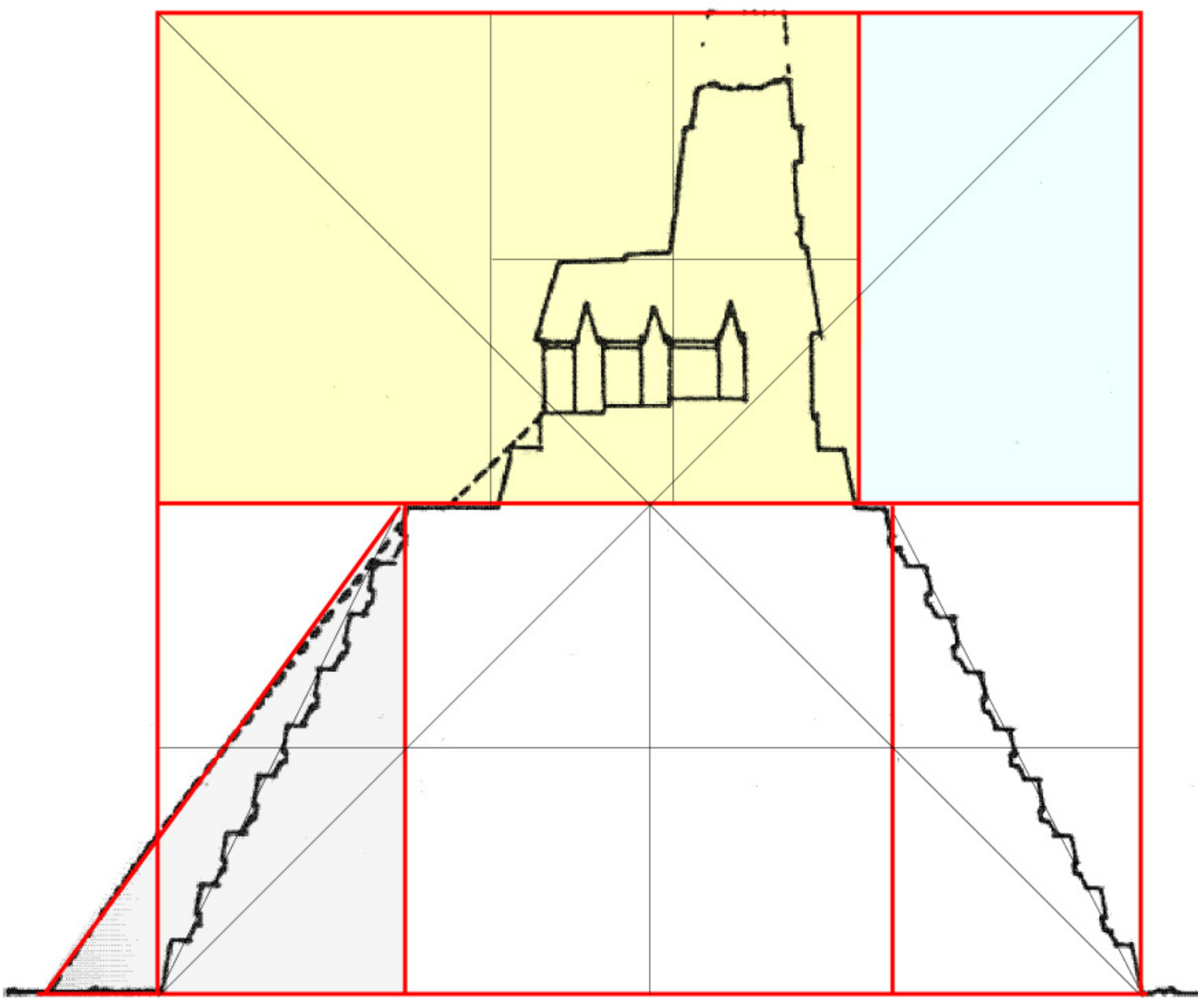
Section of Temple II (a), Tikal, Guatemala
Measured Drawing by J.A. Gomez R. (Marquina 1951)



The side section of the pyramidal base of Temple II is inscribed by a square root of four rectangle subdivided by a central square that is equal to the width of the top of the pyramid and two smaller square root of four rectangles to either side of this central square, the diagonals of which (fine black lines) define the angle of the slope of the pyramid.
Identically to Temple I, the section of the temple proper is inscribed by a quartered Pythagorean 3,4,5, rectangle, and the staircase is inscribed by a Pythagorean 3,4,5, triangle (shaded grey).

Figure 99

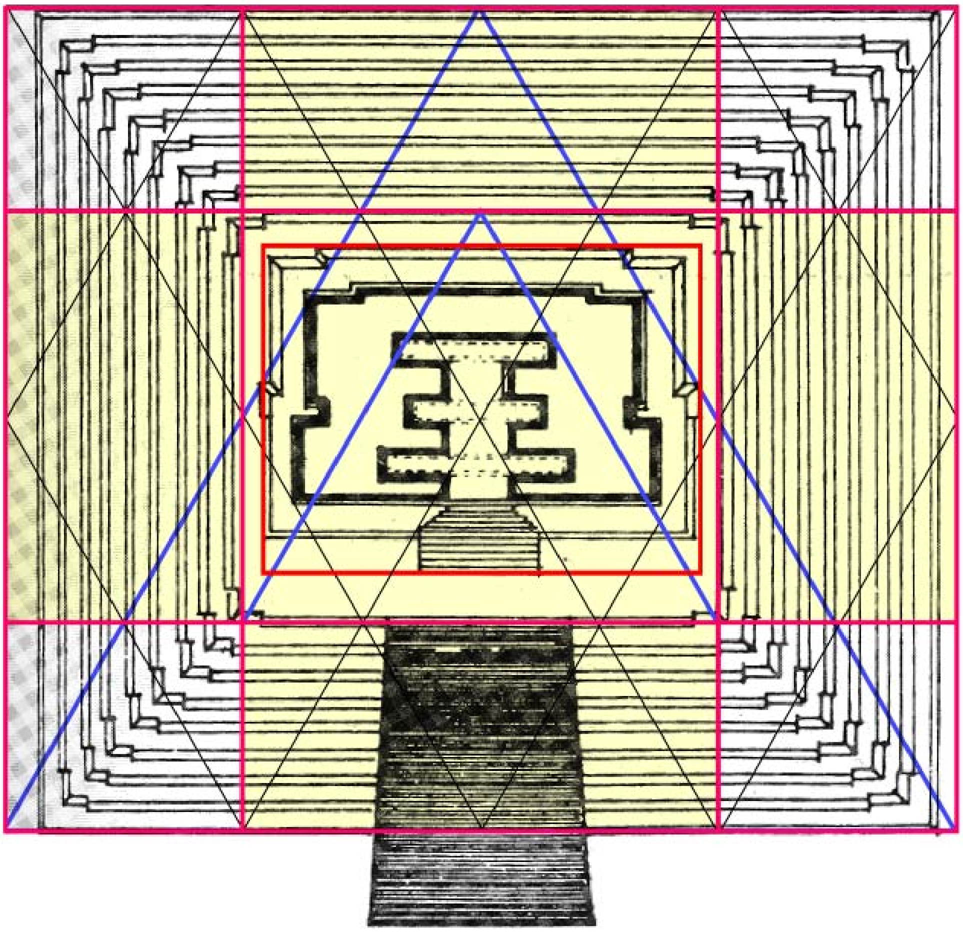
Section of Temple II (b), Tikal, Guatemala
Measured Drawing by J.A. Gomez R (Marquina 1951)



The section of Temple II may also be inscribed by a virtually perfect square that is divided horizontaly in half at the top of the pyramid. As noted in the previous diagram, the pyramid may be inscribed by a square root of four rectangle that is subdivided by a central square that is equal to the width of the top of the pyramid and two smaller square root of four rectangles to either side of this central square.
The upper half of this diagram is inscribed by a virtually perfect square root of four rectangle that is subdivided at the rear of the temple by a square root of two rectangle (shaded blue) and a square root of three rectangle (shaded yellow), as per the Formula Square Root of Four-4.

Figure 100

Section of Temple II, Tikal, Guatemala
Measured Drawing by J.A. Gomez R (Marquina 1951)



Because the front elevation of Temple II is inscribed by a two over square root of three rectangle and the side section is inscribed by a square, the plan of Temple II must also be inscribed by a two over square root of two rectangle of the same size as the front elevation. The measured drawing here is in error in that its length was drawn a bit short. I have corrected this error with the diagram that overlays the measured drawing.

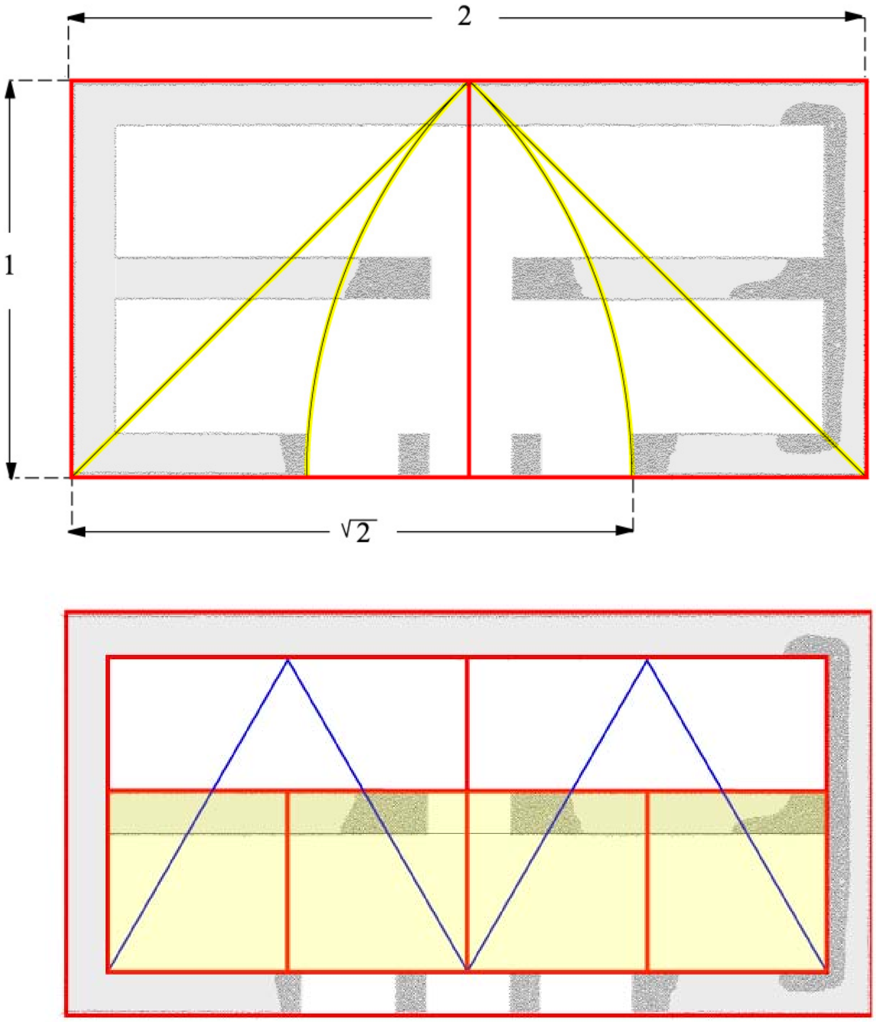
The base of the pyramid and the platform at the top of the pyramid are inscribed by two over square root of three rectangles, which in turn, are inscribed by equilateral triangles (blue lines).

The rectangles to either side of the top of the pyramid are square root of three rectangles (shaded yellow).

The temple proper, along with its staircase, is inscribed by a Pythagorean 3,4,5 rectangle.

Figure 101

Plan of Structure 2B2, Chichen Itza, Yucatan, Mexico
Measured Drawing by Karl Ruppert (1952)

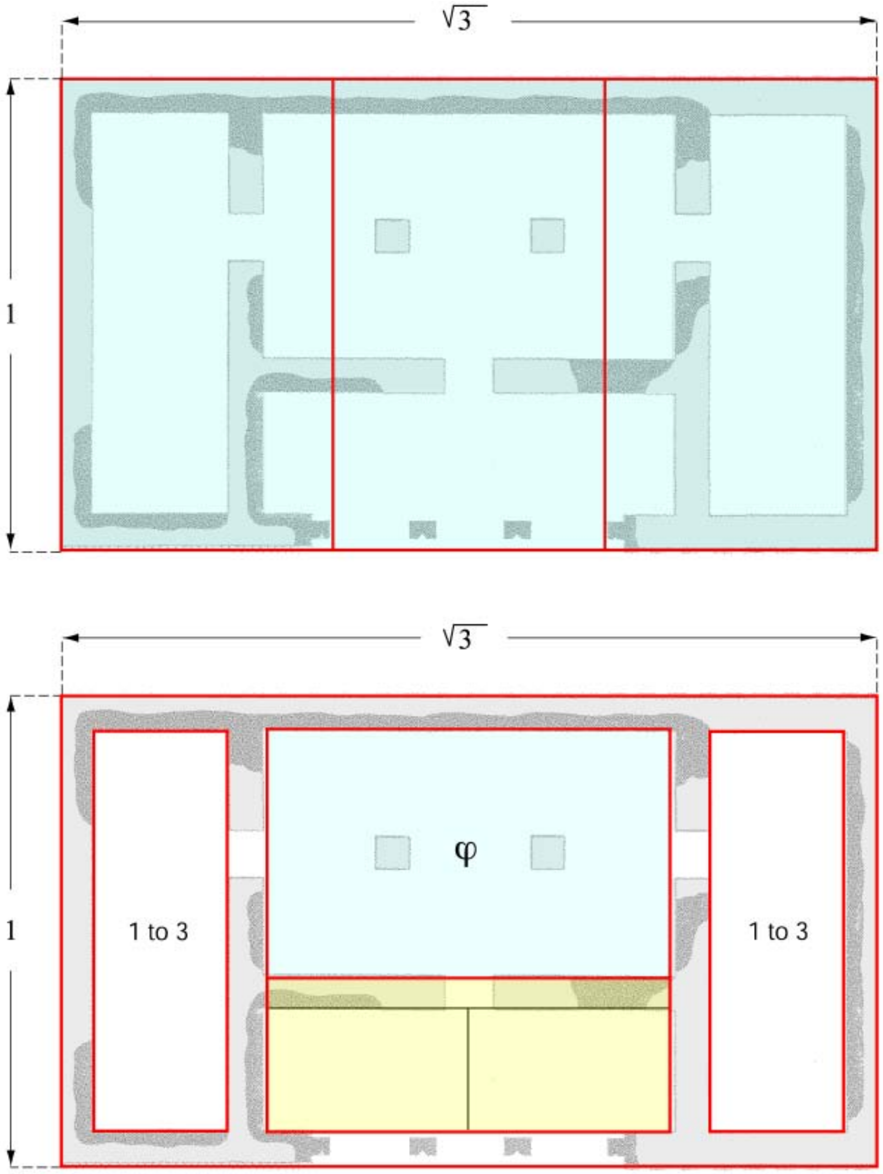


b

The proportions of the interior and exterior walls of this structure are identical to those of Houses G and H and Temple XVIIIa at Palenque. The exterior walls of Structure 2B2 are inscribed by a square root of four rectangle (one to two). The width of the entrance was probably determined by stretching a cord from the front corners of this rectangle to its center top (a forty-five degree angle) and then swinging arcs to the base of the rectangle (highlighted yellow lines). This produces a one to square root of two ratio between the entrance and the width of the structure.
The interior space of this structure (b) is subdivided into two, two over square root of three rectangles, each of which inscribe equilateral triangles (blue lines). The vault support walls further subdivide the interior space into two square root of four rectangles (shaded yellow).

Figure 102

Plan of Structure 5B21, Chichen Itza, Yucatan, Mexico
Measured Drawing by Karl Ruppert (1952)

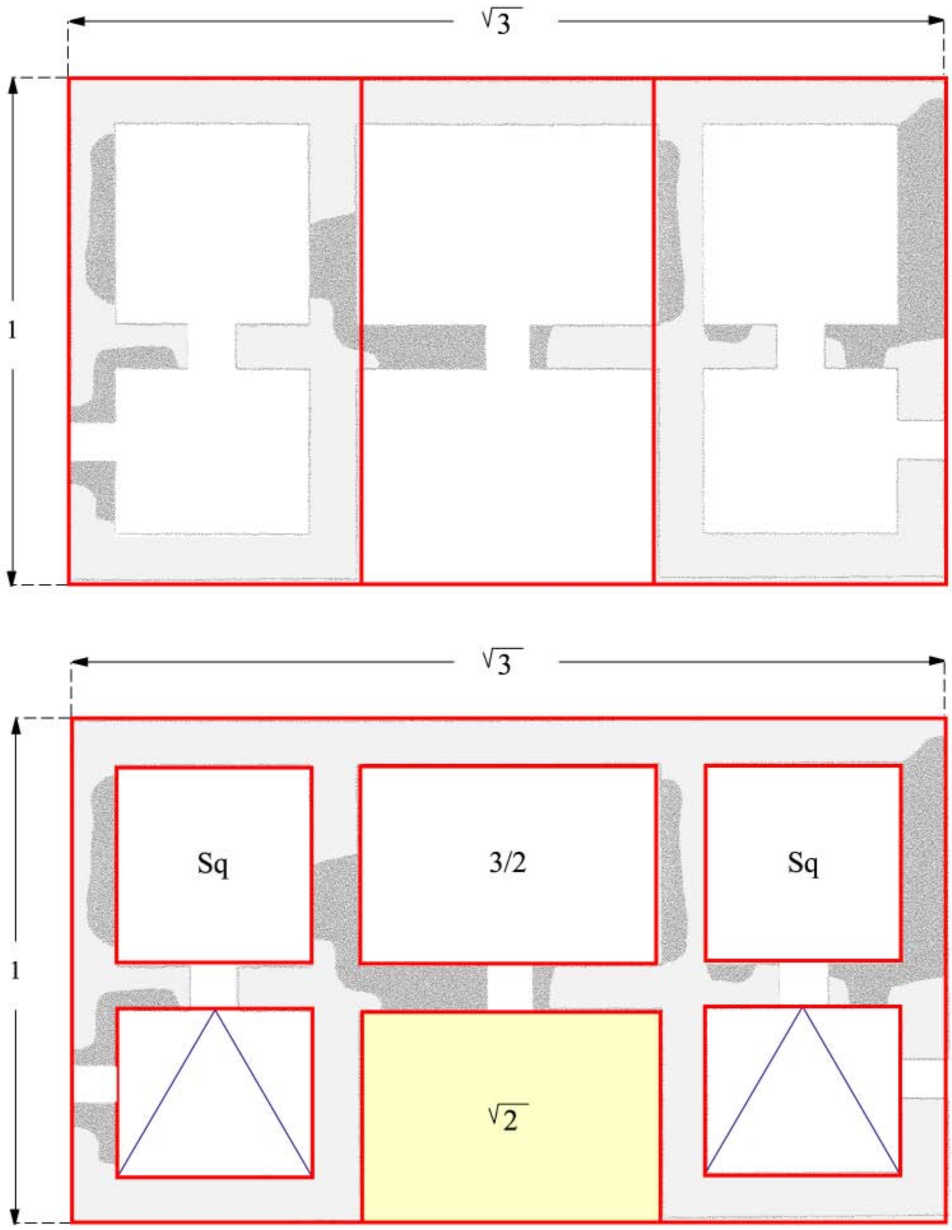


The exterior walls of structure 5B21 are inscribed by a root three rectangle. The width of the entrance of this structure is one third of the width of the exterior walls, thus subdividing the horizontal root of three rectangle into three vertical root three rectangles via Formula Square Root of Three-2

The lateral galleries are a proportion of one to three. The central galleries are inscribed by a square (in bold red lines) that is further subdivided into a phi rectangle (shaded blue) and a phi squared rectangle (shaded yellow). If the width if the front gallery is one, its length is two times phi.

Figure 103

Plan of Structure 5B10, Chichen Itza, Yucatan, Mexico
Measured Drawing by Karl Ruppert (1952)

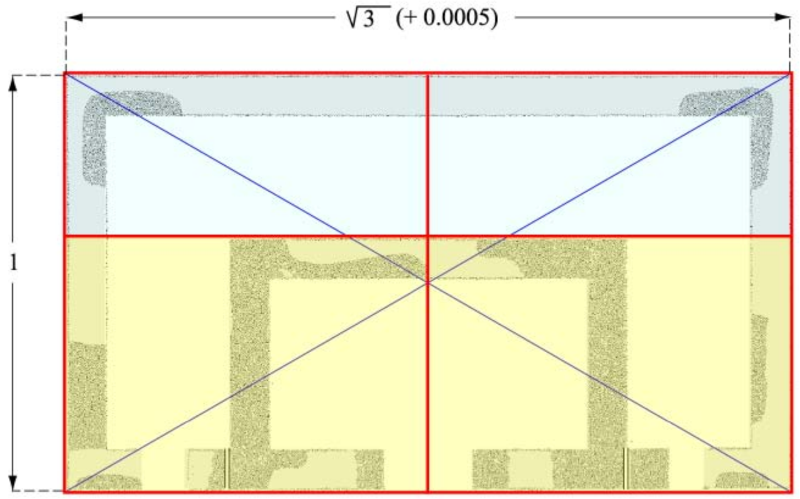


The exterior walls of structure 5B10 are inscribed by a horizontal root three rectangle subdivided by three vertical root three rectangles, as per Formula Square Root of Three-2. The interior rooms are inscribed by squares (in white), a rectangle with a ratio of two to three (also white), a root of two rectangle (shaded yellow), and two, two over root three rectangles that inscribe equilateral triangles (in blue lines).

Figure 104

Plan of Structure 3B2, Chichen Itza, Yucatan, Mexico
Measured Drawing by Karl Ruppert (1952)

a



b

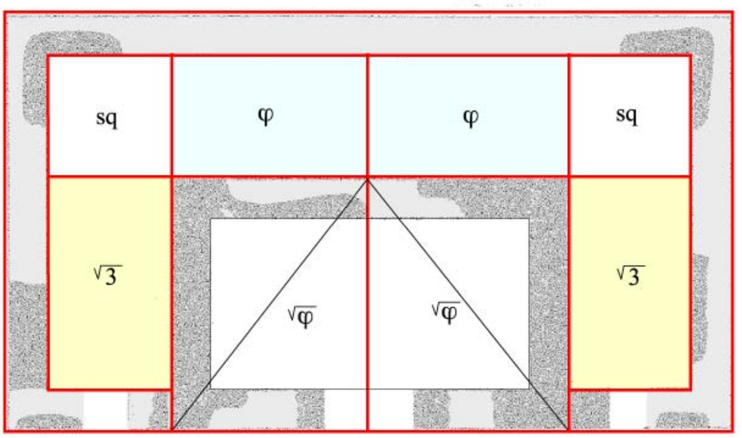
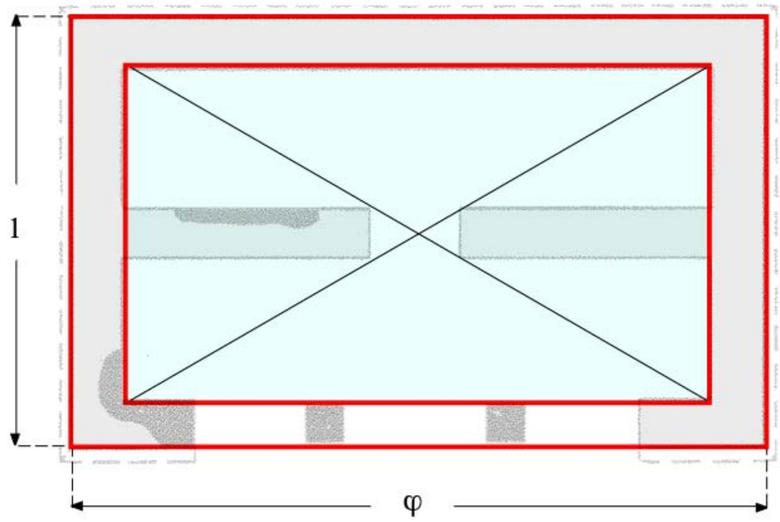


Diagram a subdivides a root three rectangle by the virtually perfect Formula Square Root of Three-3 The two square root of five rectangles are shaded blue, and the two square root of two rectangles are shaded yellow.
The interior space (b) is subdivided by square root three rectangles (shaded yellow), phi rectangles (shaded blue), two squares (in white), and two square root of phi rectangles (the exterior wall of the inner room whose diagonals are drawn in black lines).

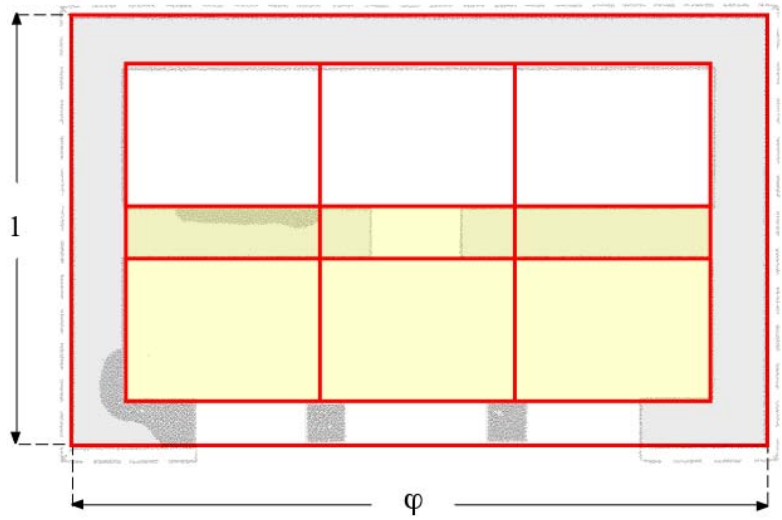
Figure105

Plan of Structure 2C4, Chichen Itza, Yucatan, Mexico
Measured Drawing by Karl Ruppert (1952)

a



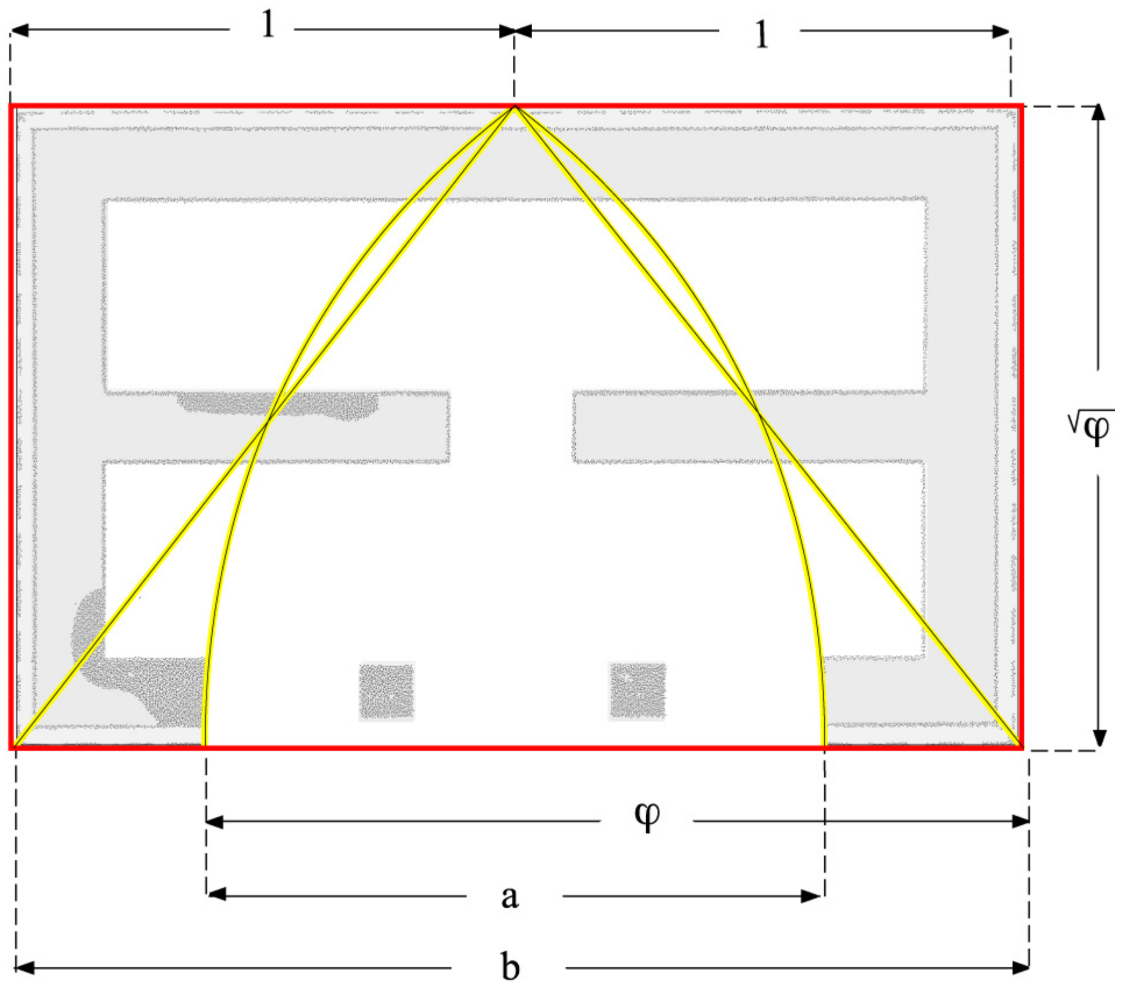
b



The exterior walls of Structure 2C4 (a) are inscribed by a phi rectangle and the interior walls by a square root of three rectangle (shaded blue).
The centered interior support walls further subdivide the interior space (b) into three squares (shaded yellow) relative to the front and rear galleries. This subdivision recalls Formula Square Root of Three-2 whereby a horizontal square root of three rectangle subdivided by three produces three vertical square root of three rectangles.

Figure 106

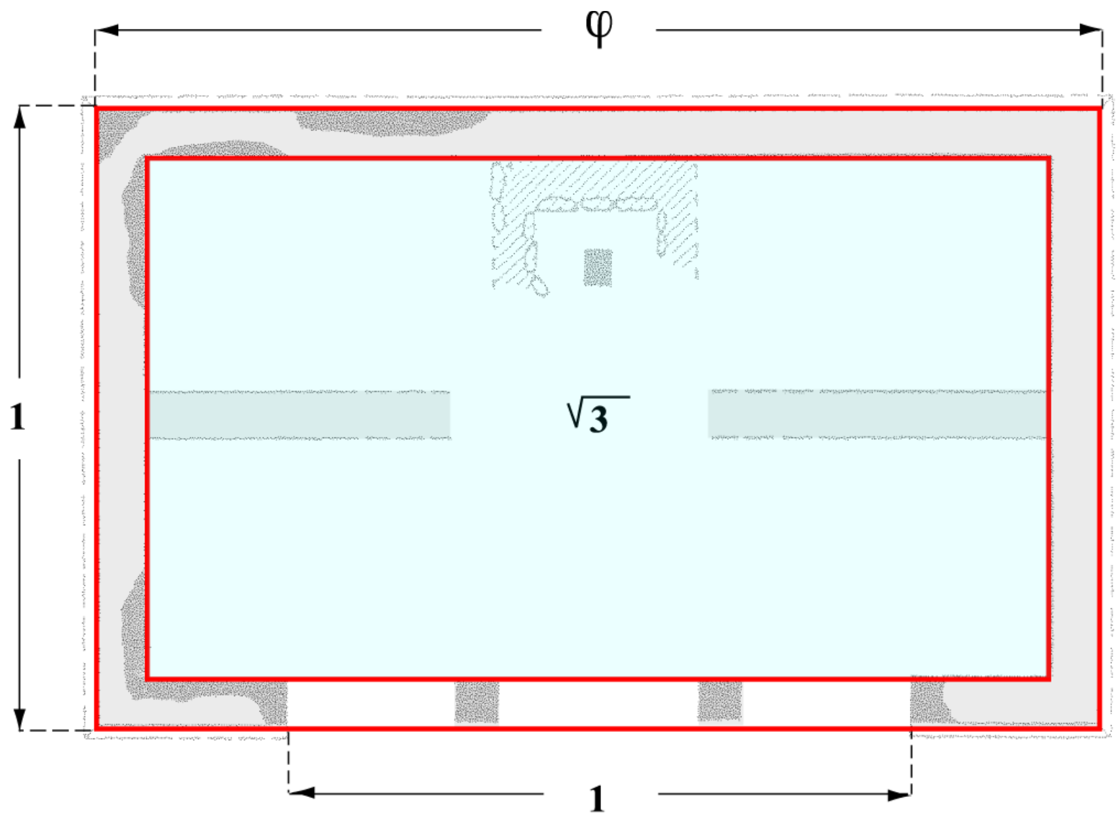
Plan of Structure 2C4 (b), Chichen Itza, Yucatan, Mexico
Measured Drawing by Karl Ruppert (1952)



The platform of Structure 2C4 is inscribed by a rectangle with a ratio of two to the square root of phi. The diagonals of the two square root of phi rectangles are used to determine the width of the entrance. If the width of the entrance (a) is one, the length of the platform (b) is phi.

Figure 107

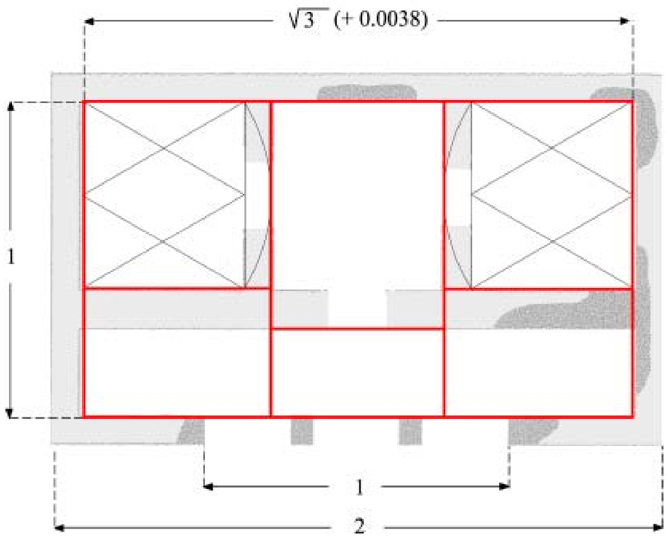
Plan of Structure 5B14, Chichen Itza, Yucatan, Mexico
Measured Drawing by Karl Ruppert (1952)

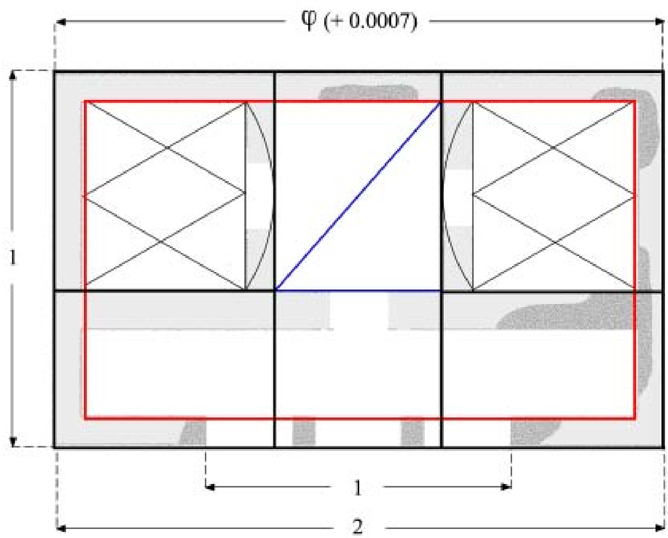


The proportions of exterior and interior walls of structure 5B14 are identical to those of Structure 2C4. The exterior walls are inscribed by a phi rectangle and the interior walls by a root three rectangle (shaded blue). The width of the entrance is equal to the width of the structure and is also in a one to phi ratio to the length of the structure.

Figure 108

Plan of Structure 2C8, Chichen Itza, Yucatan, Mexico
Measured Drawing by Karl Ruppert (1952)

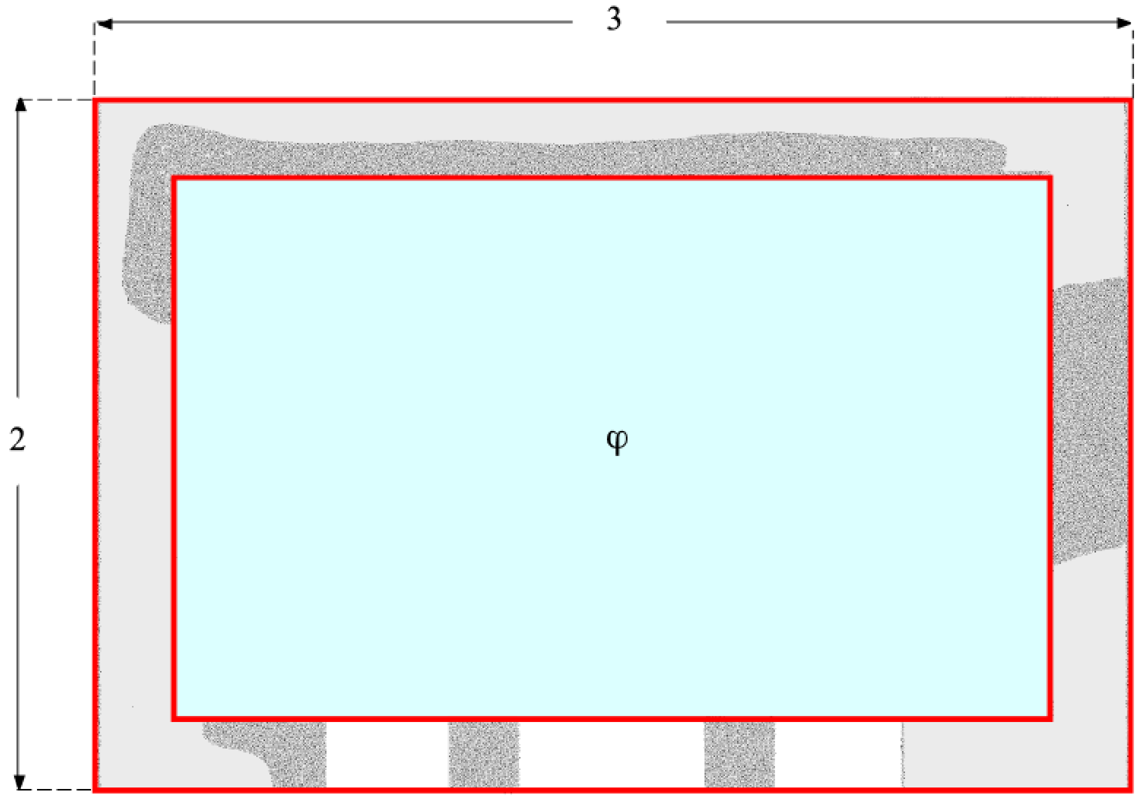




The interior space of Structure 2C8, is subdivided by the virtually perfect Formula Square Root of Three-5 (red lines). The rear lateral rooms are two over root three rectangles and inscribe equilateral triangles.
The exterior walls and interior walls may be further subdivided by the virtually perfect Formula Phi-3 (in bold black lines). The central rear room has a width of two to a length of root five and a diagonal (blue line) of three.

Figure 109

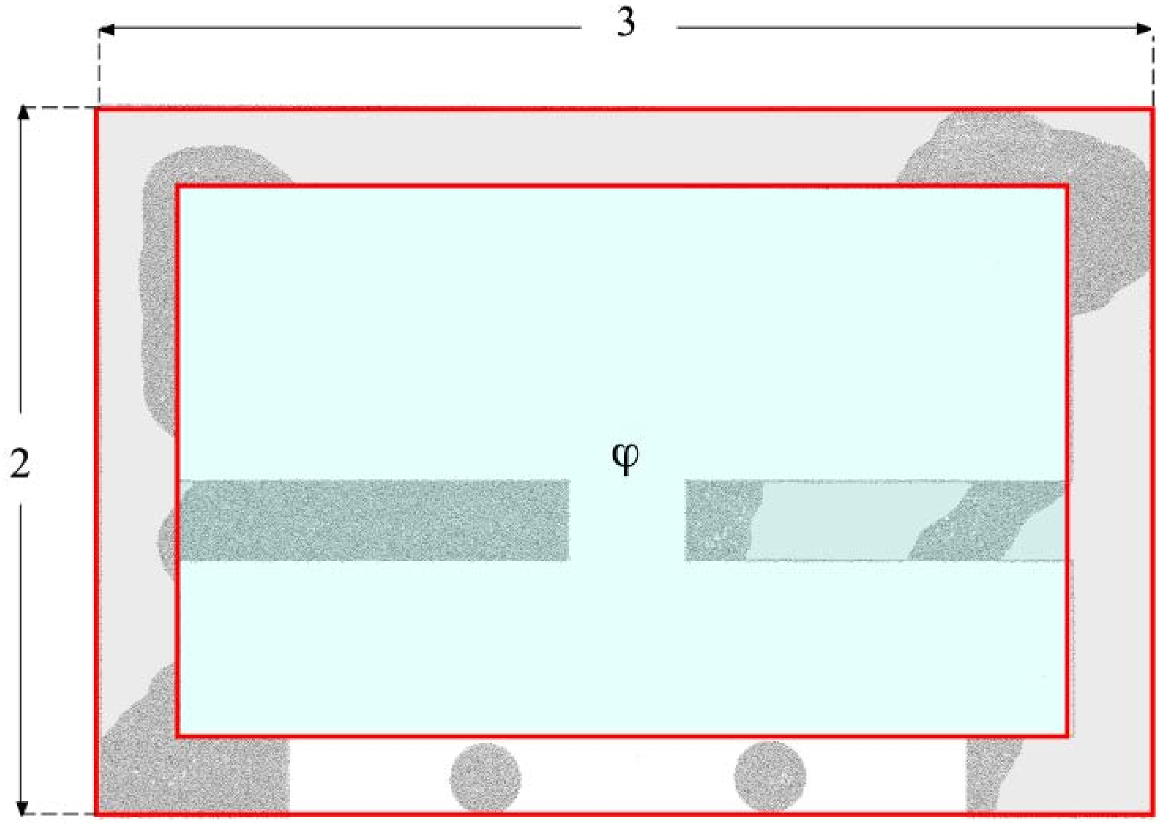
Plan of Structure 2C10, Chichen Itza, Yucatan, Mexico
Measured Drawing by Karl Ruppert (1952)



The exterior walls of Structure 2C10 are inscribed by a rectangle with a ratio of two to three. The interior space is inscribed by a phi rectangle (shaded blue).

Figure 110

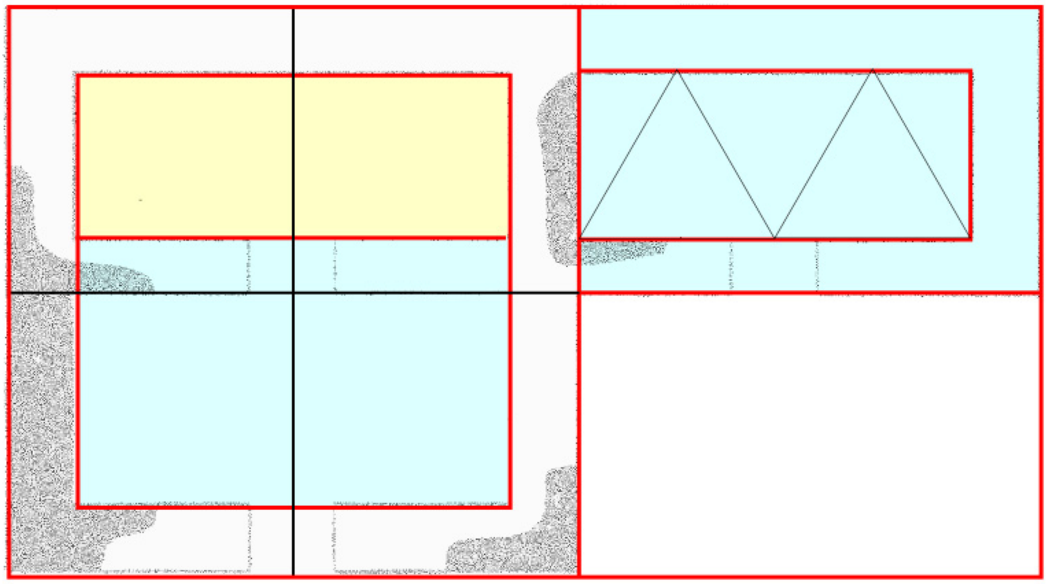
Plan of the Temple of the Painted Capstone,
Chichen Itza, Yucatan Peninsula, Mexico
Measured Drawing by Karl Ruppert (1952)



Identically to Structure 2C10 on the previous page, the exterior walls of this structure are inscribed by a rectangle with a ratio of two to three. The interior space is inscribed by a phi rectangle (shaded blue).

Figure 111

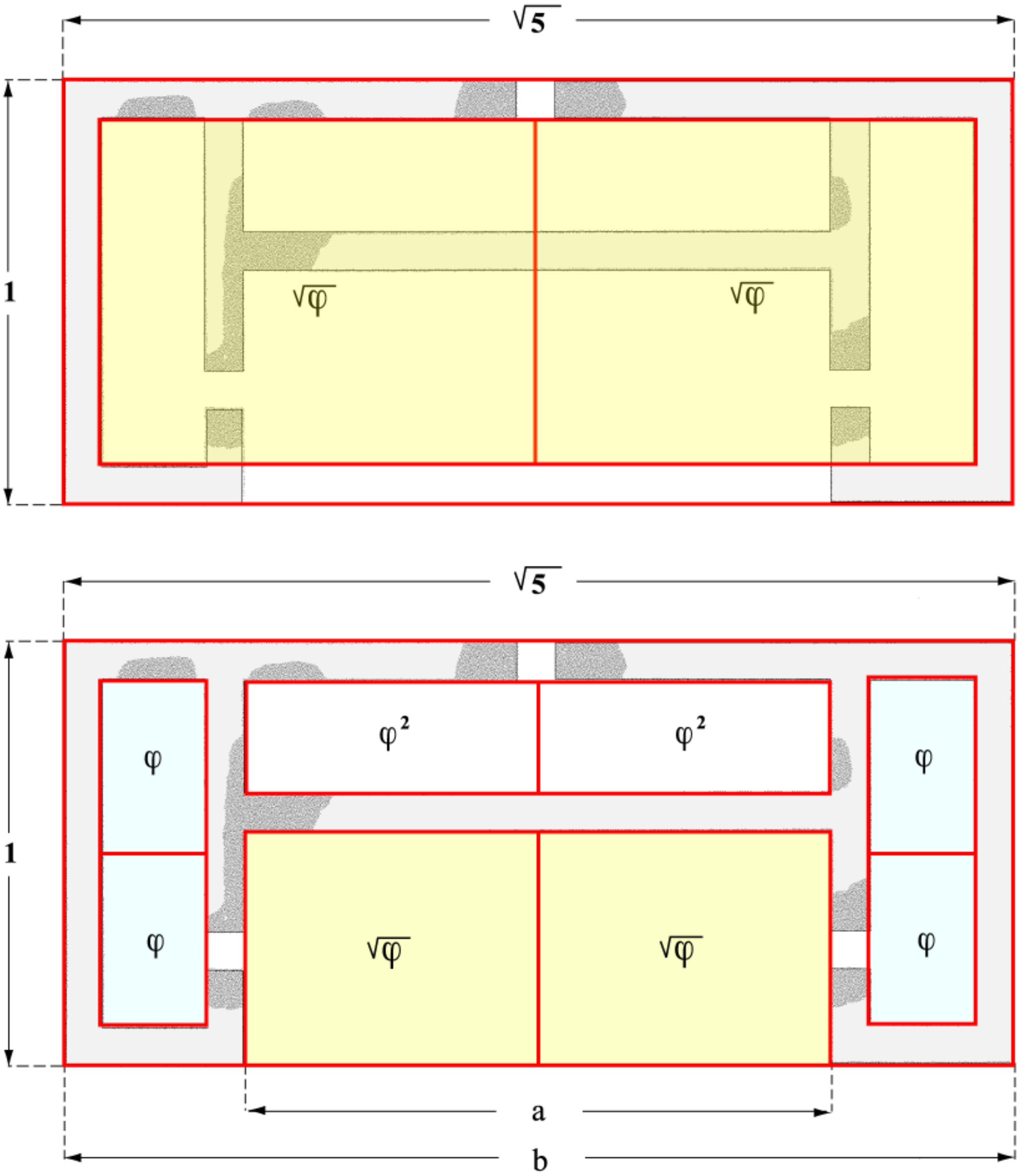
Plan of Structure 5B3, Chichen Itza,
Yucatan, Mexico
Measured Drawing by Karl Ruppert (1952)



The exterior walls of the larger of the two rooms of this structure is a square (shaded grey) and quartered (black lines). The interior of this larger room is also a square and is subdivided at the rear gallery into a phi rectangle (shaded blue) and a phi squared rectangle (shaded yellow), as per Formula Square-2. The smaller room to the right is one half the width of the larger room, and its exterior walls are inscribed by a phi rectangle (shaded blue). The empty space beneath this smaller room (in white) is also inscribed by a phi rectangle. The interior space of the smaller room can inscribe two equilateral triangles.

Figure 112

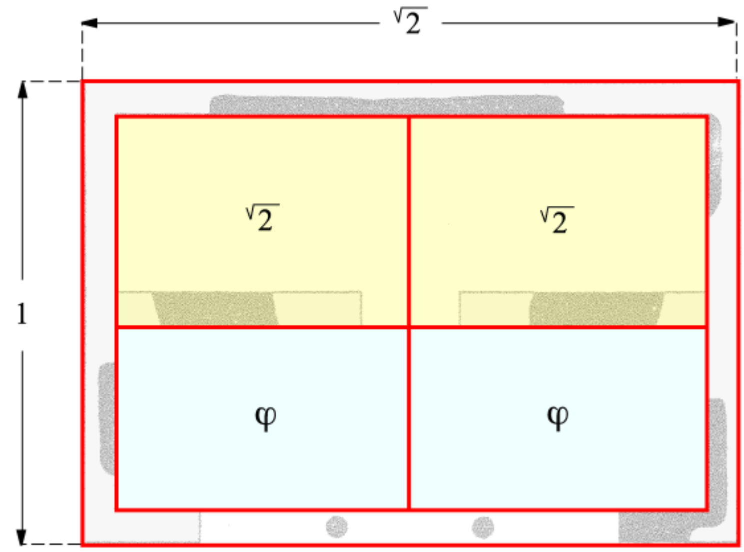
Plan of Structure 5B7, Chichen Itza,
Yucatan, Mexico
Measured Drawing by Karl Ruppert (1952)

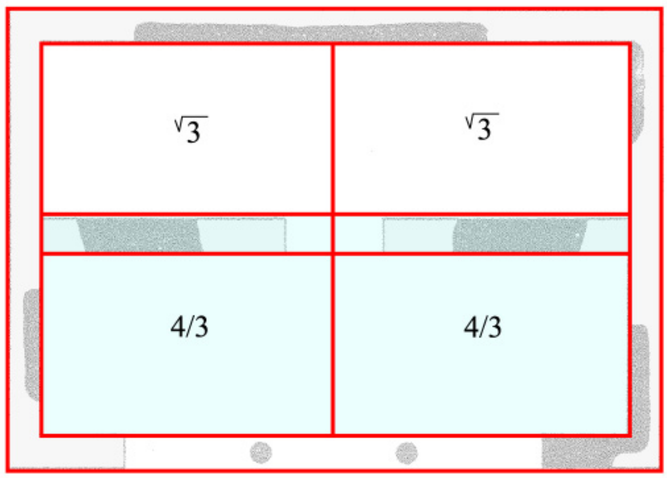


The exterior walls of Structure 5B7 are inscribed by a square root of five rectangle. The interior walls form a rectangle with a ratio of one to two times square root of phi (shaded yellow). The interior rooms are inscribed by double phi rectangles (shaded blue), phi-squared rectangles (in white), and square root of phi rectangles (shaded yellow). The width of the entrance (a) times phi equals the length of the structure (b).

Figure 113

Plan of Structure 5C1, Chichen Itza,
Yucatan, Mexico
Measured Drawing by Karl Ruppert (1952)

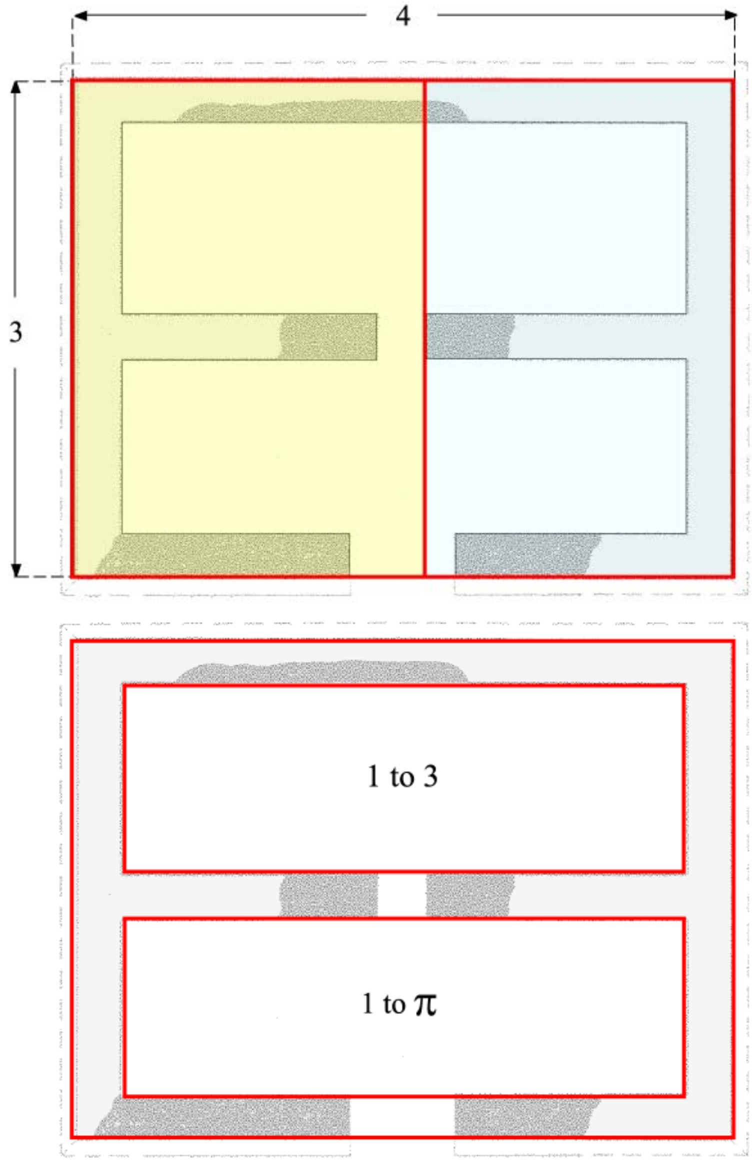
a 

b 

The exterior walls of Structure 5C1 are inscribed by two root two rectangles, and the interior space is subdivided at the front gallery into two phi rectangles (shaded blue) and two root two rectangles (shaded yellow). Divided in half, this subdivision produces two Pythagorean 3,4,5 rectangles via the virtually perfect Pythagorean 3,4,5 - 2 formula.
Figure b subdivides the interior space at the rear gallery by two root three and two Pythagorean 3,4,5 rectangles via the virtually perfect Pythagorean 3,4,5 - 3 formula.

Figure 114

Plan of Structure 5B1, Chichen Itza,
Yucatan, Mexico
Measured Drawing by Karl Ruppert (1952)

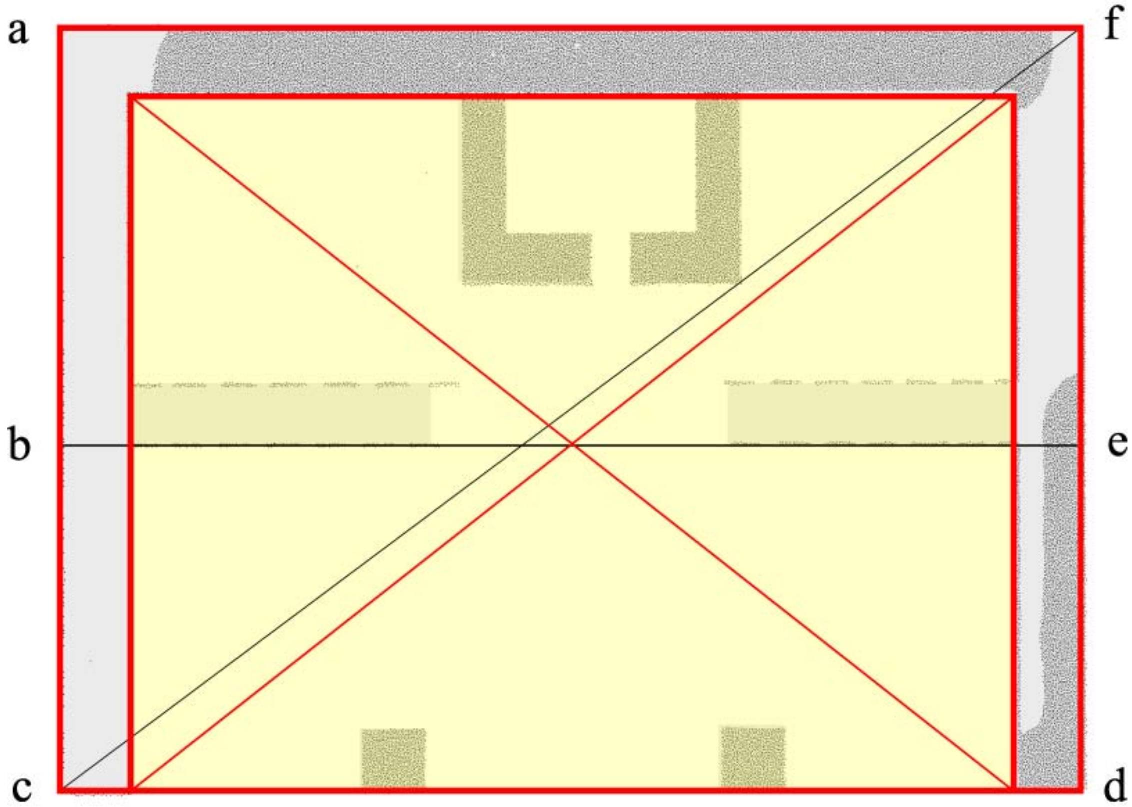


The exterior walls of Structure 5B1 are inscribed by a Pythagorean 3,4,5 rectangle. This is further subdivided into a phi rectangle (shaded blue) and a square root of three rectangle (shaded yellow), as per Pythagorean 3,4,5,-2 formula.

A rectangle with a width of one to a length of three inscribes the rear gallery, and a rectangle with a width of one to a length of pi inscribes the front gallery.

Figure 115

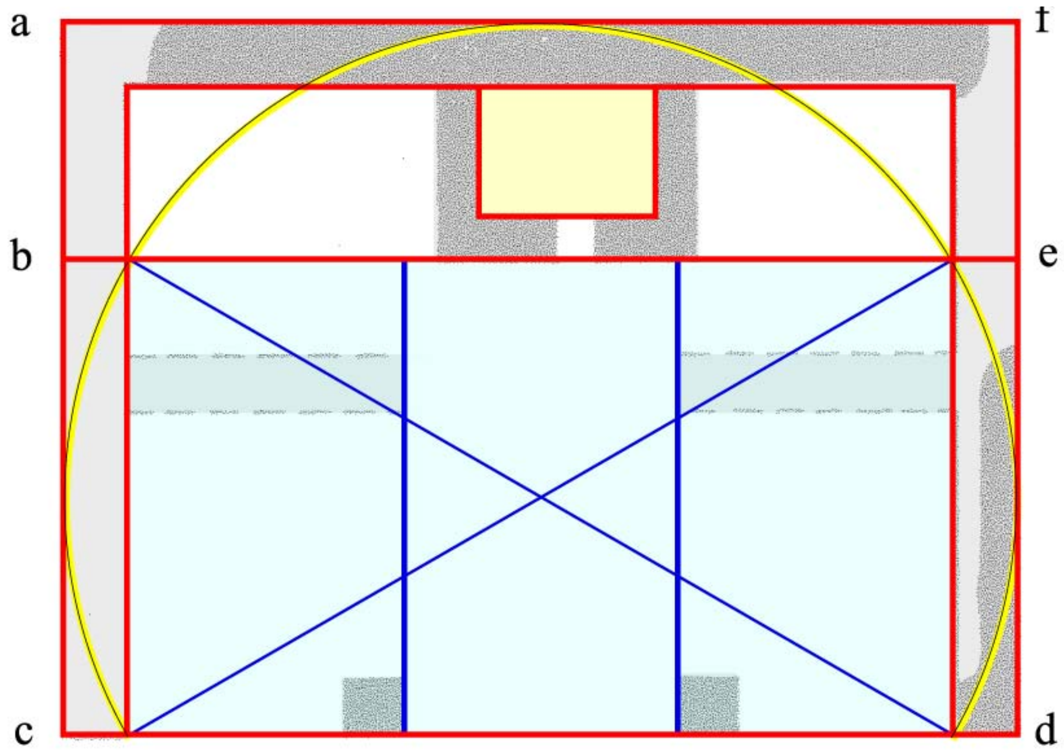
Plan of Structure 2C3 (a), Chichen Itza, Yucatan Peninsula, Mexico
Measured drawing by Karl Ruppert, 1952



Structure 2C3 is inscribed by a Pythagorean 3,4,5 rectangle. If a,c is three, a,f is four and c,f is five. The interior dimensions are a square root of phi rectangle (shaded yellow). The center of the width of this structure is demarcated by the interior wall of the front gallery (b,e).

Figure 116

Plan of Structure 2C3 (b), Chichen Itza, Yucatan Peninsula, Mexico
Measured drawing by Karl Rupert, 1952



In diagram b, a horizontal root three rectangle (shaded blue and trisected into three vertical root three rectangles) is used to determine the length and width of the Pythagorean 3,4,5 rectangle. The diagonals of this root three rectangle are also drawn in blue lines. The intersections of these diagonals, to the corners of the root of three rectangle, are the radii for the arc (highlighted in yellow) that define the widths of the walls.. If a,b is one, b,c is two, and c,d is four. Thus, b,c,d,e form a root four rectangle. This is a true formula for using a root three rectangle to create a Pythagorean 3,4,5 rectangle.

Figure 117

Plan of Structure 2C3 (c), Chichen Itza, Yucatan, Mexico
Measured Drawing by Karl Ruppert (1952)

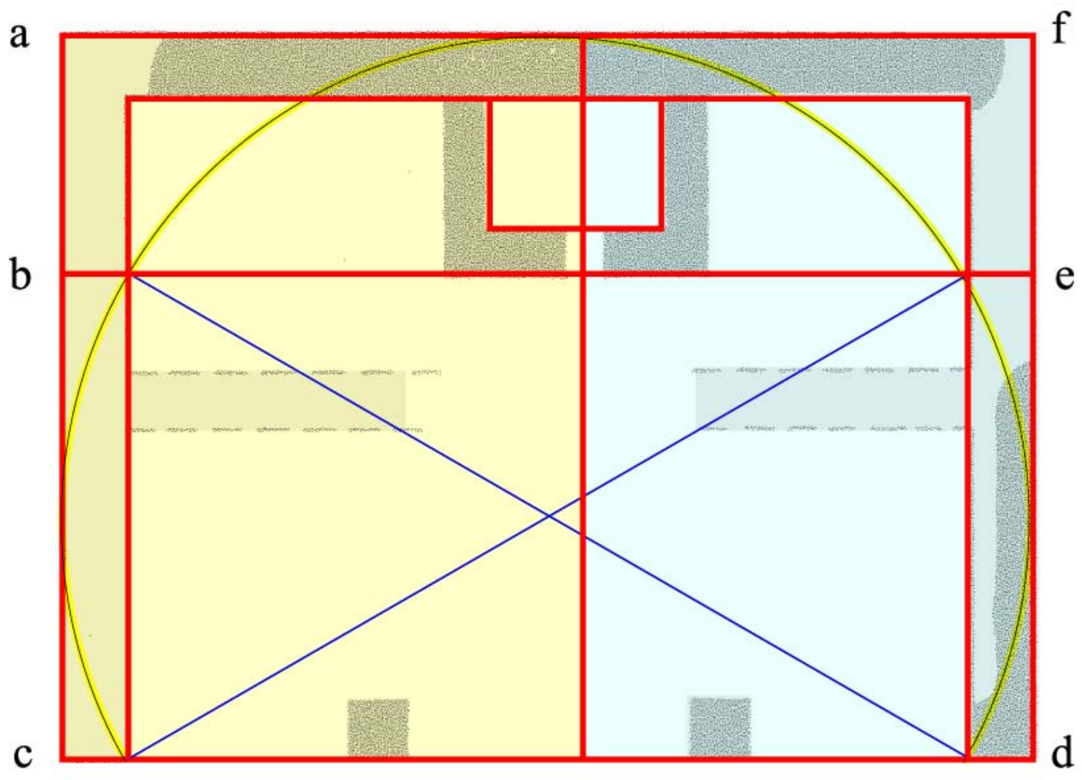
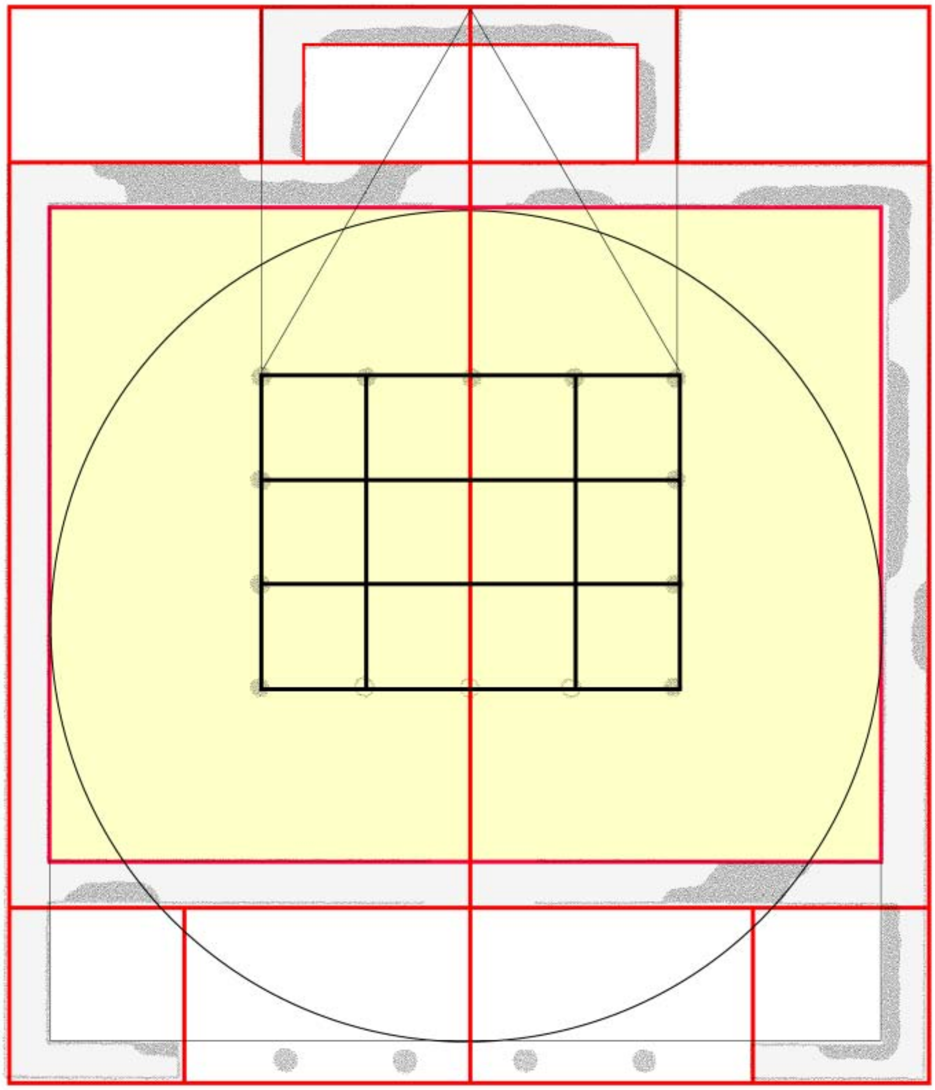


Diagram c further subdivides the Pythagorean 3,4,5 rectangle by a phi rectangle (shaded blue) and a square root of two rectangle (shaded yellow), as per the virtually perfect Formula Pythagorean 3,4,5-b. This same division divides the Pythagorean rectangle that inscribes the small room at the back wall of the structure into a root three rectangle (shaded blue) and a smaller Pythagorean 3,4,5 rectangular (shaded yellow), as per virtually perfect Formula Pythagorean 3,4,5-3.
Diagrams a,b, and c, considered together, is a good example of how more than one geometrical formula can be incorporated into a single structure.

Figure 118

Plan of Structure 5C11 (a), Chichen Itza, Yucatan, Mexico
Measured Drawing by Karl Ruppert (1952)

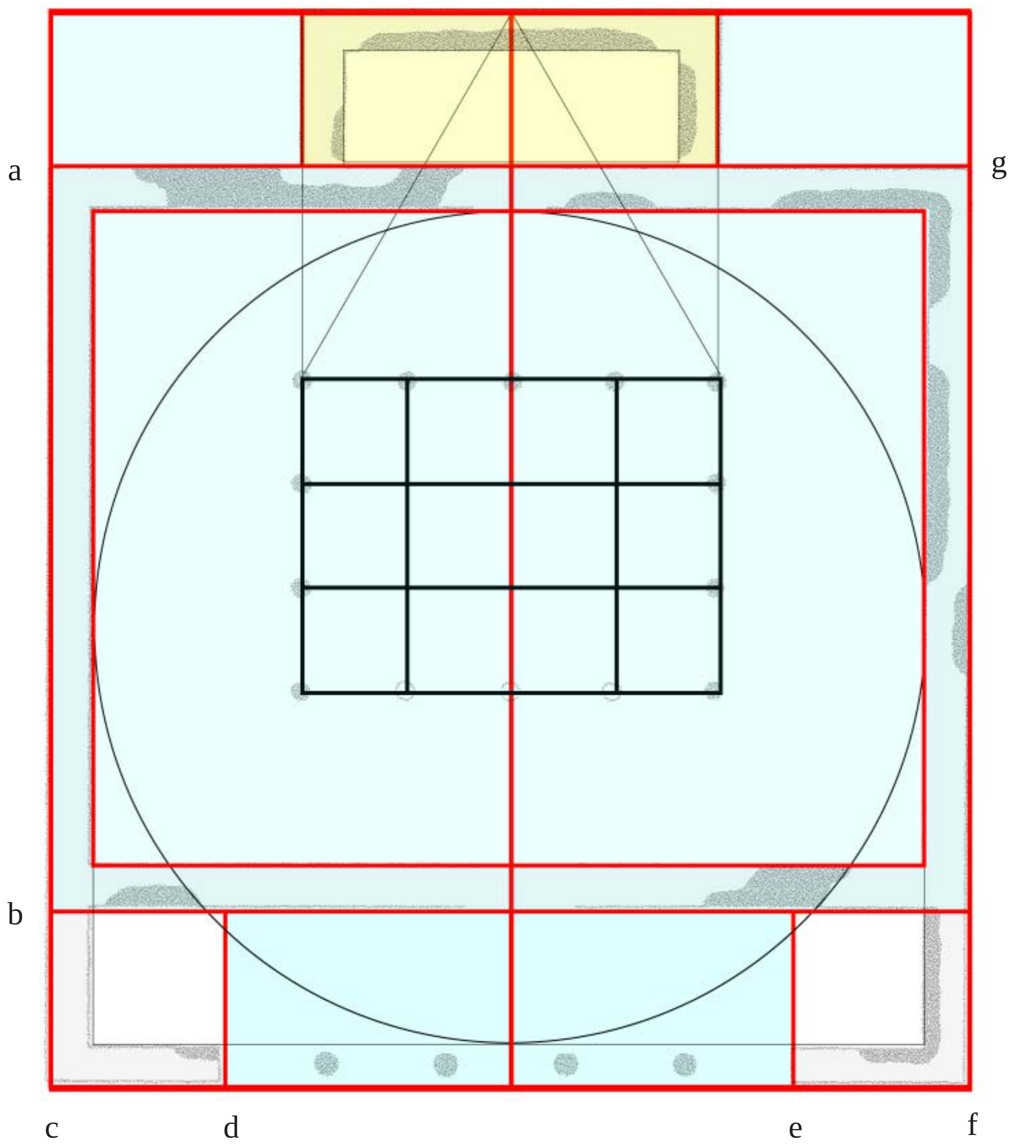


The interior room in Plan a (shaded yellow) is a square root of phi rectangle. The circle and the yellow shaded square root of phi rectangle have virtually the same area (see Formula Concentric Circles-2).

The support columns within this room form a Pythagorean 3,4,5 rectangle divided into twelve squares (bold black lines). An equilateral triangle, whose' base equals the width of the Pythagorean rectangle, terminates at the upper center of the structure.

Figure 119

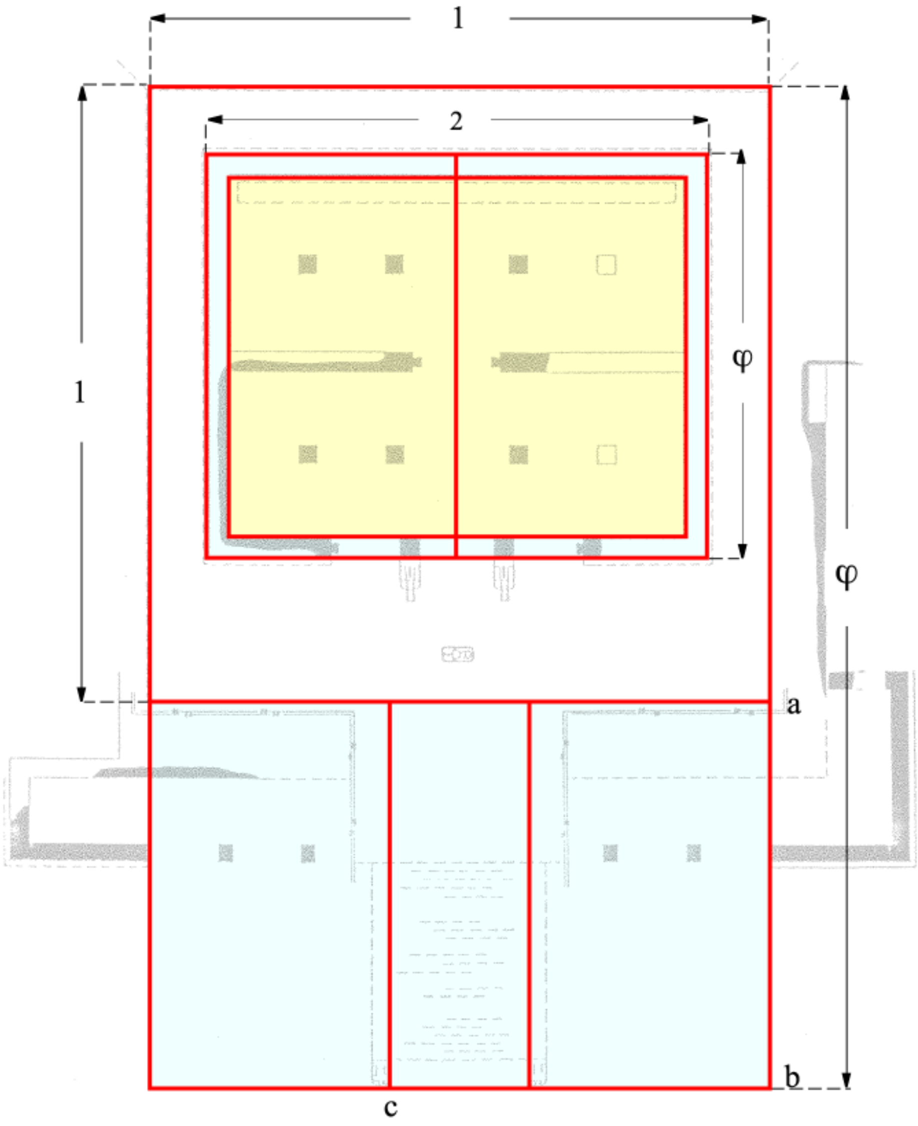
Plan of Structure 5C11 (b), Chichen Itza, Yucatan, Mexico
Measured Drawing by Karl Ruppert (1952)



In this plan (b), the square formed by a, c, f, g is subdivided into phi rectangles (shaded blue) and squares (in white). If a, g is two, a, b is phi. If d, e is one, c, f is phi.
The central upper room above a,c,f,g is inscribed in a phi-squared rectangle (shaded yellow). The spaces to either side of this room are inscribed by phi rectangles (shaded blue).

Figure 120

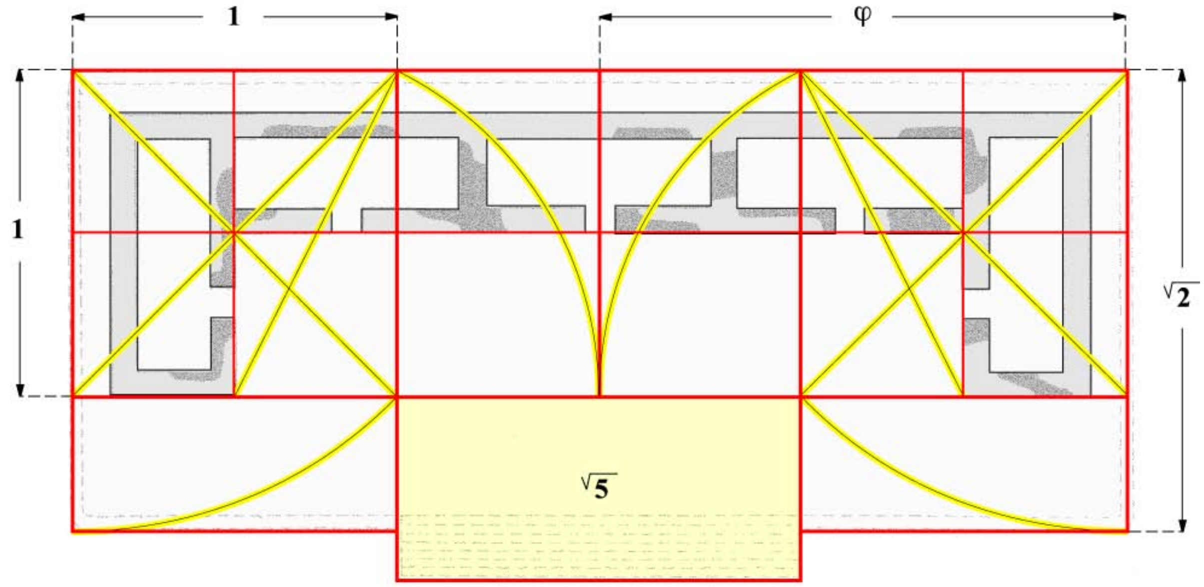
Plan of Structure 3D8, Chichen Itza, Yucatan, Mexico
Measured Drawing by Karl Ruppert (1952)



The upper platform forms a square (diagramed and in white). The platform staircase and balustrade form a phi rectangle (shaded blue). Also, a,b and c,d equal one.
The exterior walls of the upper temple are inscribed by a rectangle with a length of two and a width of phi. The interior walls of the temple define a square root of phi rectangle (shaded in yellow).

Figure 121

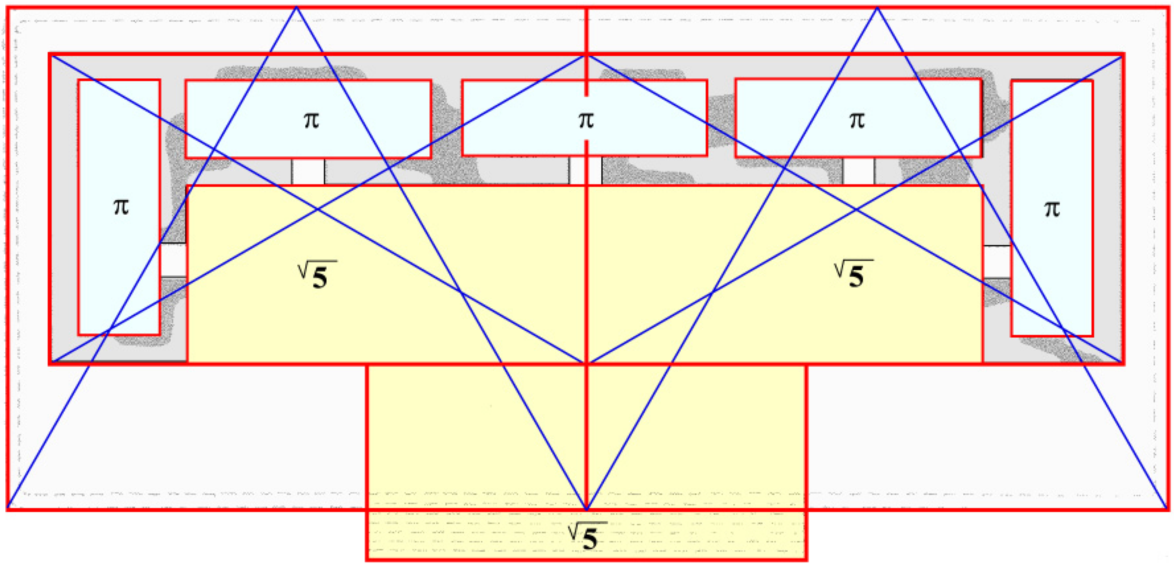
Plan of Structure 5D2 (a), Chichen Itza, Yucatan Peninsula, Mexico
Measured Drawing by Karl Ruppert (1952)



In Plan a, the suggested geometrical formula that relates the dimensions of the platform to the placement of the temple is generated by first drawing straight lines from both sides of the staircase across the width of the platform and a straight line across the length of the platform at the front edge of the U-shaped temple. The intersections of these lines create two squares (diagramed as one to one above). The diagonals of these squares are used as radii for the arcs that terminate at the front corners of the platform, producing two square root of two rectangles (yellow highlighted lines). These same squares are then quartered, and lines are drawn from the center bases of these squares to the upper inside corners of the squares and are used as radii to draw the arcs that define two phi rectangles that define the length of the platform. The remaining space, from the front edge of the staircase to the front edge of the temple, forms a square root of five rectangle (shaded yellow).

Figure 122

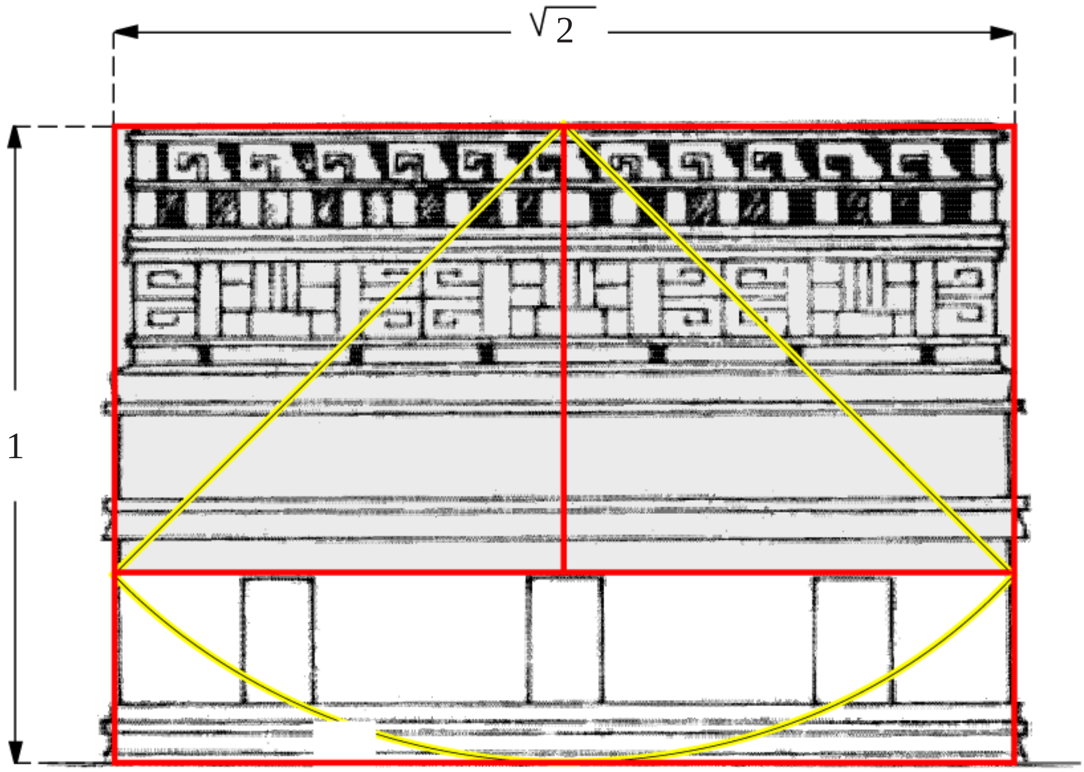
Plan of Structure 5D2 (b), Chichen Itza, Yucatan Peninsula, Mexico
Measured Drawing by Karl Ruppert (1952)



The U-shaped temple of structure 5D2 is inscribed by a rectangle with a width of one to a length of two times the square root of three. The diagonals of these square root of three rectangles form equilateral triangles and are drawn in blue lines to visually emphasize these rectangles.
Square root of five rectangles are shaded yellow, and the interior rooms of the temple are inscribed by rectangles with widths that equal one to their lengths that equal pi (shaded blue).
Because the platform has a ratio of square root of two to two times phi, two virtually perfect equilateral triangles may be inscribed (blue lines), per Formula Two over Square Root of Three -5.

Figure 123

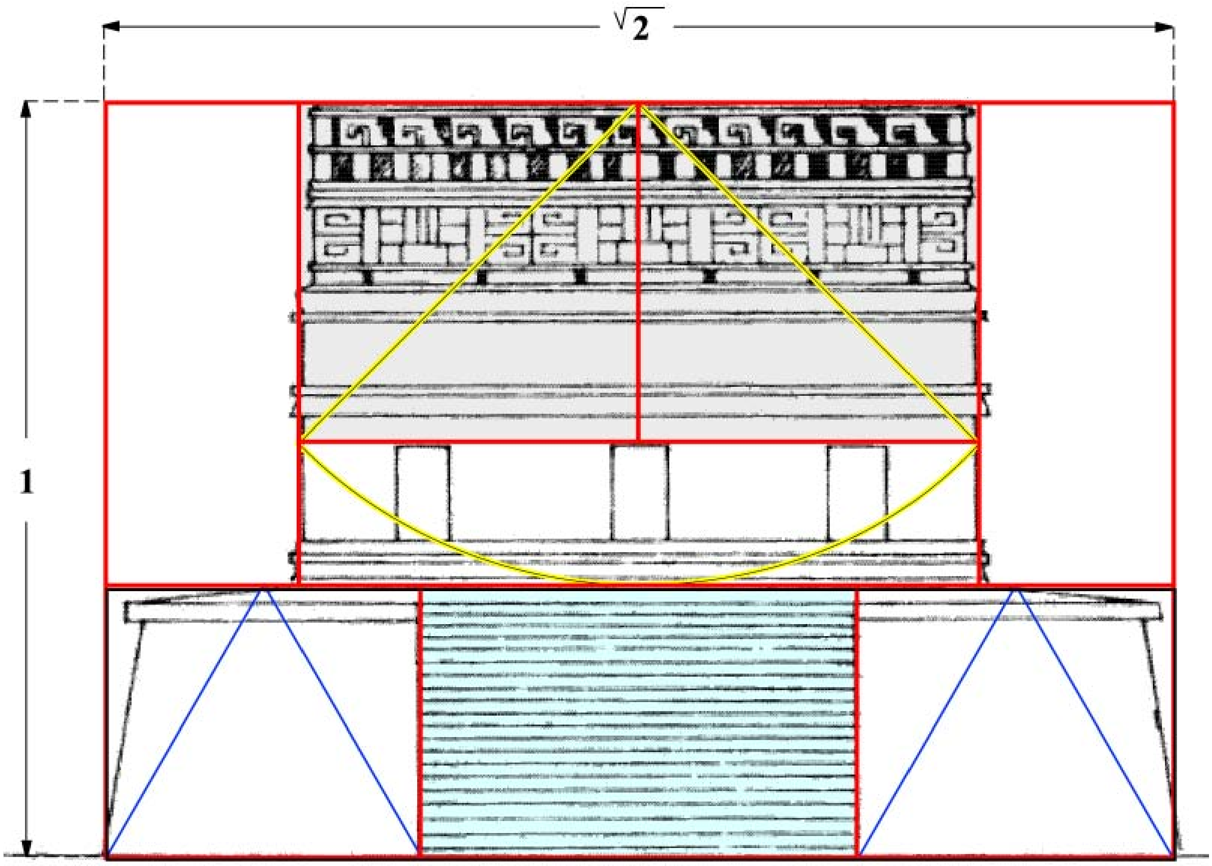
Front Elevation of Structure Chichanchob (a), Chichen Itza, Yucatan, Mexico
Measured Drawing by M. A. Fernandez (Marquina 1951)



Structure Chichanchob is inscribed by a root two rectangle. A root four rectangle (double squares) is inscribed from the top of the doorways to the top of the roof comb to the length of the structure (shaded grey). The forty-five degree diagonals of these squares are the radii for the arc that defines the height of the structure.

Figure 124

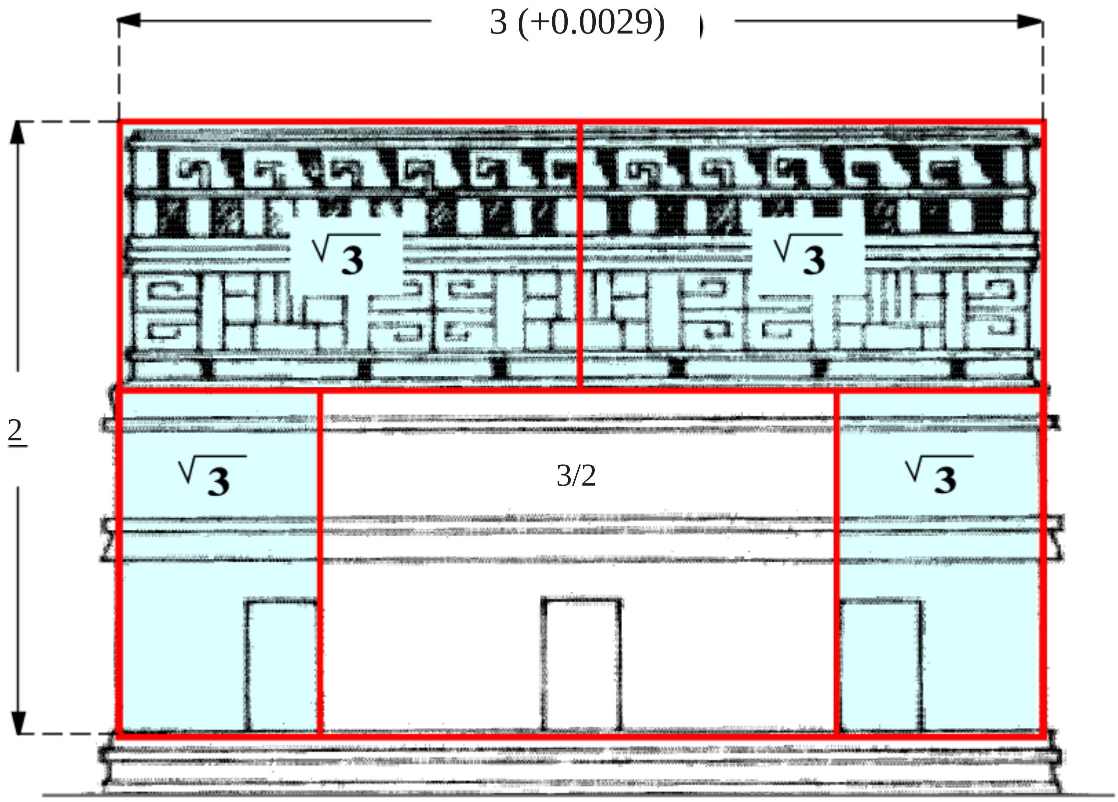
Front Elevation of Structure Chichanchob (b), Chichen Itza,
Yucatan, Mexico
Measured Drawing by M. A. Fernandez (Marquina 1951)



Structure Chichanchob (b) and its raised platform are inscribed by a root two rectangle. The staircase is inscribed by a phi rectangle (shaded blue). The raised platform to either side of the staircase is inscribed by two over root three rectangles that inscribe equilateral triangles (blue lines).

Figure 125

Front elevation of Structure Chichanchob (c), Chichen Itza,
Yucatan Peninsula, Mexico
Measured drawing by M. A. Fernandez, (Marquina 1951)

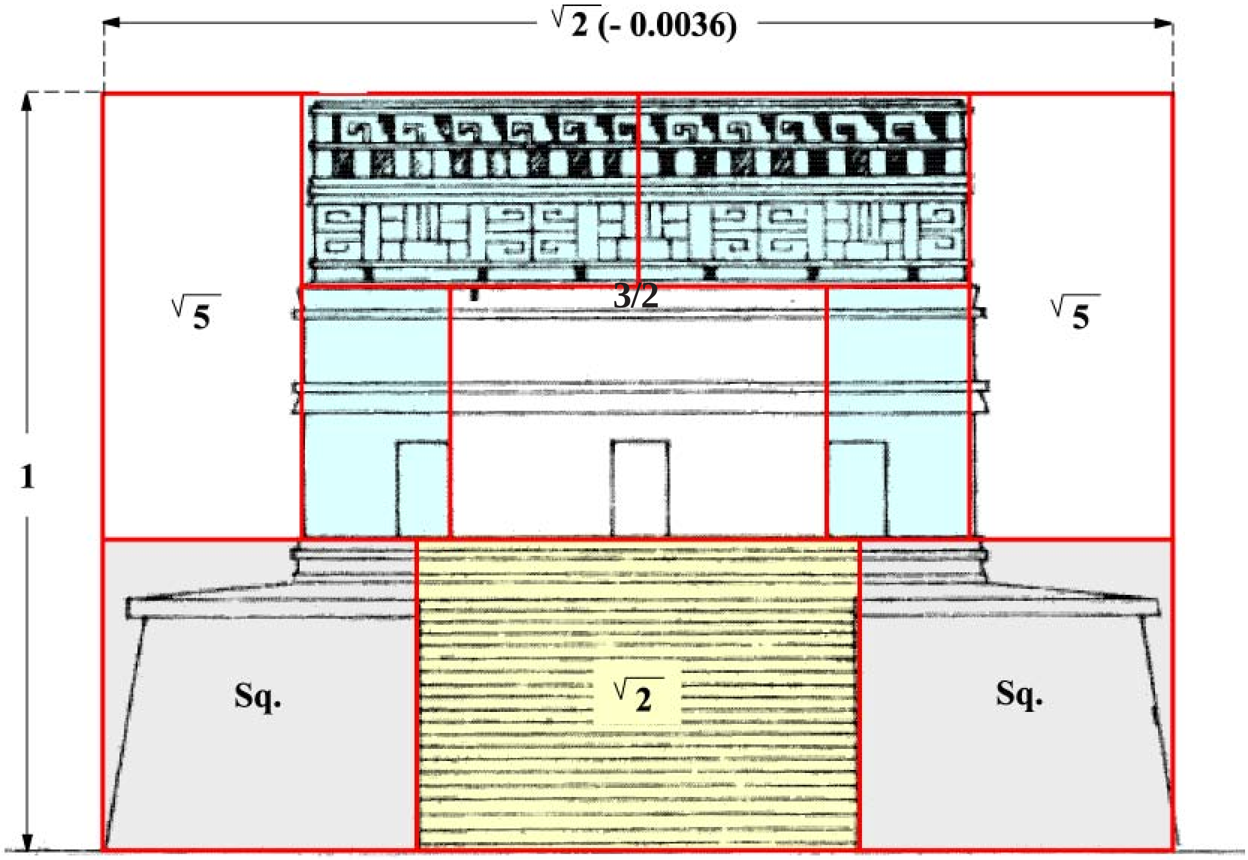


From the base of the walls of Structure Chicanchob, to the top of the roof comb to the length of the walls is inscribed by a virtually perfect three over two rectangle. The formula for this virtually perfect three over two rectangle begins with a smaller three over two rectangle that inscribes the height of the roof to the inside edges of the lateral doorways (in white). Root three rectangles are added to the sides and the top of this three over two rectangle to produce a larger, virtually perfect three over two rectangle.

Figure 126

Front Elevation of Structure Chichanchob (d), Chichen Itza,
Yucatan, Mexico
Measured Drawing by M. A. Fernandez (Marquina 1951)

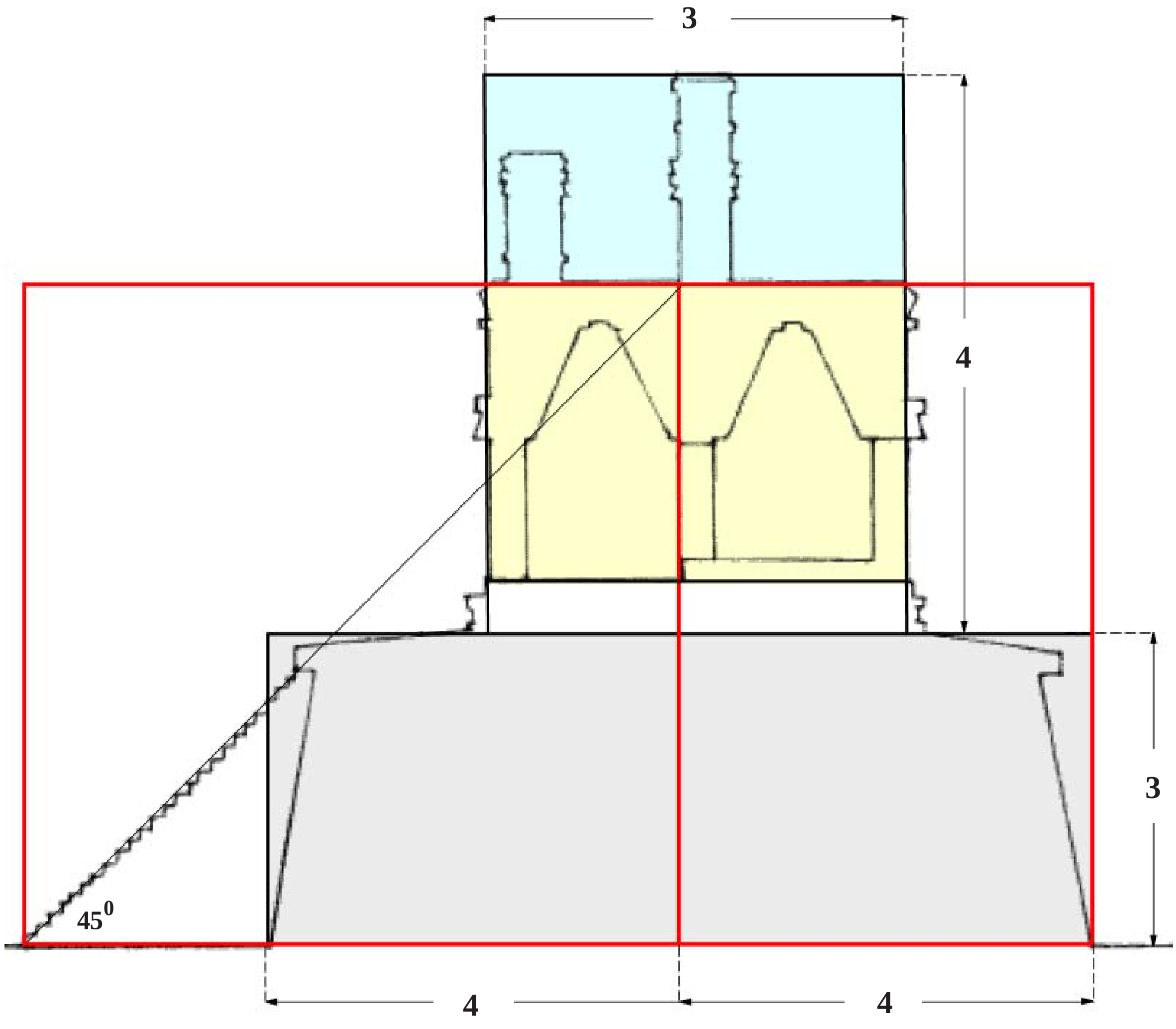
d



Here, Structure Chichanchob and its raised platform is inscribed by a virtually perfect root two rectangle. The temple proper, as in the previous diagram, is inscribed by a virtually perfect three over two rectangle (shaded blue and white). The staircase is inscribed by a root two rectangle (shaded yellow). The rased platform to either side of the staircase is inscribed by two squares (shaded grey), and the empty space to either side of the temple is inscribed by root five rectangles (in white).

Figure 127

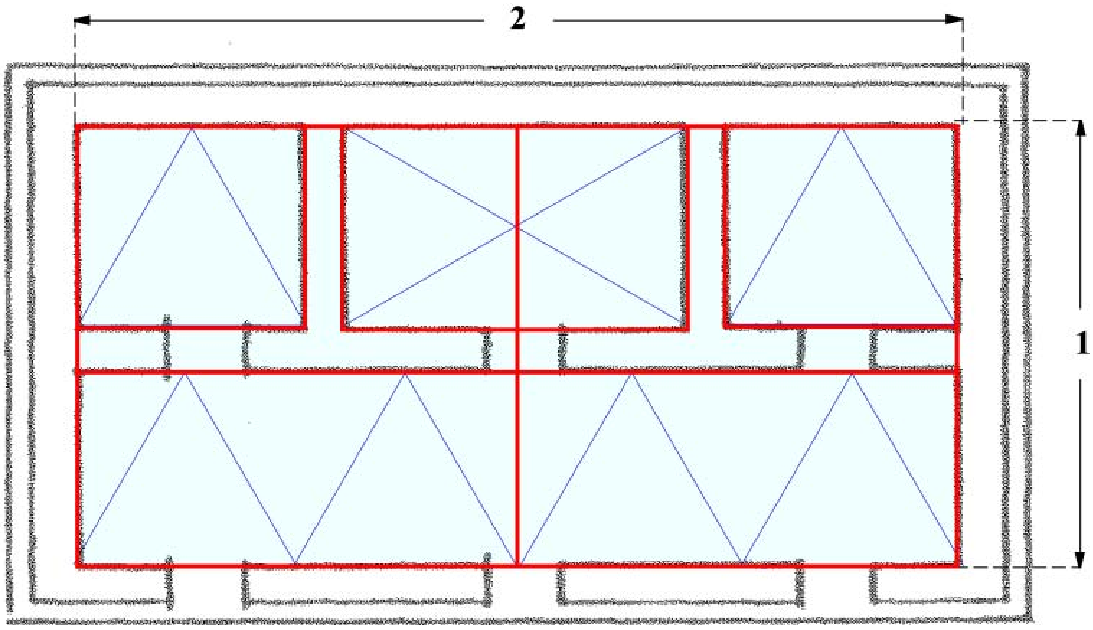
Side Elevation of Structure Chichanchob, Chichen Itza,
Yucatan, Mexico
Measured Drawing by M. A. Fernandez (Marquina 1951)



In this side elevation of Structure Chichanchob, the temple proper, including its plinth, is inscribed by a Pythagorean 3,4,5 rectangle. The height of the walls to the width of the temple is inscribed by a root two rectangle (shaded yellow), and the roof comb is inscribed by a root four rectangle (shaded blue). The raised platform is inscribed by two Pythagorean 3,4,5 rectangles (shaded grey). The bold red lines form a phi rectangle, subdivided by a smaller phi rectangle and a square, that inscribes the width of the entire structure, from the base of the staircase to the top of the roof. Note that the forty-five degree diagonal of this square is an extension of the forty-five degree diagonal of the staircase and that the right edge of this square divides the temple at the rear of the front gallery and also divides the raised platform precisely in half.

Figure 128

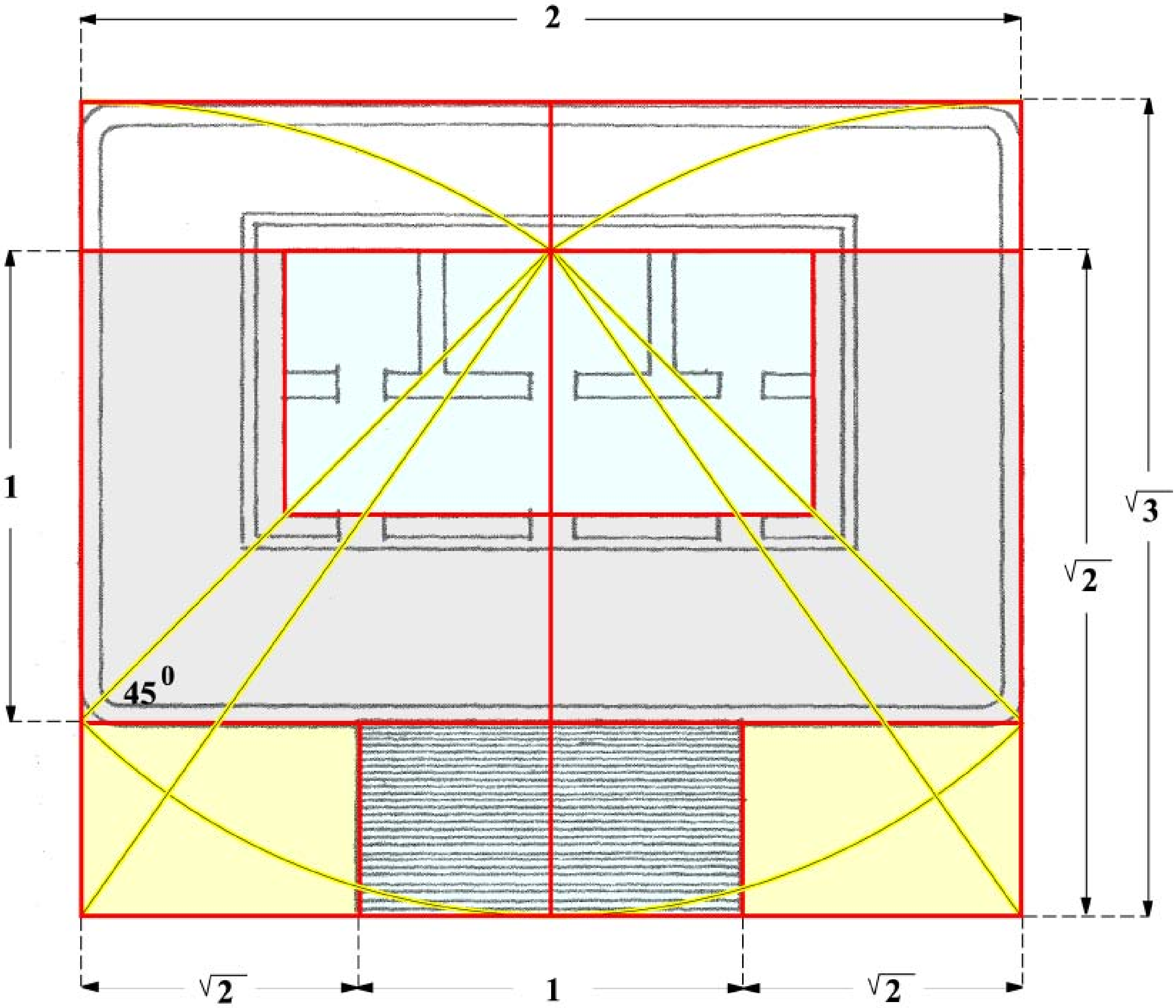
Plan of Structure Chichanchob (a), Chichen Itza,
Yucatan, Mexico
Measured Drawing by M. A. Fernandez (Marquina 1951)



The interior walls of the temple are inscribed by a root four rectangle. The front gallery and the rear rooms inscribe equilateral triangles (blue lines).

Figure 129

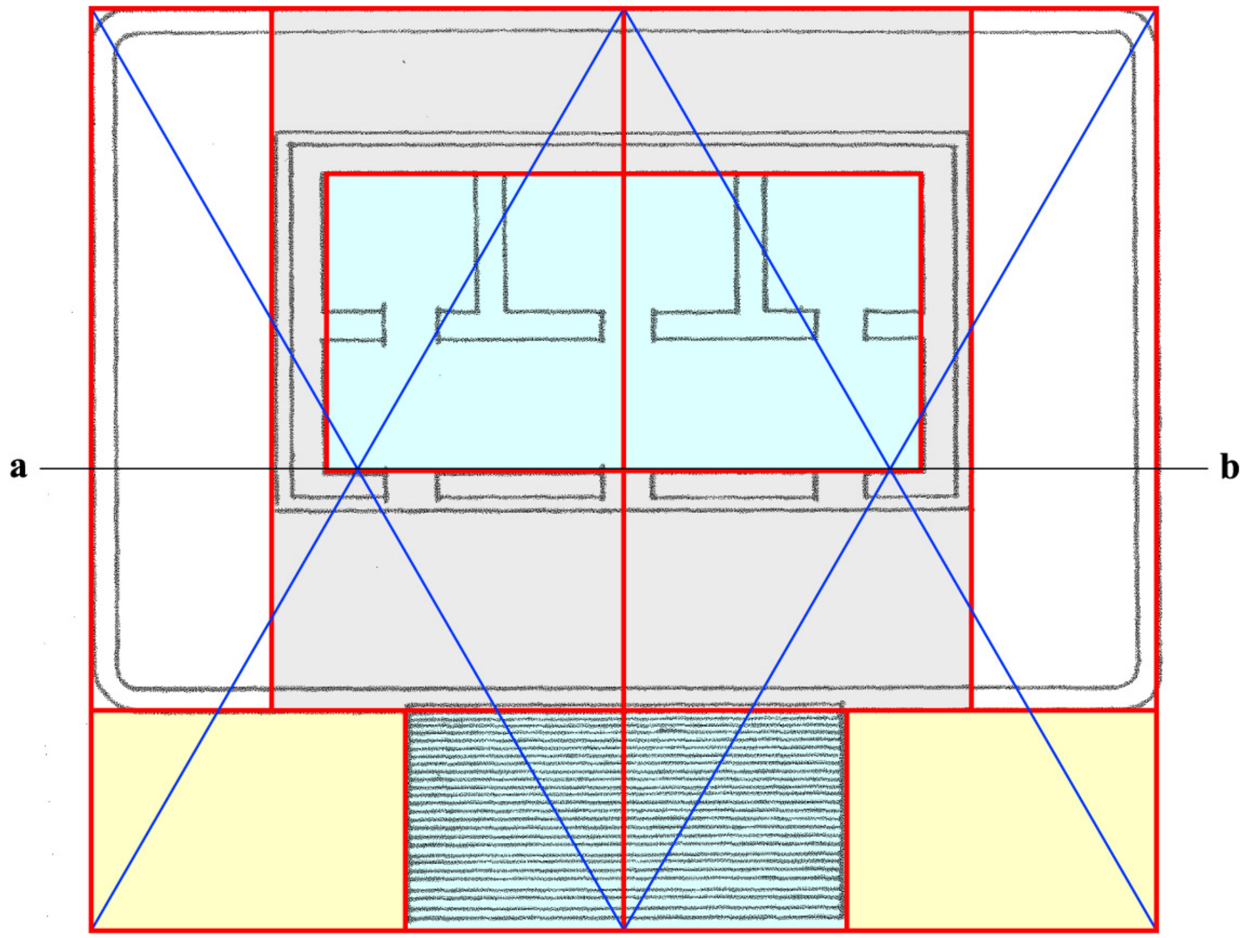
Plan of Structure Chichanchob (b), Chichen Itza,
Yucatan Peninsula, Mexico
Measured Drawing by M. A. Fernandez (Marquina 1951)



The upper platform, to the rear interior wall, is inscribed by a root four rectangle or double square (shaded grey). The forty-five degree diagonals of these squares are the radii for the arc that terminates at the front of the staircase, which is also inscribed by a root four rectangle (shaded blue). The empty space to either side of the staircase is inscribed by root two rectangles. Thus, the plan of this structure, from the rear wall of the temple proper to the front edge of the staircase, is inscribed by a root two rectangle that is subdivided by two root four rectangles and two root two rectangles, as per Formula Square Root of Two-2. Because a root two rectangle divided in half produces two smaller root two rectangles, the diagonals from the lower corners of the diagram to the center of the rear wall of the temple are the radii for the arcs at the top of the diagram that terminate at the rear wall of the raised platform and produce two root three rectangles.

Figure 130

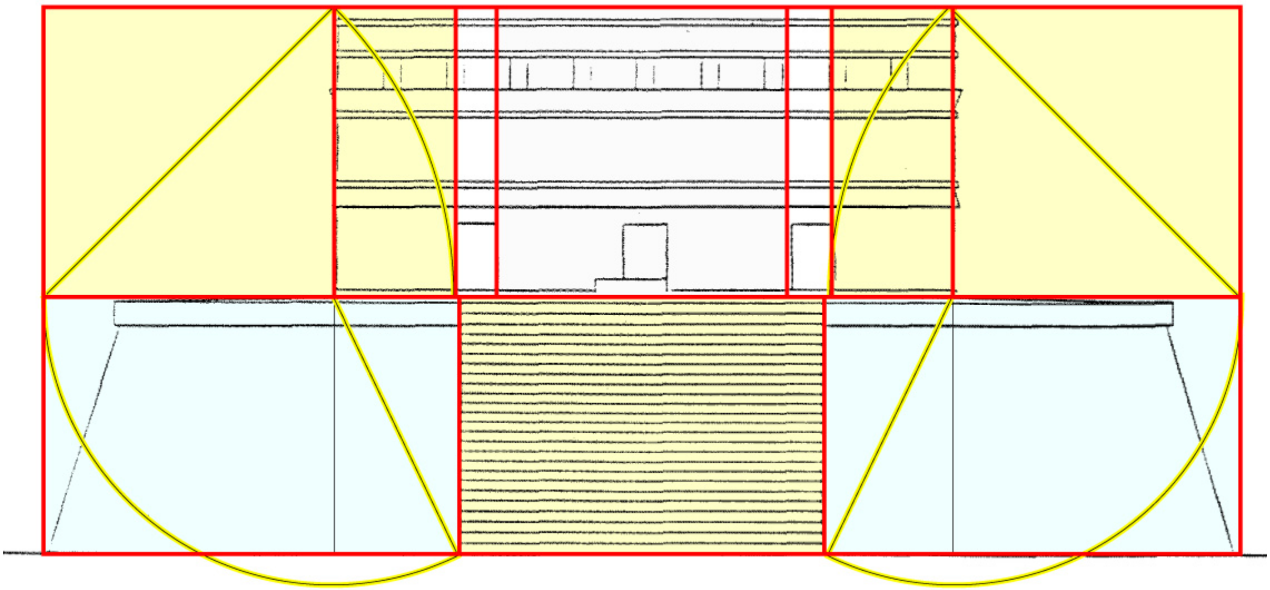
Plan of Structure Chichanchob (c), Chichen Itza,
Yucatan, Mexico
Measured Drawing by M. A. Fernandez (Marquina 1951)



The length of the temple proper, including its plinth, is equal to the width of the raised platform and is inscribed by a square (shaded grey). The entire plan is inscribed by a two over root three rectangle that inscribes opposing equilateral triangles (blue lines). The horizontal center of the plan cuts across the front interior wall of the temple (line a,b).

Figure 131

South Elevation (a) of Structure 3C7, Chichen Itza,
Yucatan, Mexico

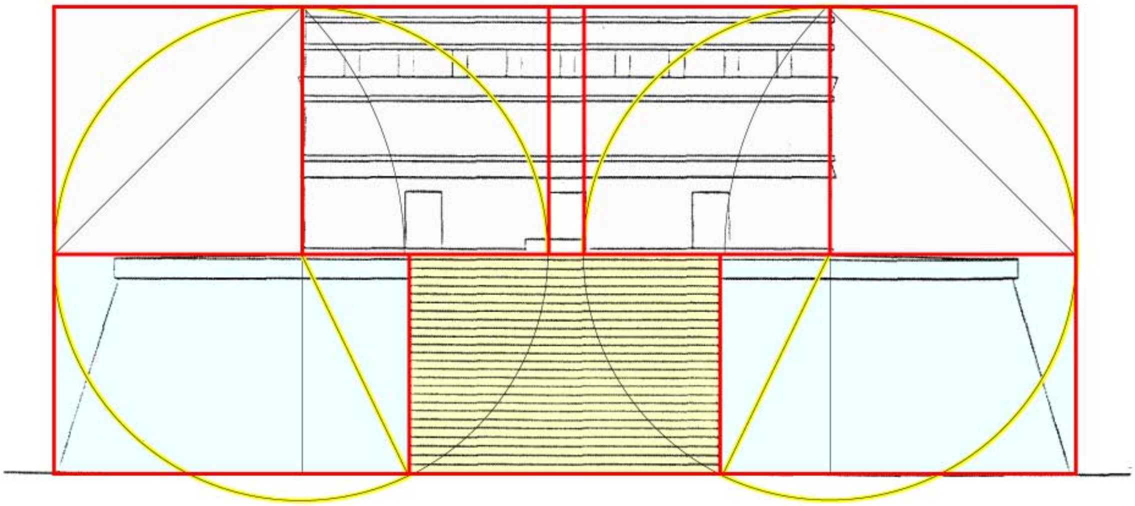


In south elevation a, the height and width of the structure is inscribed by a rectangle. The staircase and the right and left sides of the upper platform are inscribed by square root of two rectangles (shaded yellow). The upper two square root of two rectangles are subdivided by squares that terminate at the outer edges of the temple, the diagonals of which are the radii of the arcs that terminate at the outer edges of the lateral doorways (highlighted yellow lines).
The height of the platform, from the edges of the staircase to the lower corners of the platform, are inscribed by phi rectangles (shaded blue). Vertical lines drawn from the lower corners of the temple to the base of the platform create square root of four rectangles, the diagonals of which are the radii for the arcs (yellow highlighted lines) that determine the lengths of the phi rectangles.
A square is inscribed between the lateral doors and the top of the roof (shaded gray).

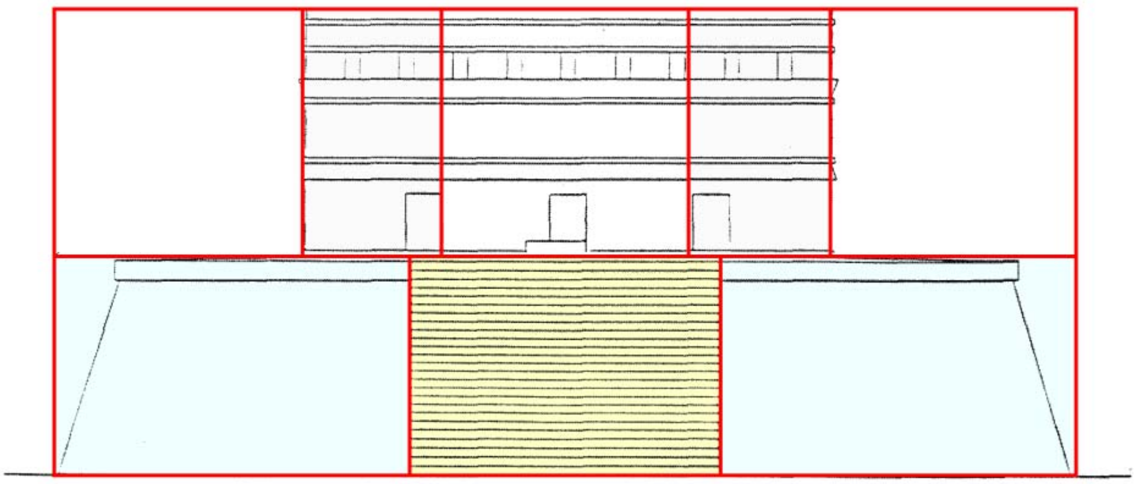
Figure 132

South Elevations b and c of Structure 3C7, Chichen Itza,
Yucatan, Mexico
Measured Drawing by Karl Ruppert (1952)

b



c



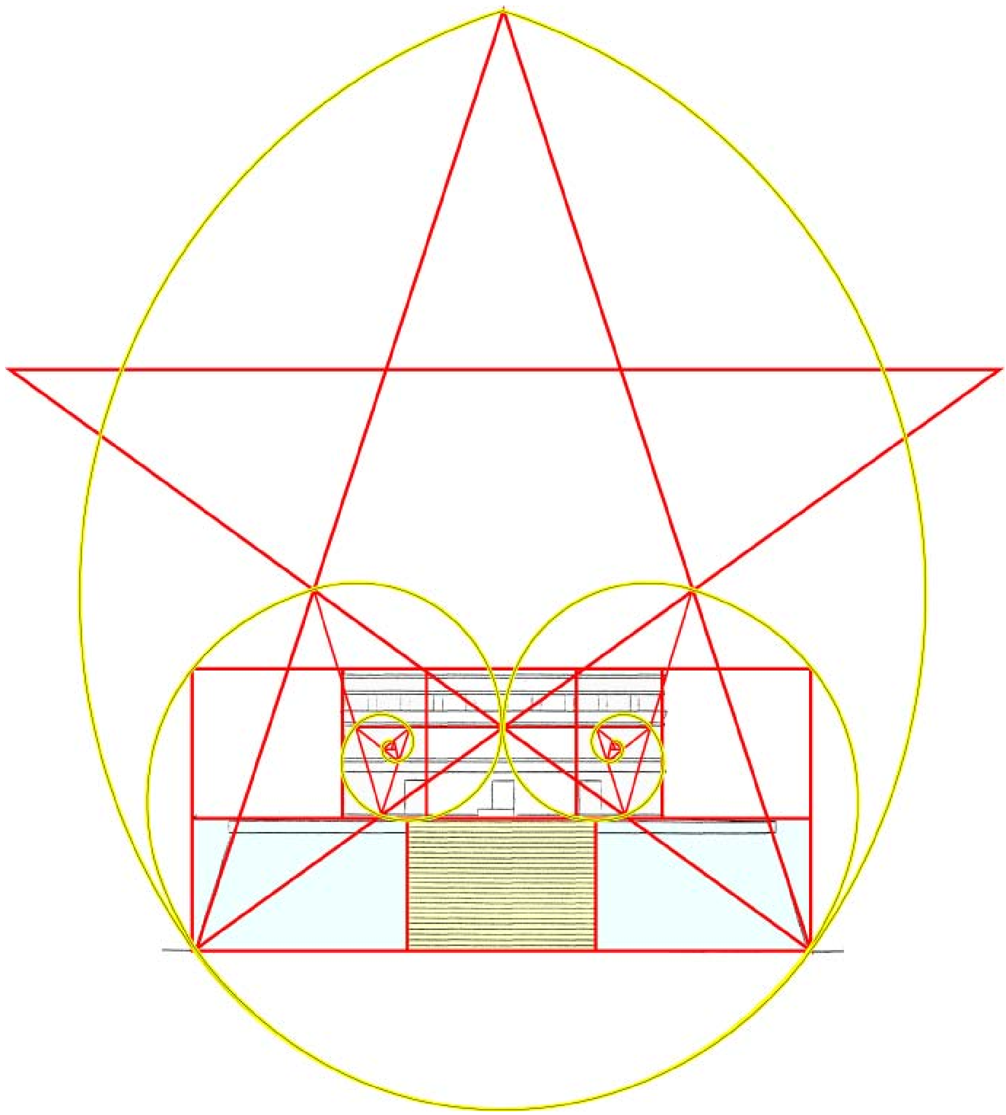
In south elevation b, the same radii of the arcs that determine the length of the phi rectangles (shaded blue) are used as radii to create double squares (shaded grey) that terminate at the lateral exterior edges of the central doorway.

In south elevation c, note the square root of three rectangles (shaded grey) that are framed by three squares (in white).

Figure 133

South Elevation of Structure 3C7 (d), Chichen Itza,
Yucatan, Mexico
Measured Drawing by Karl Ruppert (1952)

d

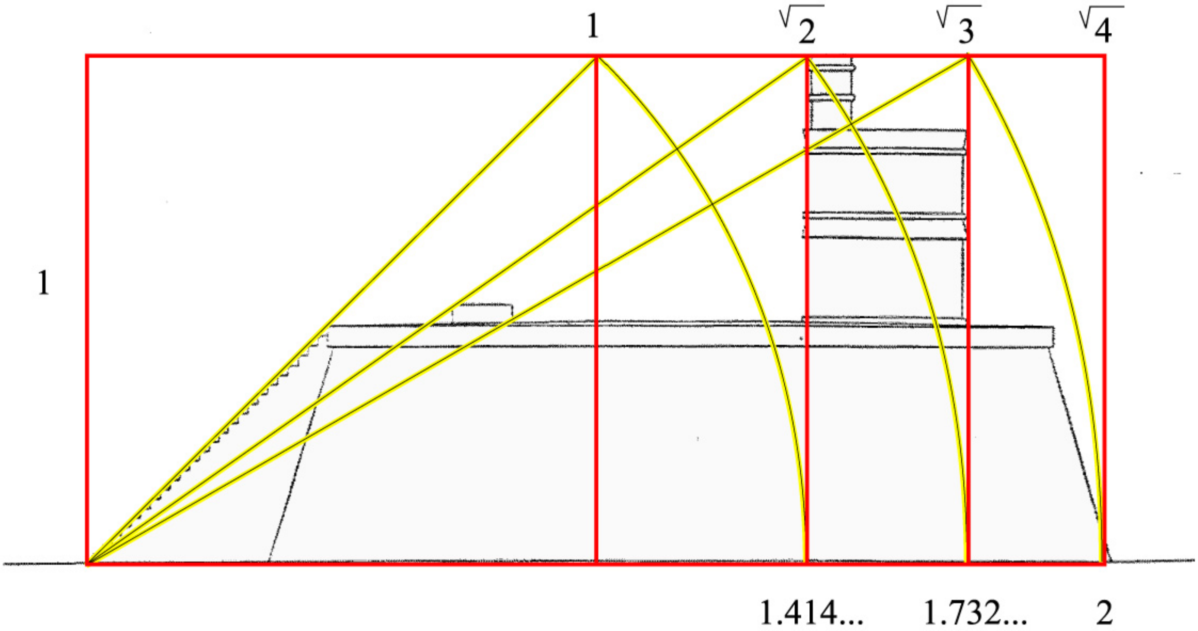


I consider the diagram that overlays Structure 3C7 speculative. Because the angle of the platform and the angle of the base of a pentagram are seventy-two degrees, I decided to place the base of the pentagram, with its equiangular spirals, at the base of the platform. Note how the spirals fit neatly within the temple and frame the doorways. Also note how the rectangle that inscribes the structure fits neatly within the spirals.

Figure 134

East Elevation of Structure 3C7 (a), Chichen Itza,
Yucatan Peninsula, Mexico
Measured Drawing by Karl Ruppert (1952)

a

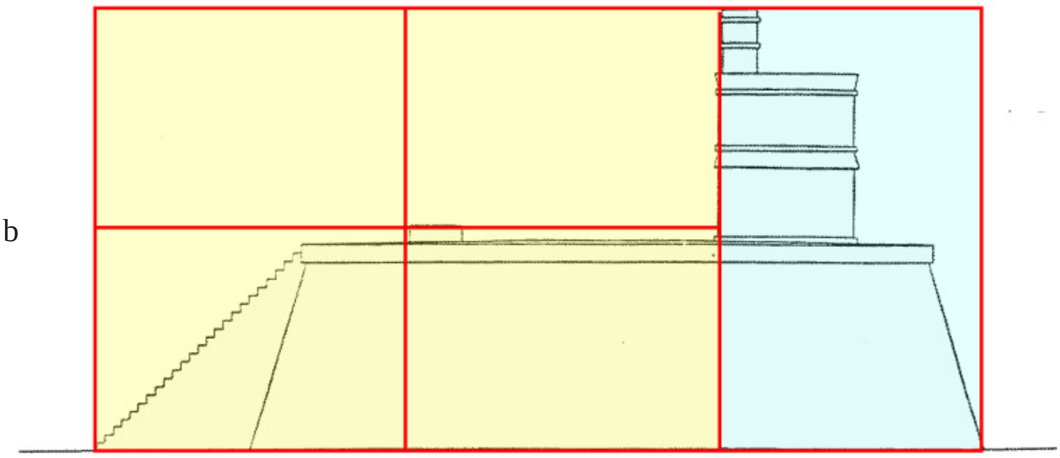


The east elevation of Structure 3C7 (a) is inscribed by a square root of four rectangle that is subdivided by a square, square root of two, and square root of three rectangles. Note how the forty-five-degree diagonal of the square is the angle of the staircase and how the diagonals of the square root of two and three rectangles frame the rectangular altar on the top of the platform.

Figure 135

East Elevations of Structure 3C7 (b and c), Chichen Itza,
Yucatan, Mexico

b



c

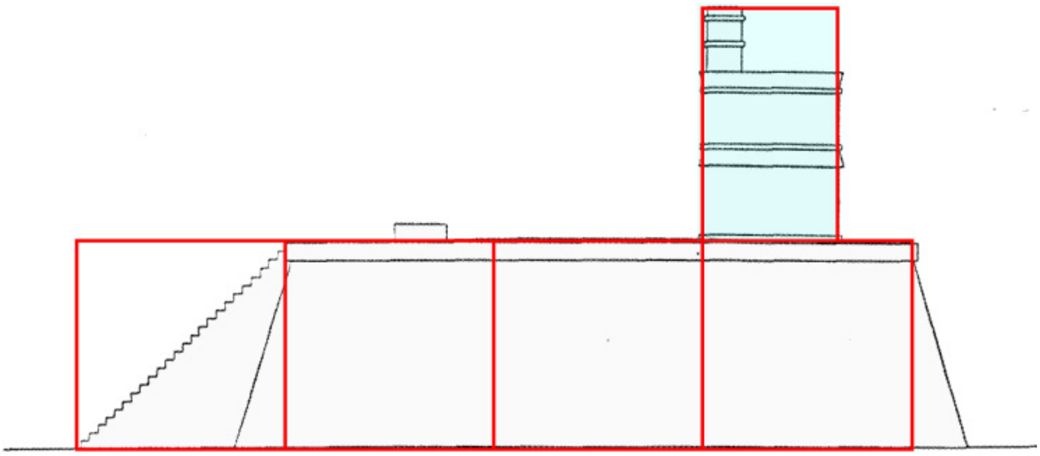
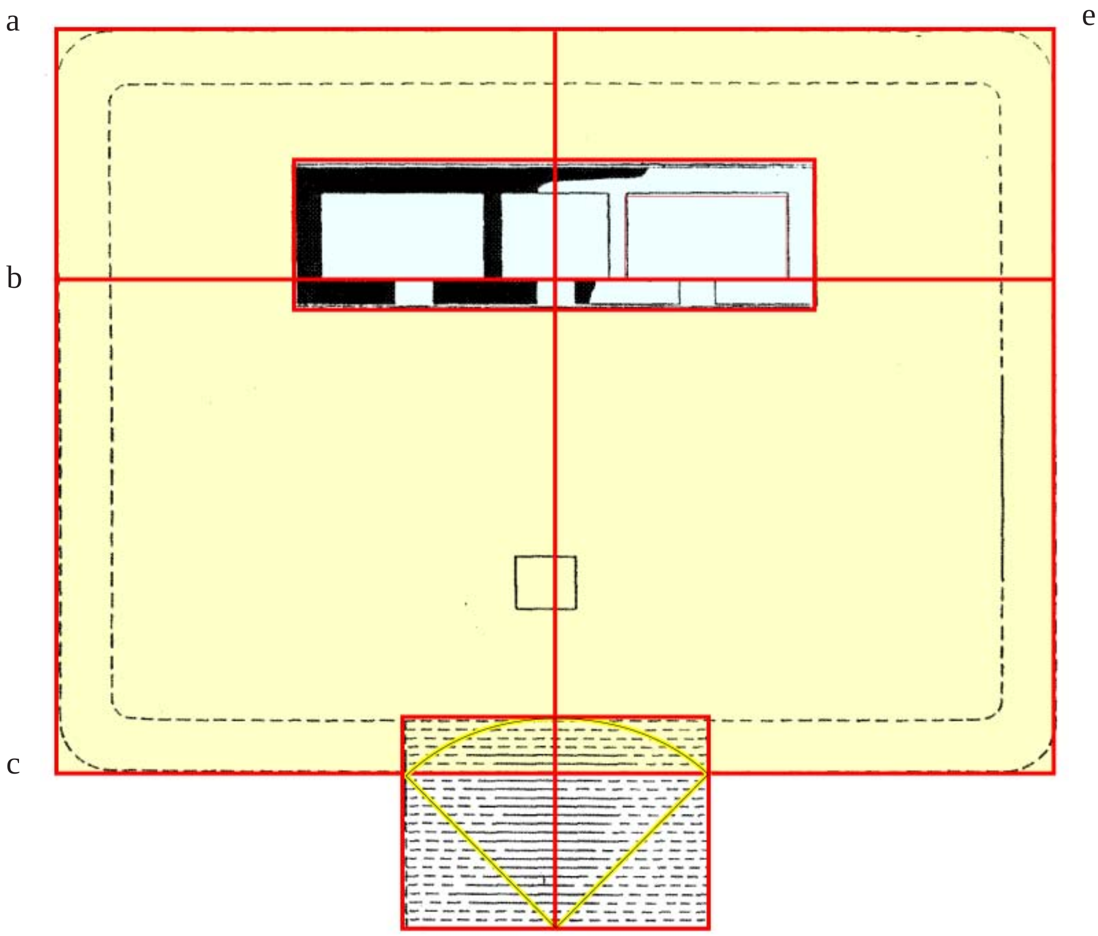


Diagram b inscribes the east elevation of Structure 3C7 by a virtually perfect square root of four rectangle, subdivided by a square root of two rectangle (quartered and shaded yellow) and a square root of three rectangle (shaded blue), as per Formula Square Root of Four-3.
Diagram c shows the relationship between four squares, the upper platform, the temple, and the staircase (red lines). The east elevation of the temple proper is inscribed by a phi rectangle (shaded blue).

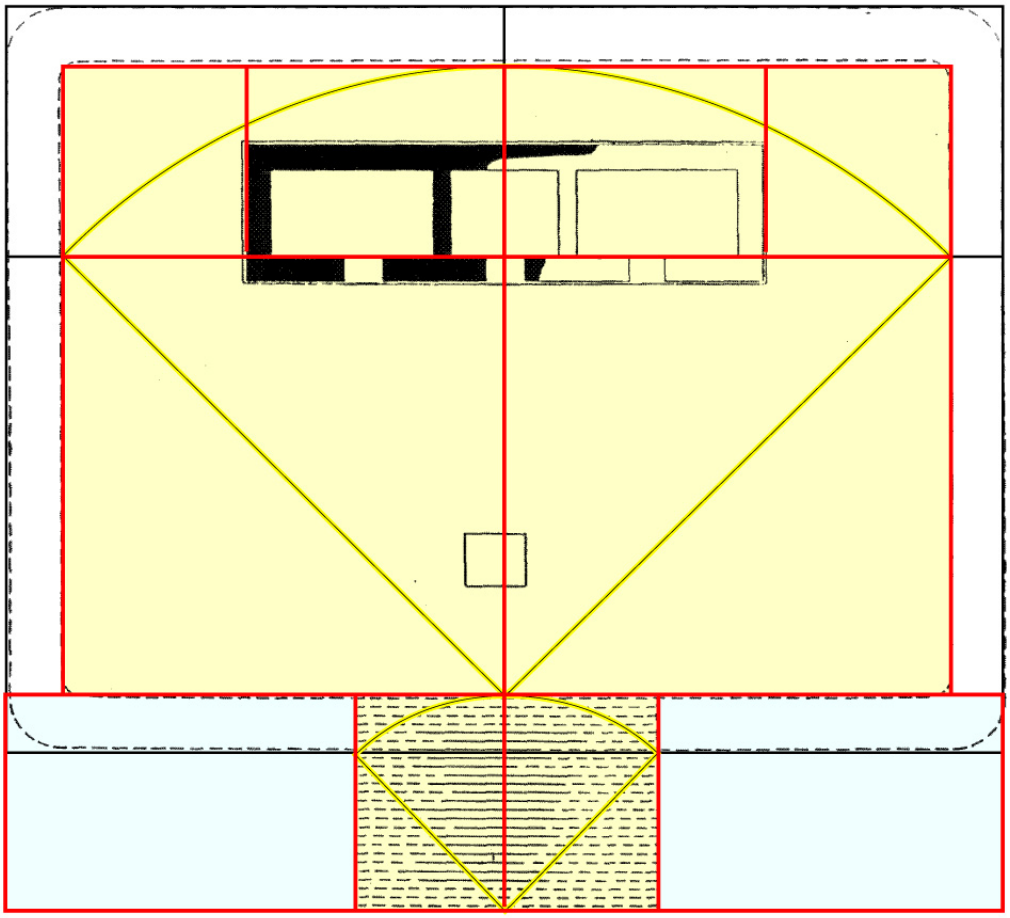
Figure 136

Plan of Structure 3C7 (a), Chichen Itza,
Yucatan, Mexico
Measured Drawing by Karl Ruppert (1952)



The plan of the bases of the platform of Structure 3C7 is inscribed by a Pythagorean 3,4,5, rectangle that is subdivided by square root of four rectangles. If a,b is one, then b,c and a,d are two and a,e is four. The portion of the staircase that extends beyond the front of the platform is also inscribed by a square root of four rectangle (in white), and the yellow highlighted diagonals of the double square are the radii for the arc that defines the width of the staircase, which is inscribed by a square root of two rectangle.
The temple at the top of the platform is inscribed by a rectangle whose width is one to a length of two times square root of three (shaded blue).

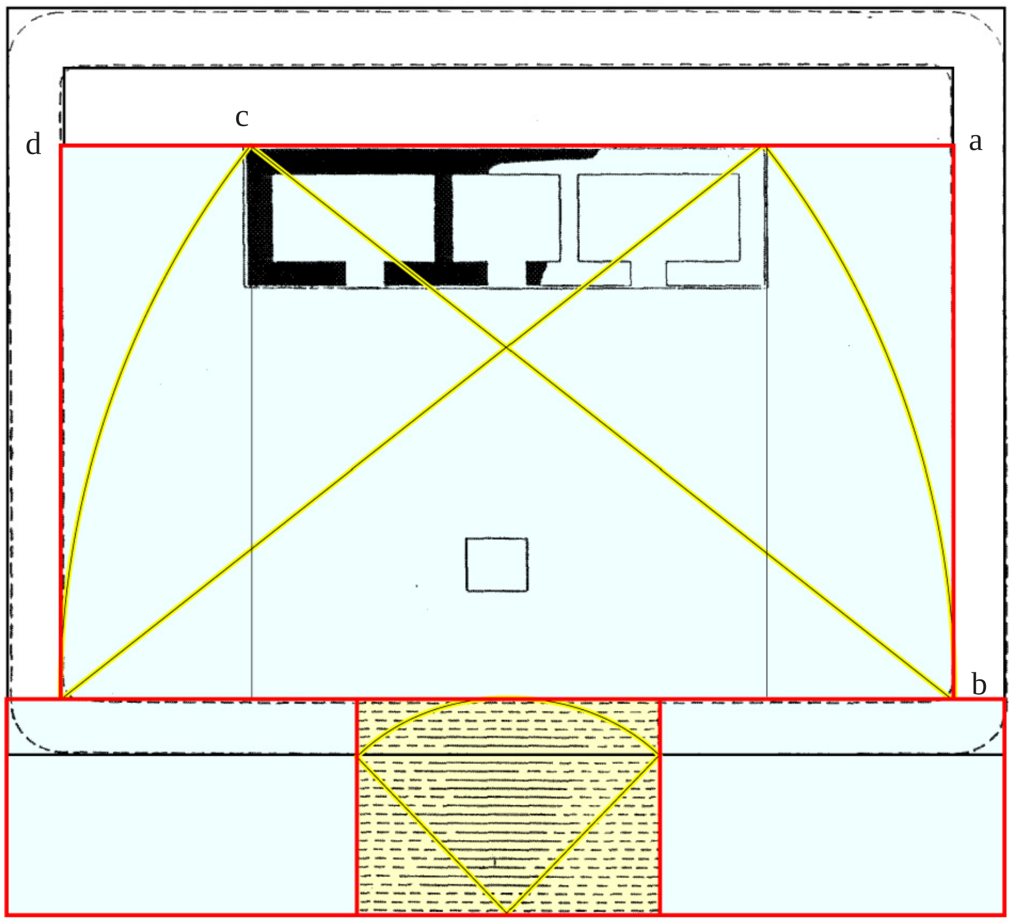
Figure 137
Plan of Structure 3C7 (b), Chichen Itza,
Yucatan, Mexico
Measured Drawing by Karl Ruppert (1952)



The top of the platform of Structure 3C7 is inscribed by a square root of two rectangle that is subdivided into squares and square root of two rectangles via Formula Square Root of Two-2.
The staircase is inscribed by a square root of two rectangle, and to either side of the staircase phi rectangles are inscribed (shaded blue).

Figure 138

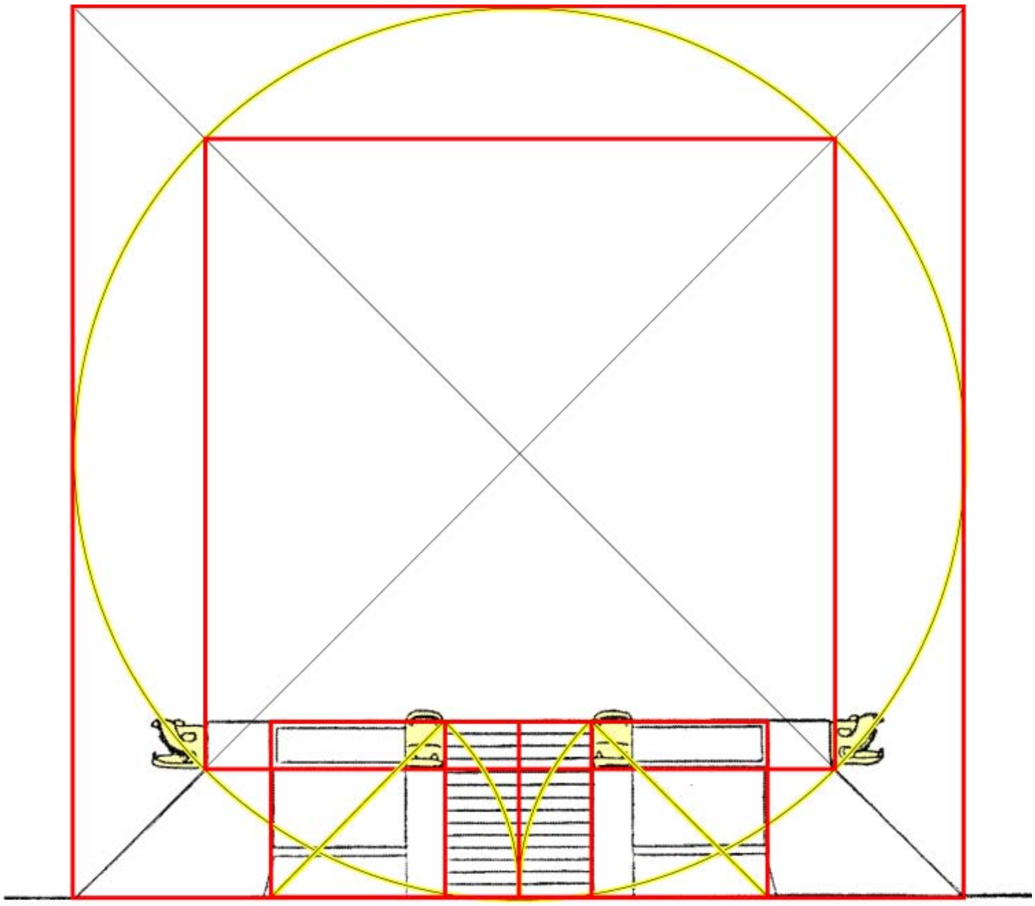
Plan of Structure 3C7 (c), Chichen Itza,
Yucatan, Mexico
Measured Drawing by Karl Ruppert (1952)



A phi rectangle is inscribed from the front of the upper platform to the rear wall of the temple (shaded blue). If a,b is one, then a.c is square root of phi, a,d is phi, and diagonal b,c is phi.

Figure 139

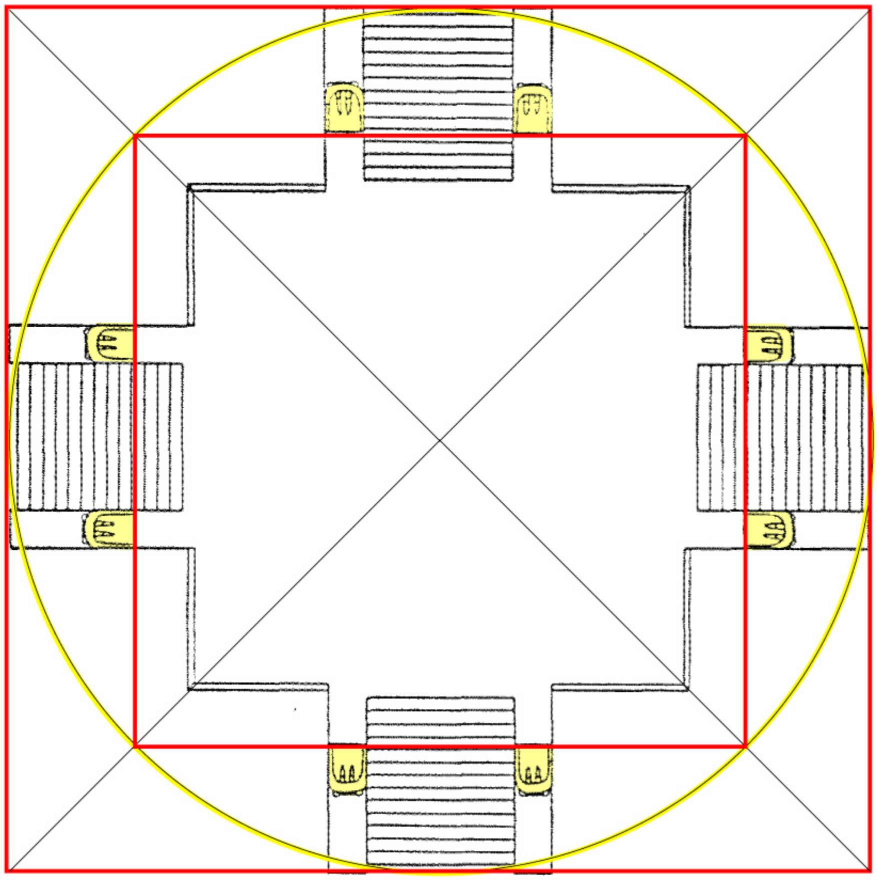
Elevation of Structure C3C, Chichen Itza,
Yucatan, Mexico
Measured Drawing by Karl Ruppert (1952)



A square is drawn from the base of the platform, and the circle inscribed within the square inscribes a smaller square. If the width of the smaller square is one, the width of the larger square is square root of two. The slope of the platform is forty-five degrees. Note the relationship of the width of the smaller square to the width of the upper platform and to the serpent heads (shaded yellow).Note that the circle appears to be grasped in the mouths of the serpent heads in profile.

Figure 140

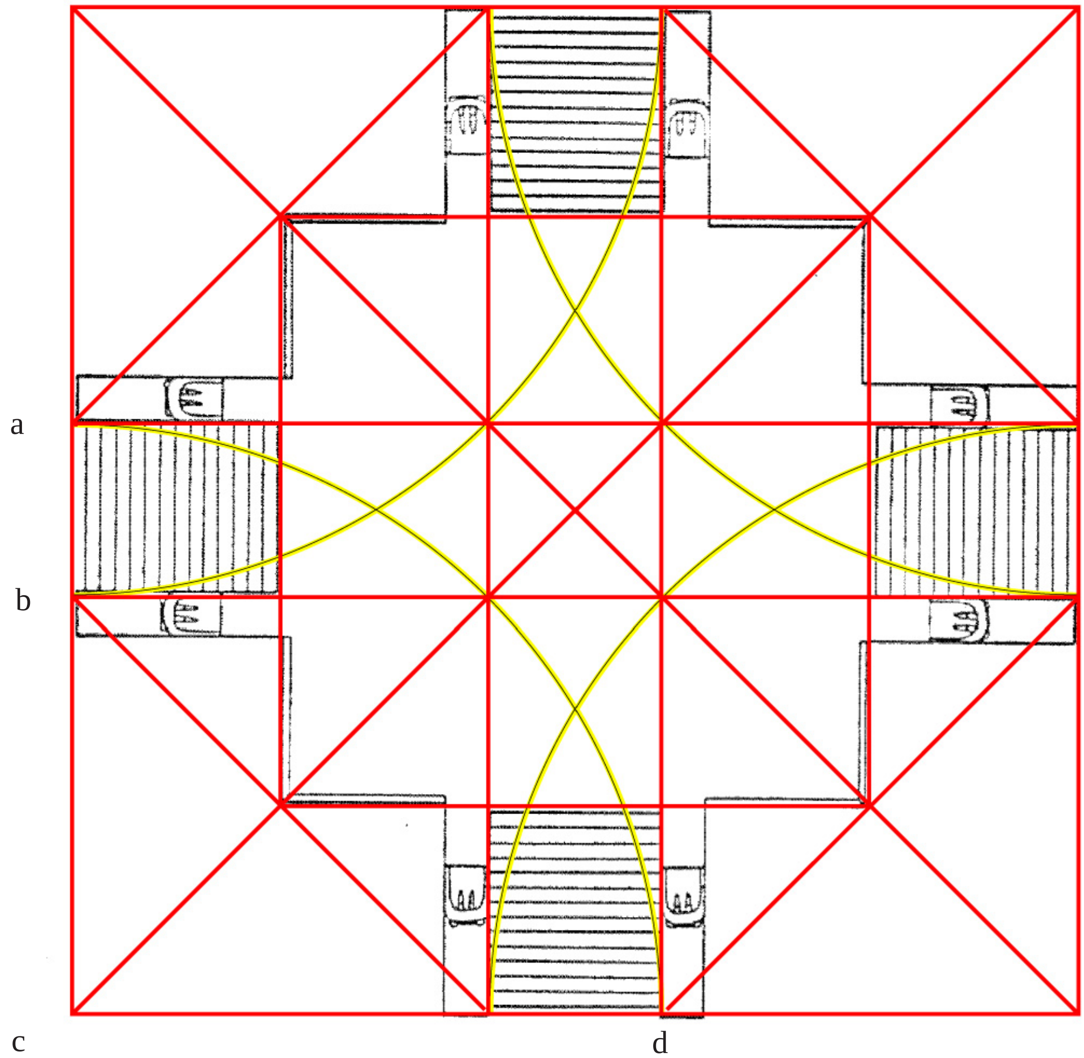
Plan of Structure C3C (a), Chichen Itza, Yucatan, Mexico
Measured Drawing by Karl Ruppert (1952)



A square is drawn from the base of the platform, a circle is inscribed within the square that inscribes a smaller square. If the width of the smaller square is one, the width of the larger square is square root of two. Note the relationship of the the width of the smaller square to the width of the upper platform and to the the serpent heads (shaded yellow).

Figure 141

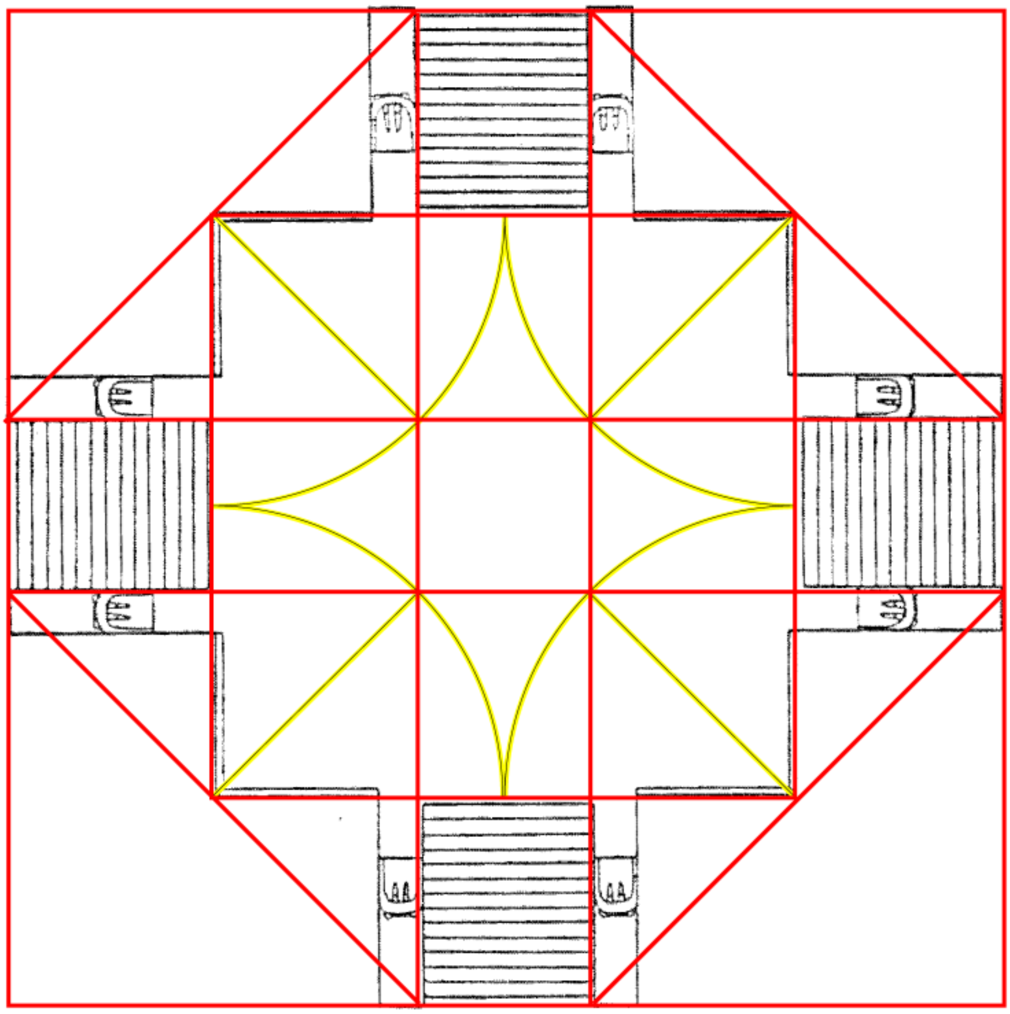
Plan of Structure C3C (b), Chichen Itza,
Yucatan, Mexico
Measured Drawing by Karl Ruppert (1952)



Structure 3C3 is a square platform with four staircases. Straight lines drawn from the edge of each staircase form four squares at the corners of the diagram. At the center of these squares are the corners of the platform. If b,c is one, then c,d is square root of two. Line c,d is also the radius for arc a,d.

Figure 142

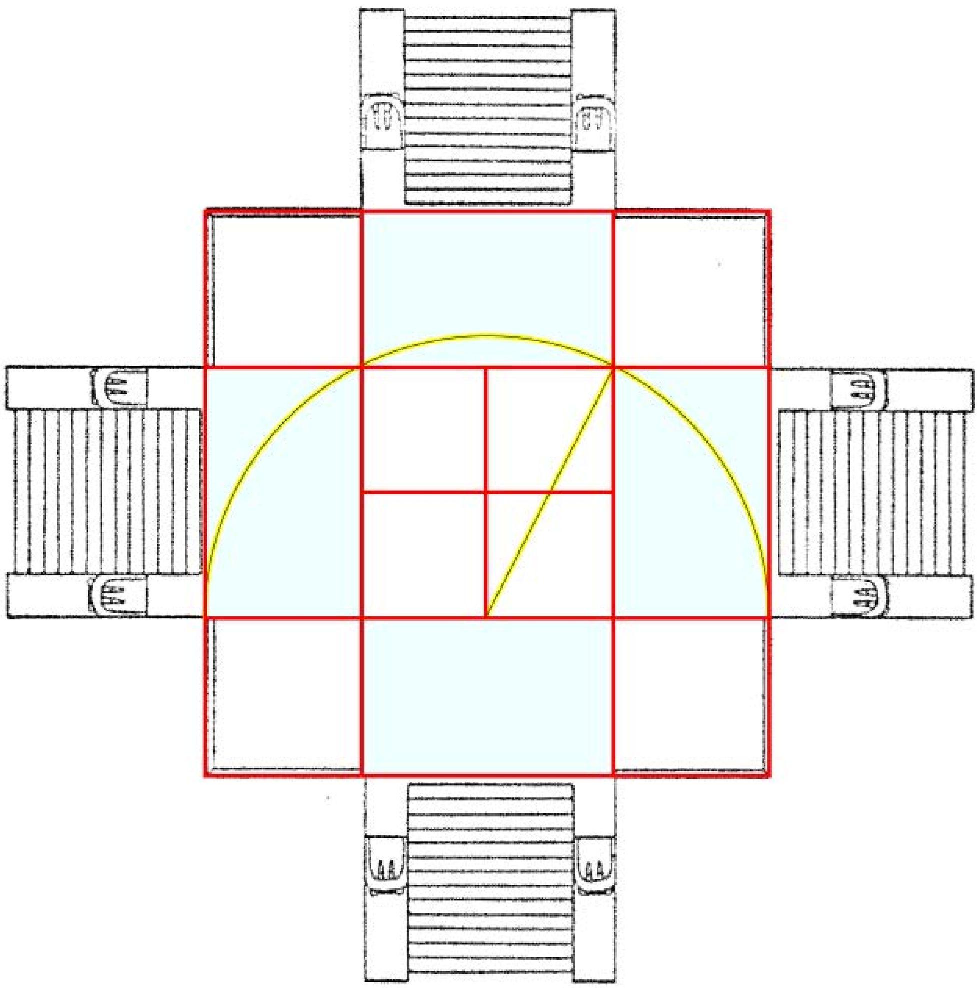
Plan of Structure C3C (c), Chichen Itza,
Yucatan, Mexico
Measured Drawing by Karl Ruppert (1952)



Straight lines drawn from the edge of each staircase form four squares at the corners of the platform. The diagonals of theses squares (highlighted in yellow) are the radii for the four arcs that terminate at the centers of the staircases.

Figure 143

Plan of Structure C3C (d), Chichen Itza, Yucatan, Mexico
Measured Drawing by Karl Ruppert (1952)

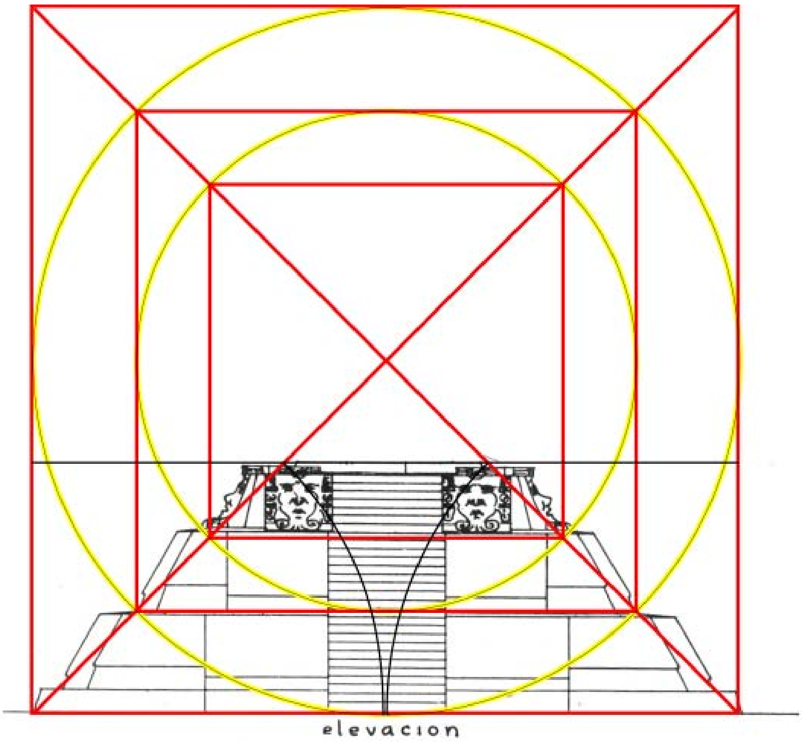


Straight lines drawn from the upper corners of each balustrade form a square in the center of the platform. This square (in white) is quartered and an arc is drawn from the center base to the upper corner to form phi rectangles (shaded blue).

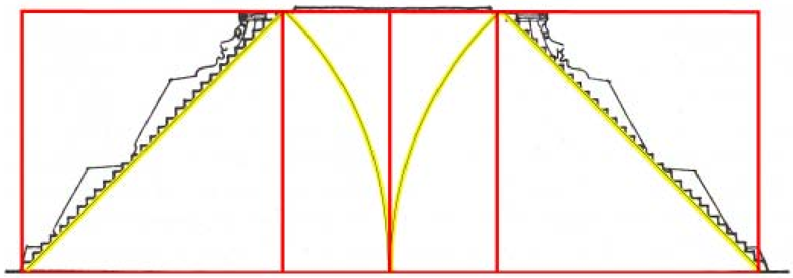
Figure 144

Elevation and Section of the Temple of the Faces, Acanceh,
Yucatan, Mexico
Measured Drawing by L. MacGregor K. (Marquina 1951)

a



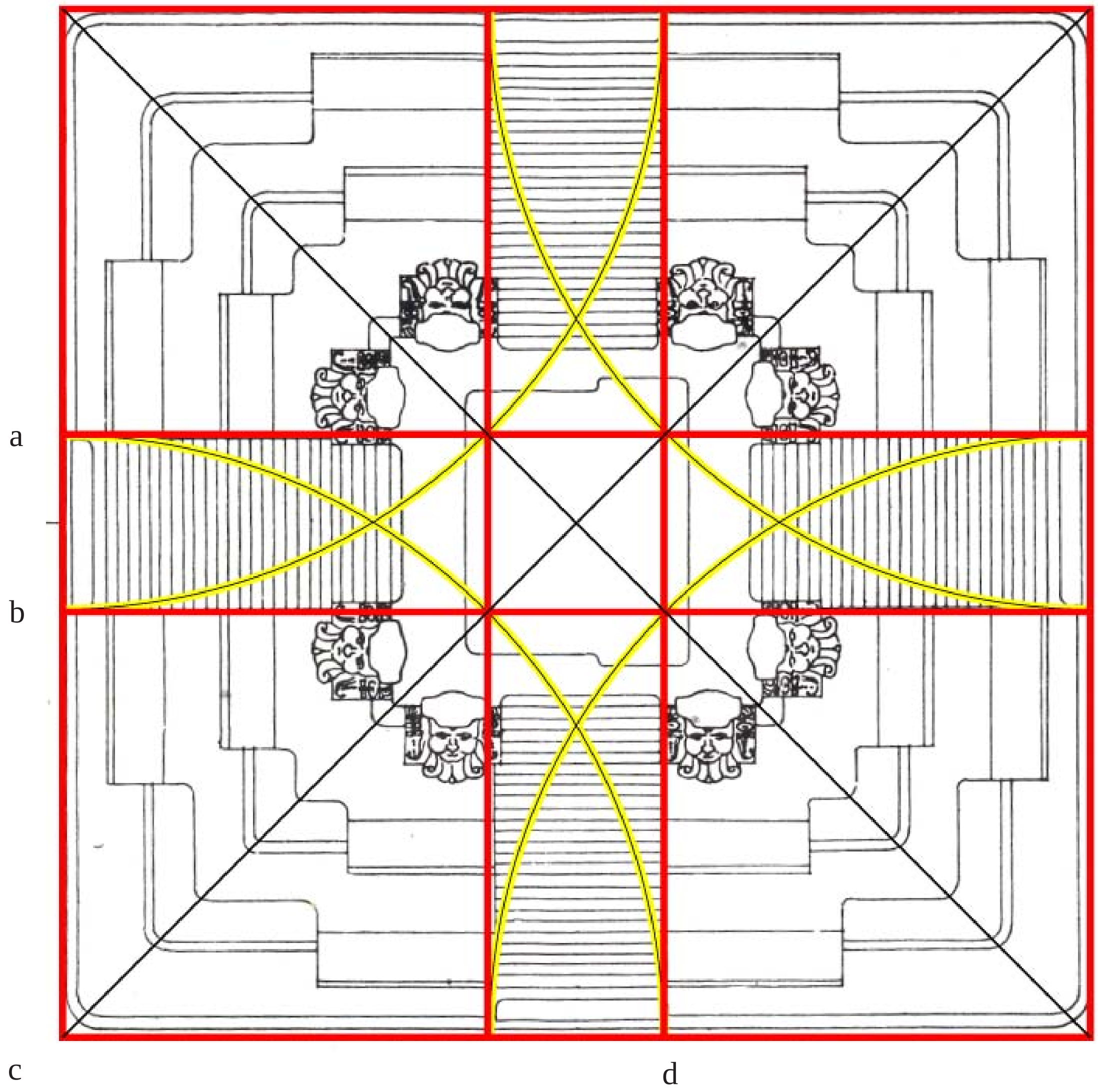
b



In the front elevation (a), the concentric circles that are overlaid determine the height and width of the platforms.
In the section (b), the forty-five degree diagonal lines that overlay the staircases are the radii for the arcs that produce two root two rectangles.

Figure 145

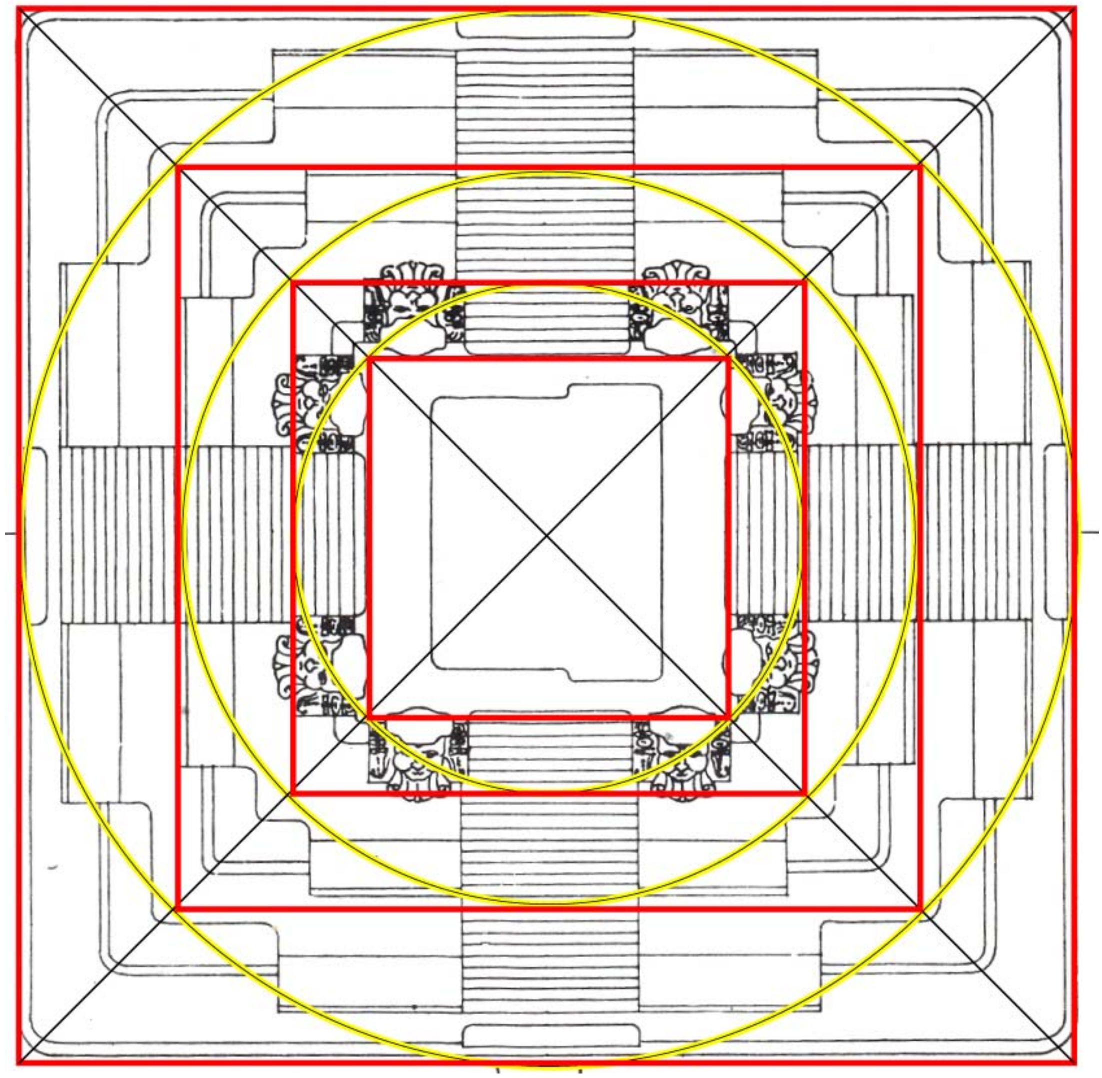
Plan of the Temple of the Faces (a), Acanceh,
Yucatan, Mexico
Measured Drawing by L. MacGregor K. (Marquina 1951)



Identically to Structure 3C3 at Chichen Itza (Figure 140), straight lines are drawn from the edges of each staircase to form four squares at the corners of the diagram. If b,c is one, then c,d is square root of two. Line c,d is also the radius for arc a,d.

Figure 146

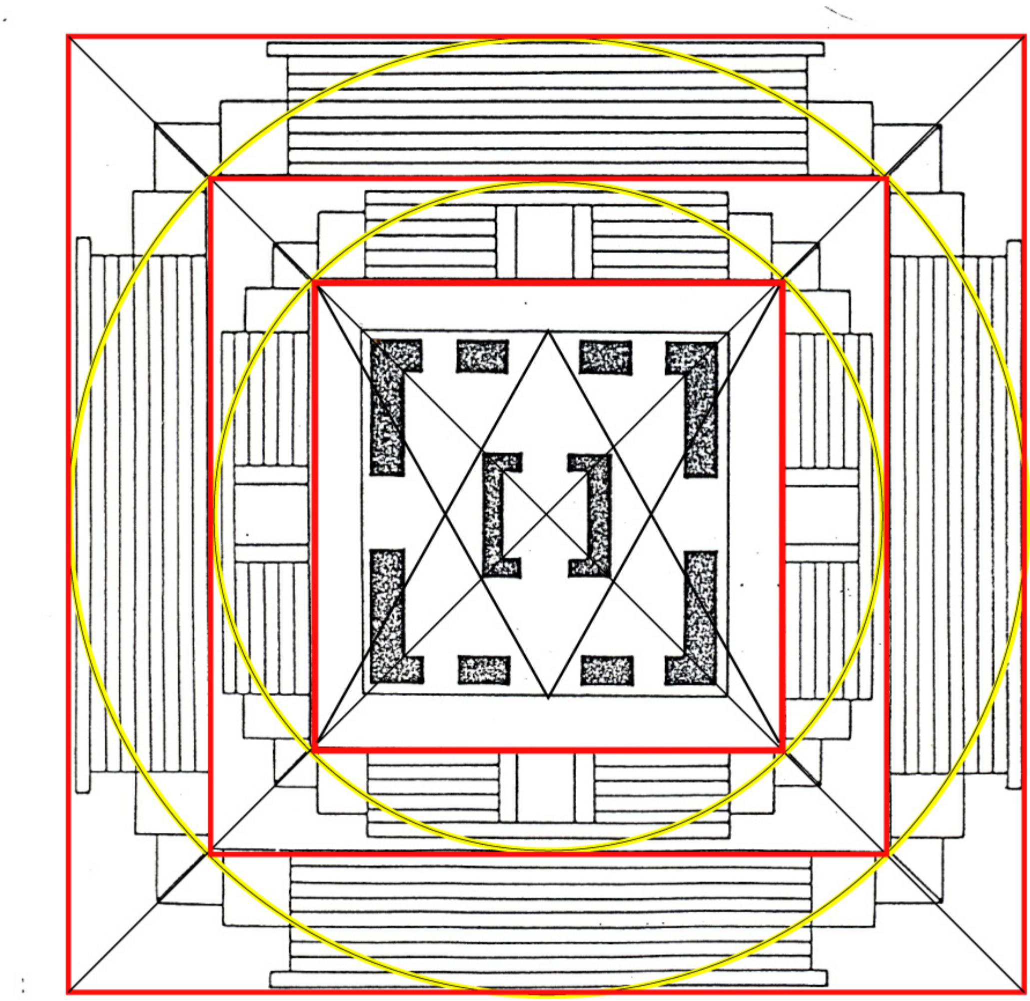
Plan of the Temple of the Faces (b), Acanceh,
Yucatan, Mexico
Measured Drawing by L. MacGregor K. (Marquina 1951)



Concentric circles and squares overlay the plan of the Temple of the Faces, and each square inscribes the three raised platforms and the uppermost platform of the pyramid. From the smallest to the largest, each square has a root two relationship to the previous square.

Figure 147

Plan of the Temple of the Seven Dolls, Dzibilchaltun,
Yucatan, Mexico
Measured Drawing by L. MacGregor K. (Marquina 1951)



As with the plan of the Temple of the Faces from Acanceh, concentric circles and squares overlay the plan of the Temple of the Seven Dolls and each square inscribes the raised platforms and the uppermost platform of the pyramid. From the smallest to the largest, each square has a root two relationship to the previous square.

Beyond the Maya area, at the Late Post-Classic site of Zempoala, in Veracruz, the Temple of the Gods of the Wind combines some of the proportions we have already seen at a fascinating level of complexity. On the plaza in front of the principal structure sits a circular raised platform with a single staircase and a square raised platform with four staircases. The main structure consists of a steep platform that is rectangular in the front and semicircular in back and supports a circular temple at the top. The rear of this steep platform rests on a low, U-shaped, rectangular platform. At the front of the platform, a staircase and balustrade access a slightly higher platform that supports a rectangular temple with a columnar altar at the center of the front entrance.

The rectangular portion of the steep platform is inscribed by a phi-squared rectangle (shaded blue), and its length is equal to the width of the low U-shaped platform. Straight lines drawn from the front corners of this steep platform to the rear wall of the U-shaped platform thus forms a square, the remainder of which is a phi rectangle (shaded light yellow). The center of the round temple at the top of the steep platform is also the center of this square. The diagonal lines from the center of this square to to its upper corners are used as radii to form the arcs (highlighted in yellow) that define the width of the balustrades, which in turn define the interior part of the U- shaped platform, creating a square root of two rectangle. The diagonals from the center of the initial square to the corners of this square root of two rectangle are used to form the arcs that define the length of the U-shaped platform and produce a square root of three rectangle.

The rectangular raised platform at the front of the steep platform is also inscribed by a phi rectangle (shaded light yellow). The plan of the balustrade that accesses the platform is inscribed by a square root of four rectangle. The diagonals of this upper balustrade are used to form arcs that terminate at the edges of the front balustrade of the lower platform and cross one another at the base of the columnar altar at the center of the front entrance of the rectangular temple (yellow highlighted lines) creating a phi rectangle between the two balustrades (shaded a darker yellow).

The staircase that accesses the steep platform (not including the balustrade) is inscribed by a Pythagorean 3,4,5, rectangle (in white).

Finally, another phi-squared rectangle is drawn from the front edge of the front platform to the front edge of the small square platform centered before it (shaded blue). The width of the small square platform subdivides this phi-squared rectangle into two squares and a central phi rectangle. This phi-squared rectangle, added to the phi rectangle of the front raised platform, also form a square. Note that the horizontal center line of this square terminates at the lower edges of the lateral balustrades of the front platform and that the center of this square is at the center of the columnar altar at the entrance of the rectangular temple (fine black lines).

Figure 148

Plan of the Temple of the Gods of the Wind
Zempoala, Veracruz, Mexico

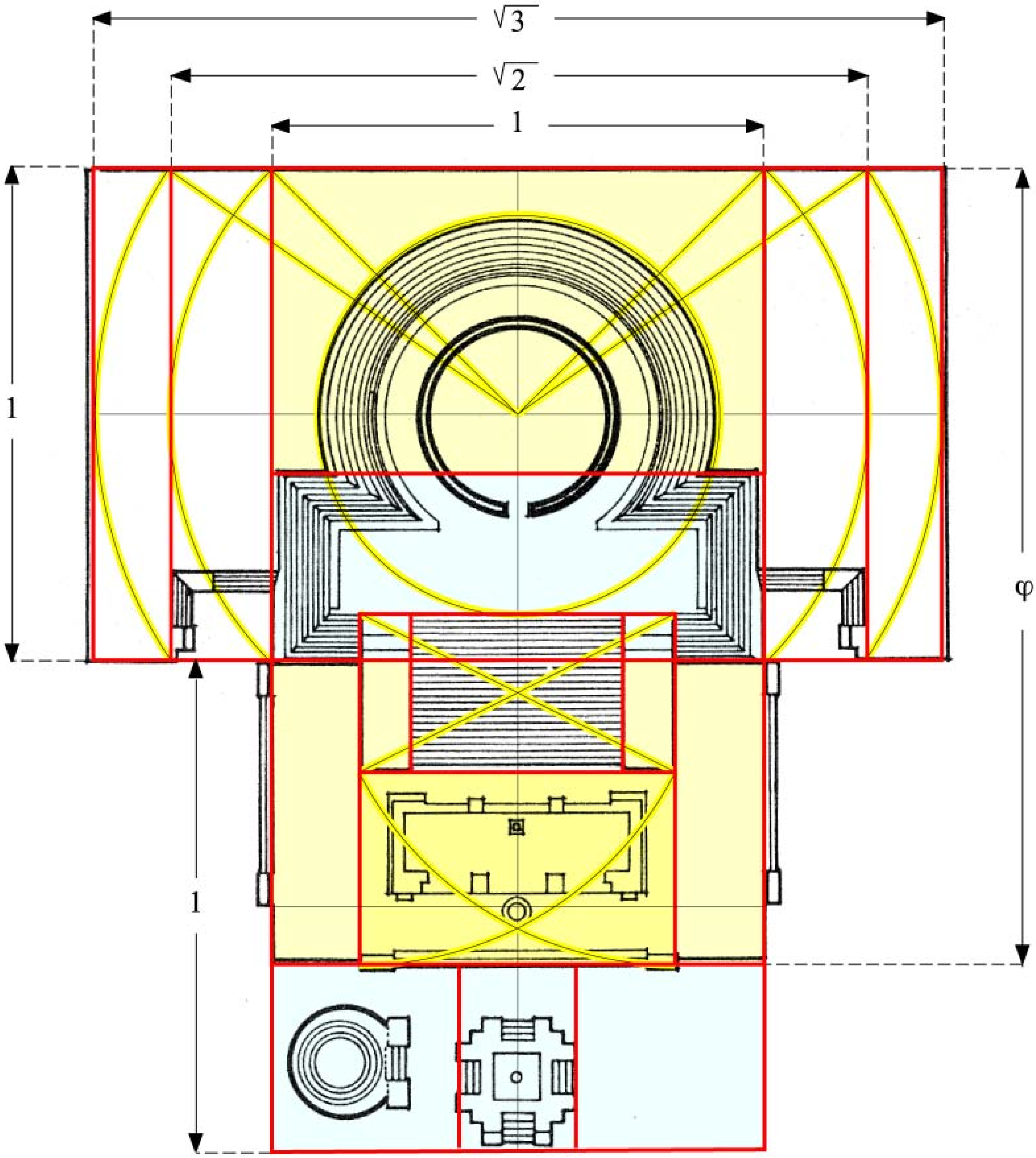
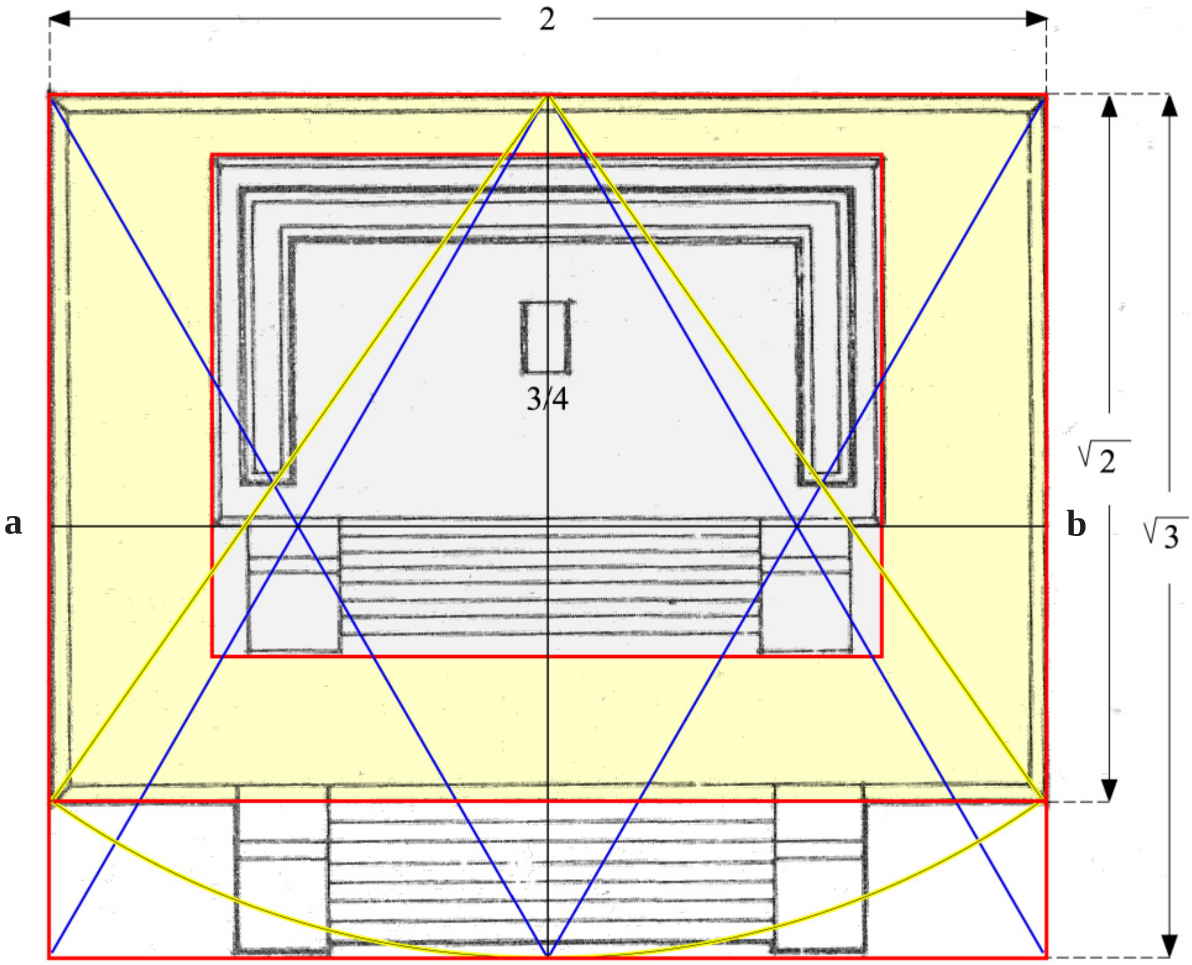


Figure 149

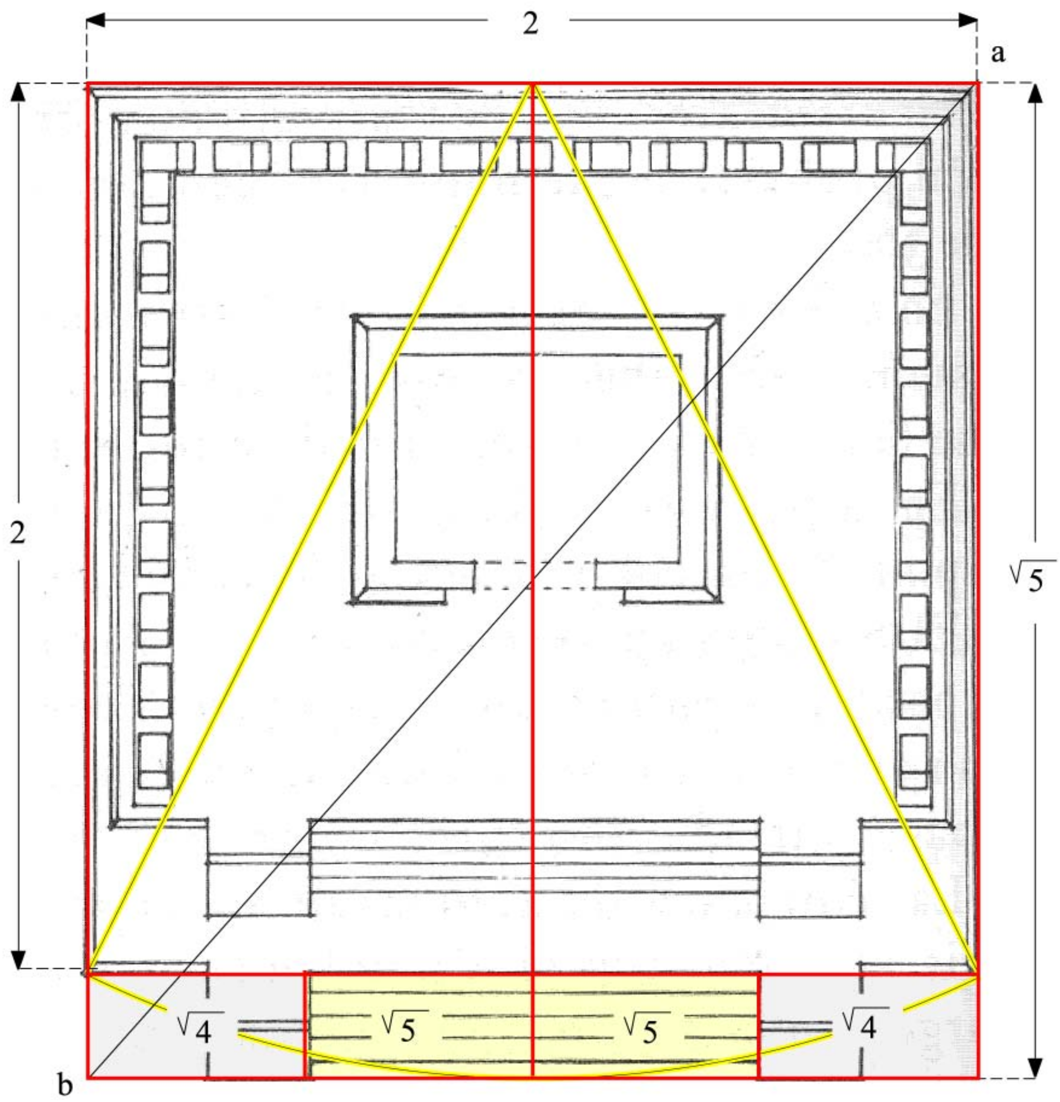
Plan of the Temple of the Faces
Zempoala, Veracruz, Mexico
Measured Drawing by I. Marquina (1951)



The plan of the Temple of the Faces is inscribed by a two over root three rectangle that inscribes two opposing equilateral triangles (blue lines). The horizontal center of the plan is line a,b at the front edge of the uppermost platform.
The principal raised platform is inscribed by a root two rectangle (shaded yellow), and the diagonals from its top center to its lower corners are the radii for the arc that determines the width of the plan.
The uppermost platform and balustrade is inscribed by a Pythagorean 3,4,5 rectangle.

Figure 150

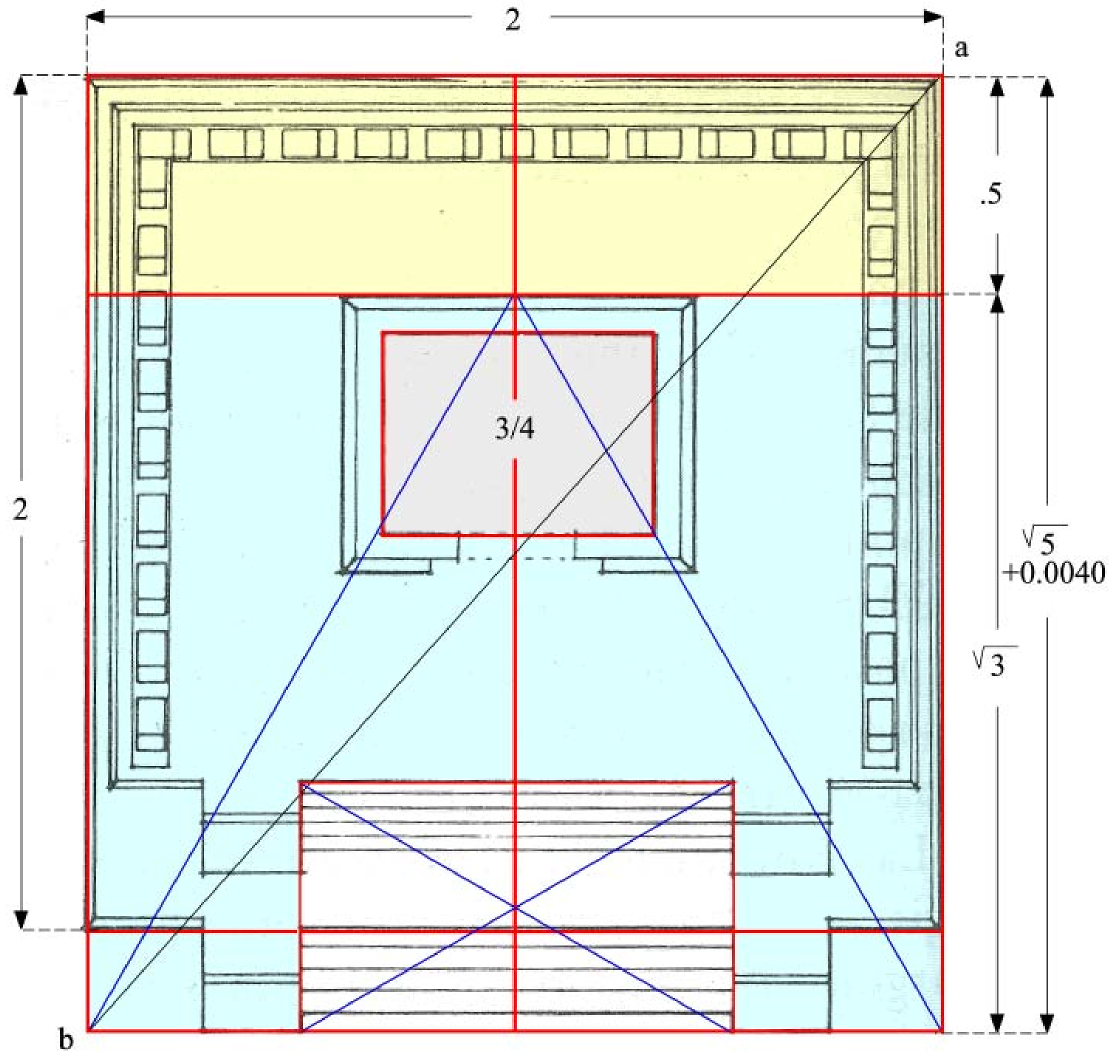
Plan of the Temple of Moctezuma (a)
Zempoala, Veracruz, Mexico
Measured Drawing by I. Marquina (1951)



Plan a of the Temple of Moctezuma is inscribed by a root five over two rectangle. If the width of the plan is two, then the length is root five and the diagonal (a,b) is three.
The raised platform is inscribed by a square, and the diagonals from the rear center of the platform to the front corners are the radii for the arc that determines the length of the platform.
The staircase is inscribed by two root five rectangles (shaded yellow), and the balustrade and the remaining space to either side of the balustrade are inscribed by root four rectangles, as per Formula Square Root of Five-1.

Figure 151

Plan of the Temple of Moctezuma (b)
Zempoala, Veracruz, Mexico
Measured Drawing by I. Marquina (1951)

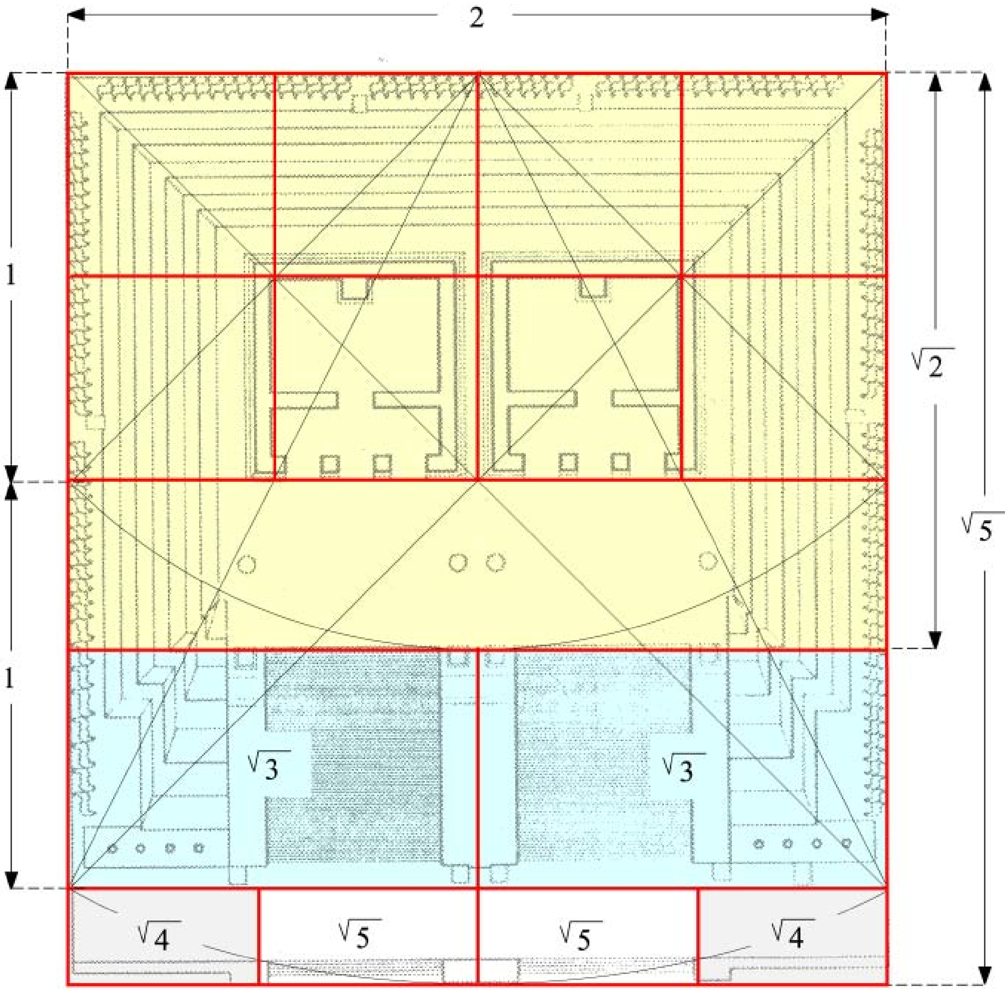


Plan b of the Temple of Moctezuma is inscribed by a virtually perfect root five over two rectangle, as per Formula Square Root of Five-4. A two over root three rectangle is inscribed from the rear of the temple to the front of the staircase (shaded blue). The remaining space to the rear of the temple is inscribed by two root four rectangles.

The interior of the temple is inscribed by a Pythagorean 3,4,5 rectangle (shaded grey) and the staircases are inscribed by a root three rectangle (in white) The blue lines form equilateral triangles.

Figure 152

Plan of the Pyramid at Tenayuca (a)
Zempoala, Veracruz, Mexico
Measured Drawing by I Marquina (1951)

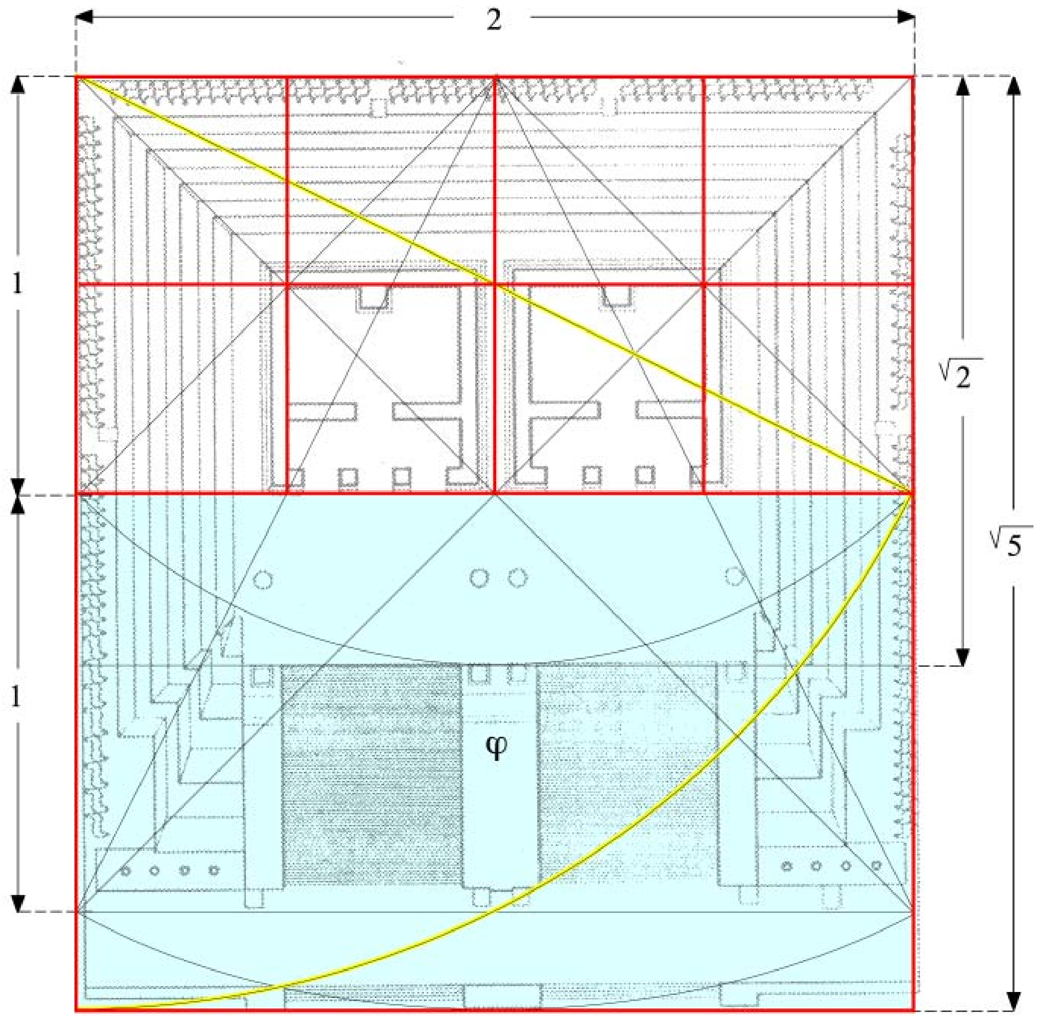


The Pyramid at Tenayuca, (not including the narrow, low platform that extends beyond the front of it), is inscribed by a square. The front edges of the twin temples at the top of the pyramid demarcate the horizontal center of this square. The upper half of the square (a double square) is quartered, and the centers of the double squares demarcate the inner rear corners of the rear galleries of the temples. The diagonals of the double square are the radii for the arc that terminates at the front edge of the platform at the top of the staircase and forms a square root of two rectangle (shaded yellow). Two virtually perfect root three rectangles, as per Formula Square Root of Four-3, are shaded blue.

The entire plan is inscribed by a root five over two rectangle that is subdivided by root four and five rectangles, as per Formula Square Root of Five-1

Figure 153

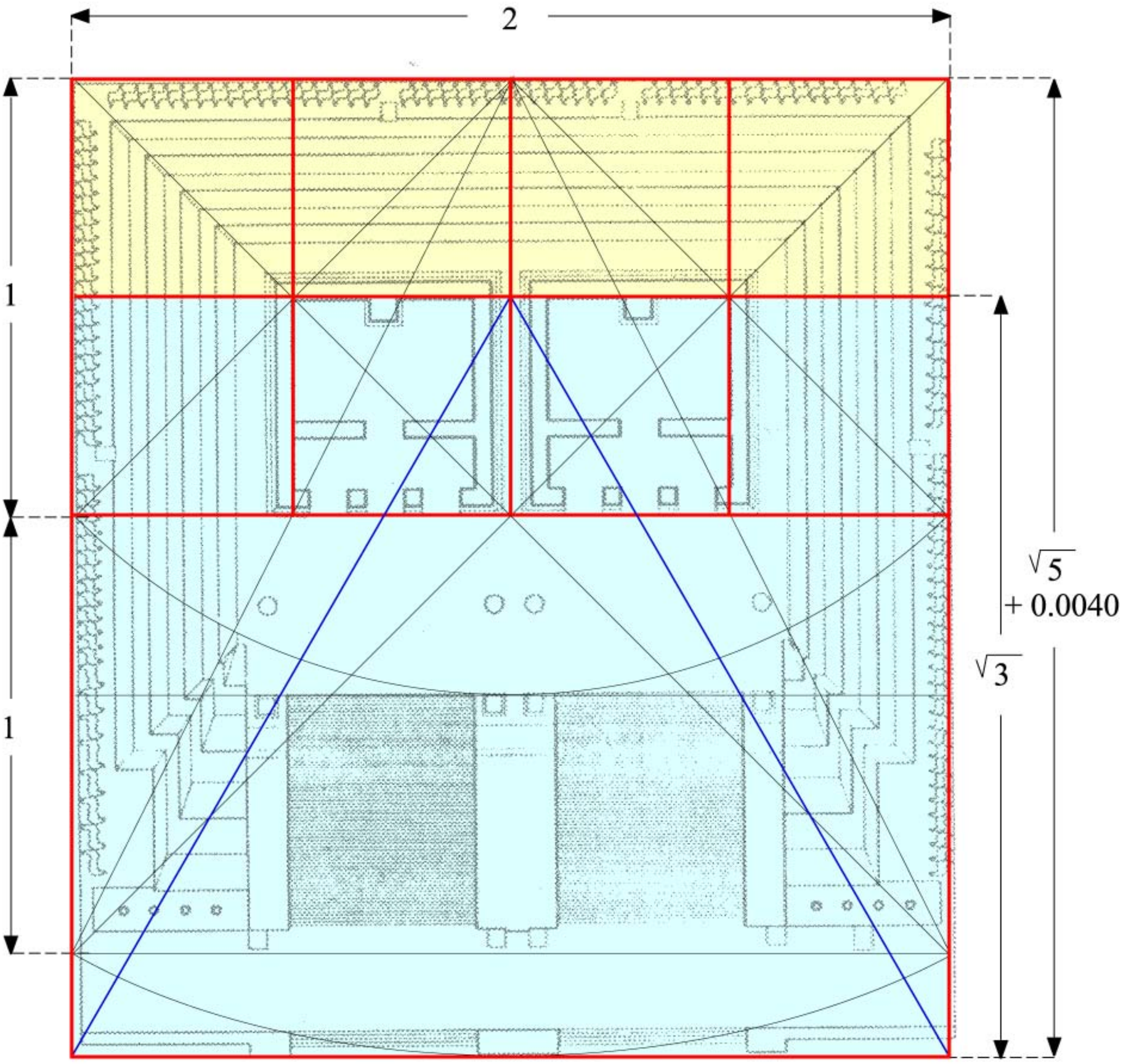
Plan of the Pyramid at Tenayuca (b)
Zempoala, Veracruz, Mexico
Measured Drawing by I. Marquina (1951)



In Plan b, the diagonal (highlighted in yellow) of the double square (in white), demarcated by the exterior of the front walls of the twin temples, is the radius of the arc that terminates at the lower left-hand corner of the diagram and forms a phi rectangle (shaded blue).

Figure 154

Plan of the Pyramid at Tenayuca (c)
Zempoala, Veracruz, Mexico
Measured Drawing by I. Marquina (1951)

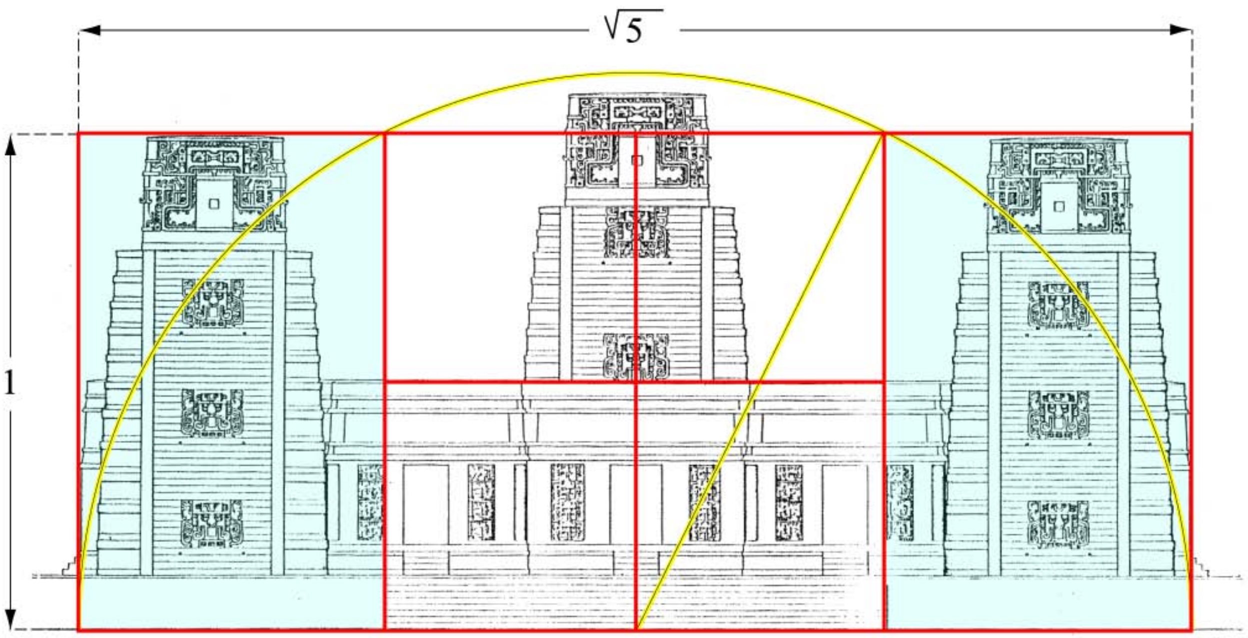


Plan c of the Pyramid of Tenayuca is inscribed by a root five over two rectangle that is subdivided by a two over root three rectangle (shaded blue), which inscribes an equilateral triangle (blue lines), as well as four squares (shaded yellow), as per virtually perfect Formula Square Root of Five-4

Figure 155

Front Elevation, Xpuhil (a),
Quintana Roo, Mexico
Measured Drawing by J. A. Gomez R. (Marquina 1951)

a

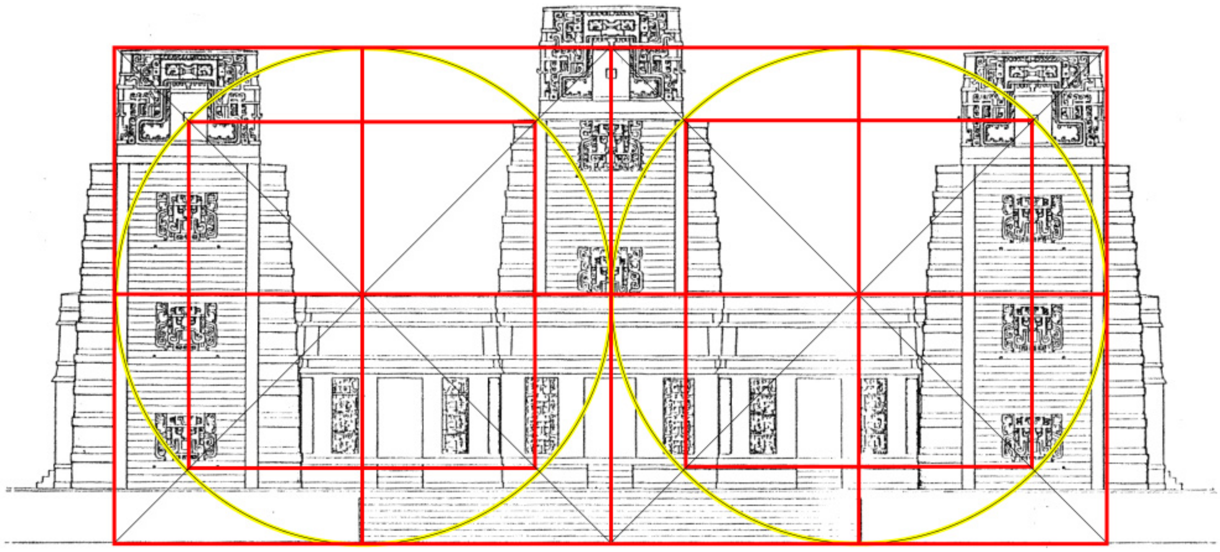


A quartered square is inscribed from the base of the staircase to the top of the lateral towers and the top of the doorway of the central tower. The horizontal center of the square demarcates the height of the roof of the temple. The diagonal, from the center base of this square to its upper corner, is the radius for the arc that terminates at the lateral edges of the temple and creates a root five rectangle that is subdivided by a square and two phi rectangles (shaded blue), as per Formula Square Root of Five-3

Figure 156

Front Elevation, Xpuhil (b),
Quintana Roo, Mexico
Measured Drawing by J. A. Gomez R. (Marquina 1951)

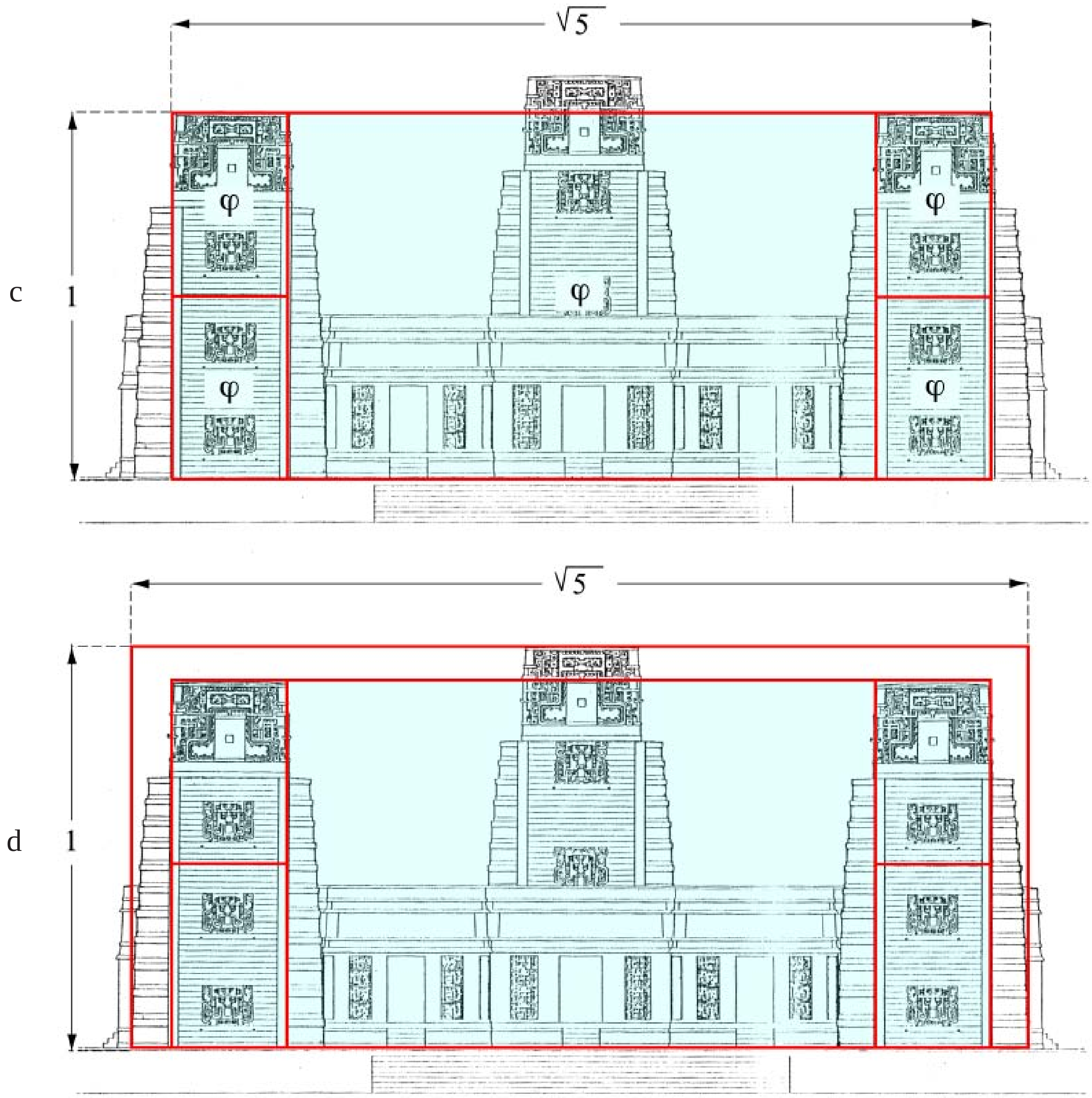
b



Beginning with the same square that produced the root five rectangle in Elevation a, a root four rectangle inscribes the balustrades and the tops of the roofs of the lateral towers. Circles are inscribed within the double squares and squares are then inscribed within these circles. The inner squares to the outer squares have a ratio of one to root two. Note that the bases of the inner squares rest on the top of the central staircase, the tops of the squares terminate at the height of the central tower, the inner sides of the squares terminate at the balustrade of the central tower, and the outer sides of the squares terminate at the center lines of the staircases of the lateral towers.

Figure 157

Front Elevations, Xpuhil (c and d),
Quintana Roo, Mexico
Measured Drawing by J. A. Gomez R. (Marquina 1951)



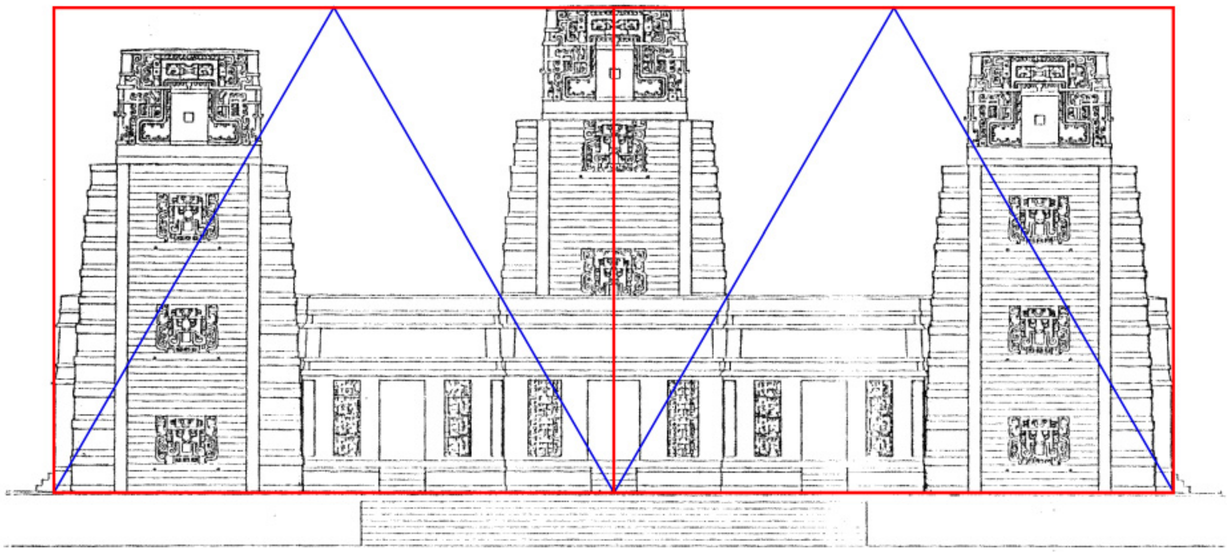
In Elevation c, the temples and balustrades of the lateral towers are inscribed by vertical phi rectangles and the space between them is inscribed by a horizontal phi rectangle. Together, these five phi rectangles precisely subdivide a root five rectangle.

In Elevation d, a root five rectangle inscribes the width of the lateral towers and the height of the central tower.

Figure 158

Front Elevation, Xpuhil (e),
Quintana Roo, Mexico
Measured Drawing by J. A. Gomez R. (Marquina, 1951)

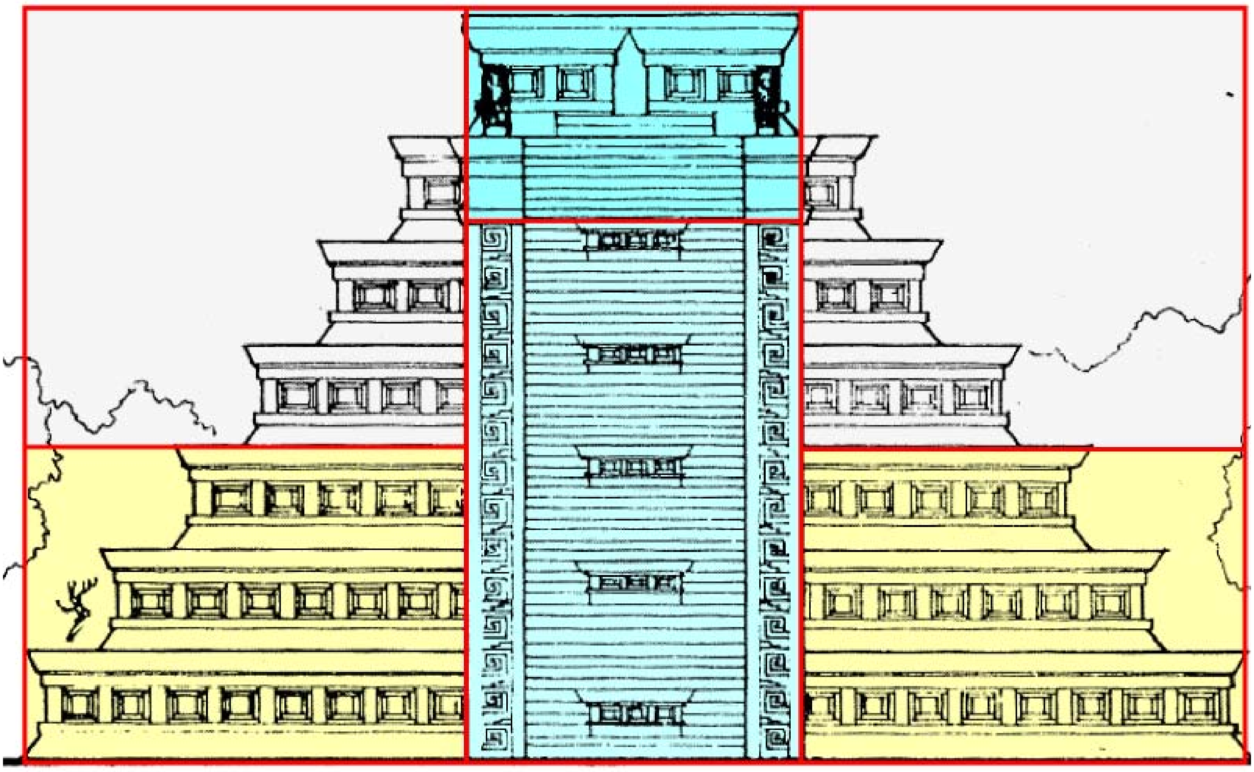
e



The width of the temple to the height of the roof of the central tower is inscribed by two, two over root three rectangles that inscribe equilateral triangles (blue lines)

Figure 159

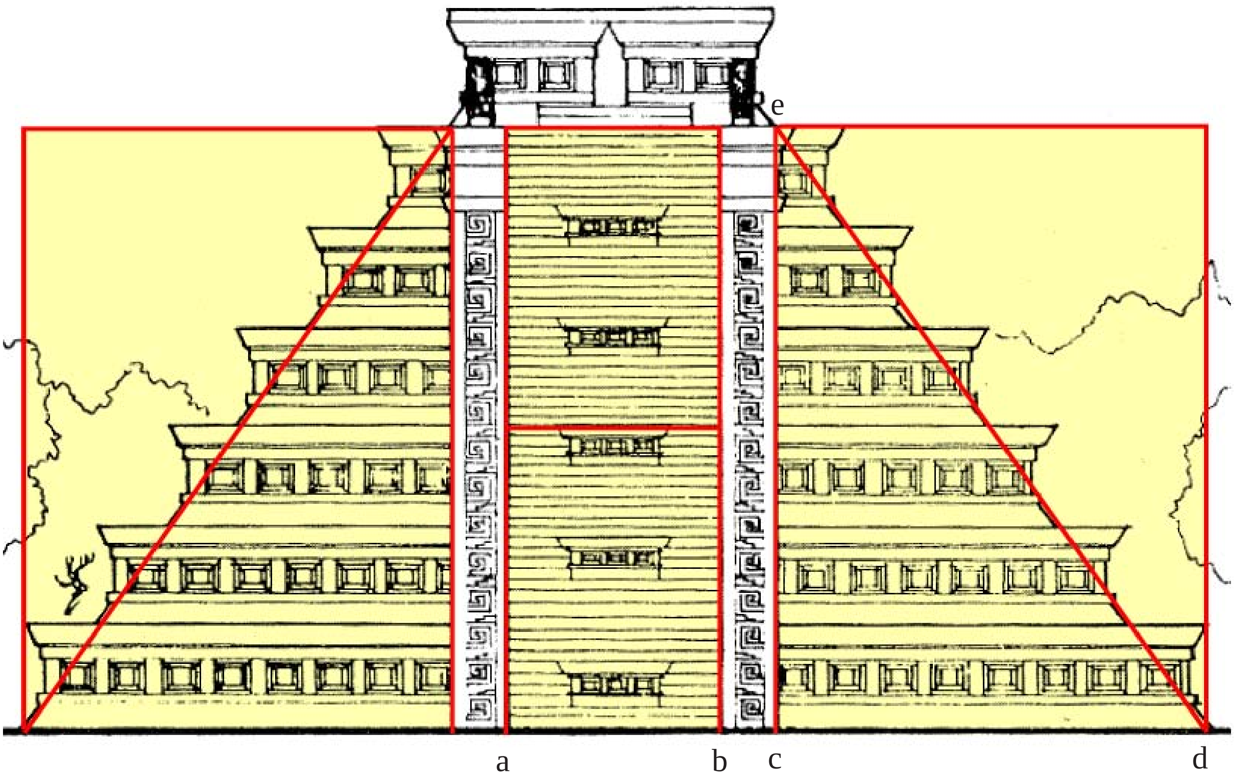
Elevation of the Pyramid of the Niches (a)
Veracruz, Mexico
Measured Drawing by L. MacGregor K. (Marquina 1951)



The front elevation of the Pyramid of the Niches is inscribed by a virtually perfect phi rectangle that is subdivided via the virtually perfect Formula Phi-3. The squares are shaded grey, the square root of two rectangles are shaded yellow. The square root of five rectangle in the center is subdivided into a vertical phi rectangle (the decorated balustrade, shaded light blue) and a horizontal phi rectangle (the plain balustrade and temple, shaded darker blue).

Figure 160

Elevation of the Pyramid of the Niches (b),
Veracruz, Mexico
Measured Drawing by L. MacGregor K. (Marquina 1951)

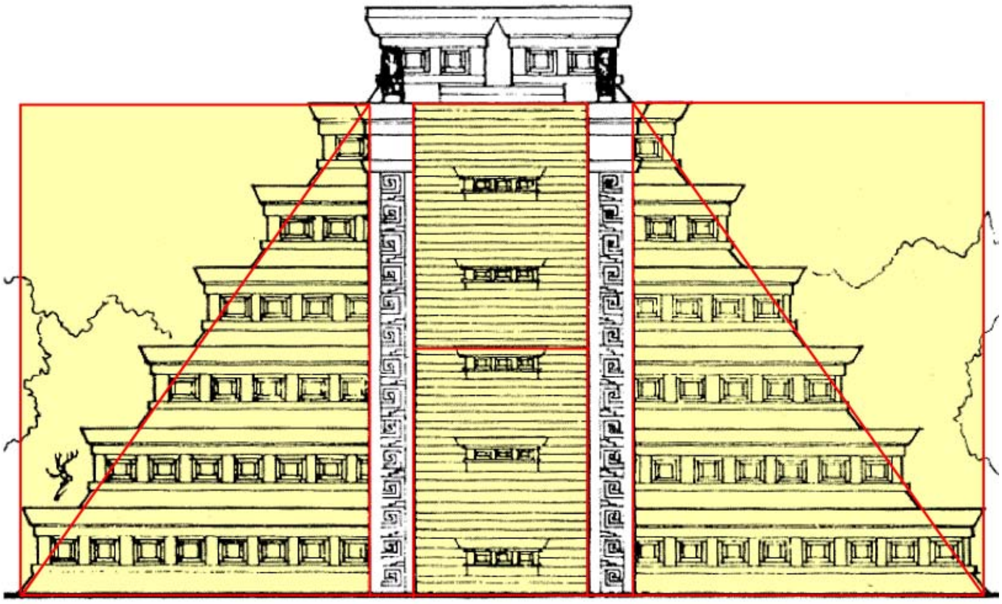


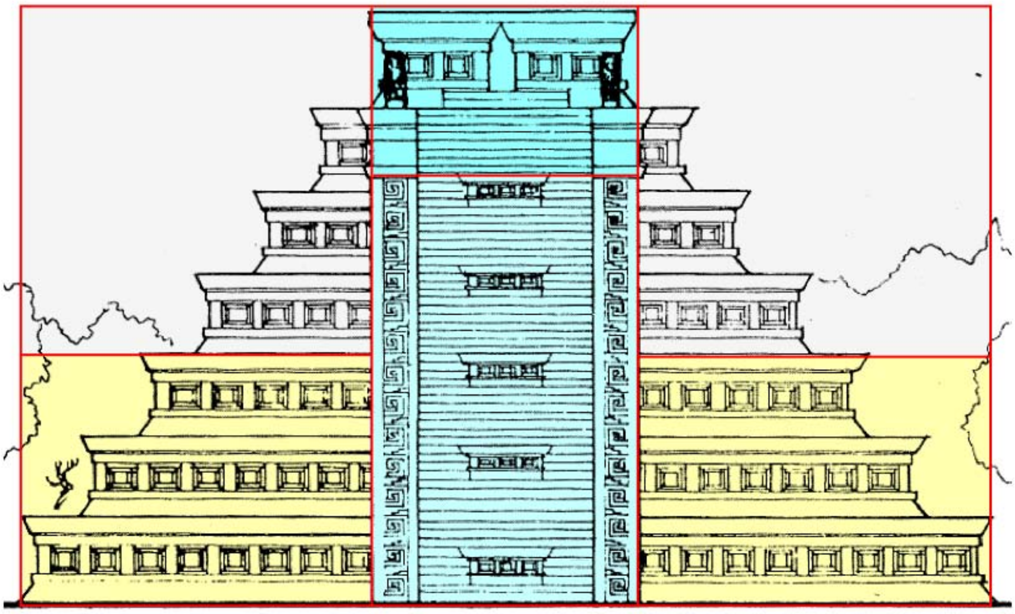
This diagram demonstrates the square root of two proportions inherent in the design of the Pyramid of the Niches. The square root of two rectangles are shaded yellow. The slope of the pyramid is determined by the diagonal of the lateral square root of two rectangles. The staircase is inscribed by two square root of two rectangles.

If the width of the staircase (a,b) is one, then from the outside lower corners of the balustrade to the corners of the pyramid (c,d) is two. Thus, whether intentionally or not, the area of the staircase is equal to the area of triangle c,d,e.

Figure 161

Elevation of the Pyramid of the Niches (c)
Veracruz, Mexico
Measured Drawing by L. MacGregor K. (Marquina 1951)

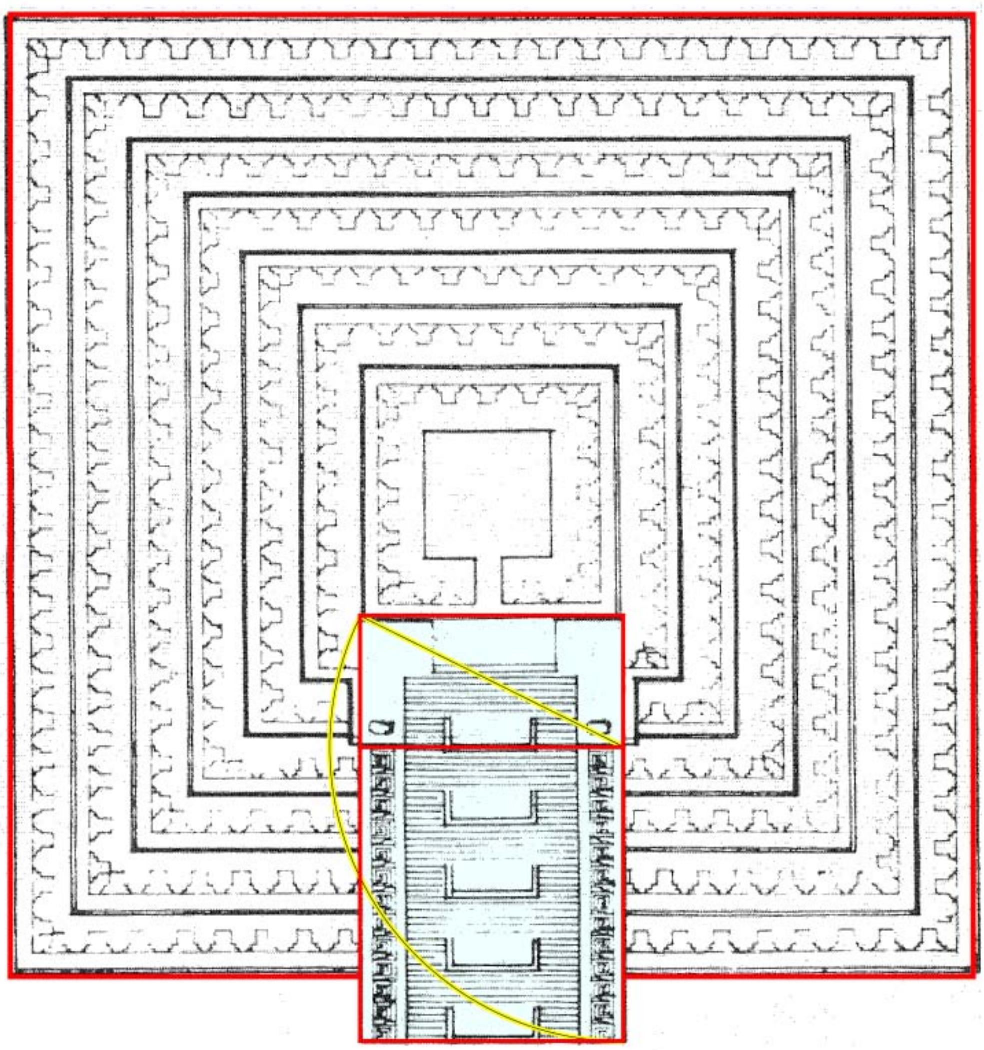




Both diagrams are shown together here to better appreciate how more than one geometrical formula can be integrated into a single architectural design.

Figure 162

Plan`of the Pyramid of the Niches,
Veracruz, Mexico
Measured Drawing by L. MacGregor K. (Marquina 1951)



The plan of the Pyramid of the Niches is inscribed by a square. The staircase and balustrade are inscribed by a phi rectangle (shaded blue) and the uppermost, undecorated portion of the staircase and balustrade are inscribed by a square root of four rectangle, the diagonal of which is the radius used to determine the length of the phi rectangle.

CHAPTER 8

GEOMETRY IN CLASSIC AND POST CLASSIC PERIOD MAYA ART

The following observations reveal some overall rules and norms that seem to characterize the composition of Maya art and the codices, It should be noted that these proposed “rules and norms” are general ones, some more strictly adhered to than others and, in some cases, ignored. After extensive examination, I have found no obvious correlation between particular geometric proportions and specific subject matter. It does appear, however, that Maya artist/geometers were guided by certain general principles.

1. The underlying geometry is not overt. On the contrary, it appears to be consciously hidden.

2. Bilateral symmetry is clearly important but not essential.

3. Intentional asymmetries are often noted; for example, one corner of Yaxchilan Lintel 11 is notably out of square.

4. Body postures often follow the principle angles and curves of the underlying geometrical composition.

5. Objects, such as bodies, arms, legs, spears, and serpent bars, generally touch or are

parallel to the principle lines and curves of the underlying geometrical composition. Rarely do these lines run through the centers of these objects.

6. The brow, in particular, as well as hands and feet are extremely important and are almost always touching or crossed by critical junctures or principle lines of the geometrical diagram. Principle lines of the geometrical program almost always divide the headdress from the head of human subjects.

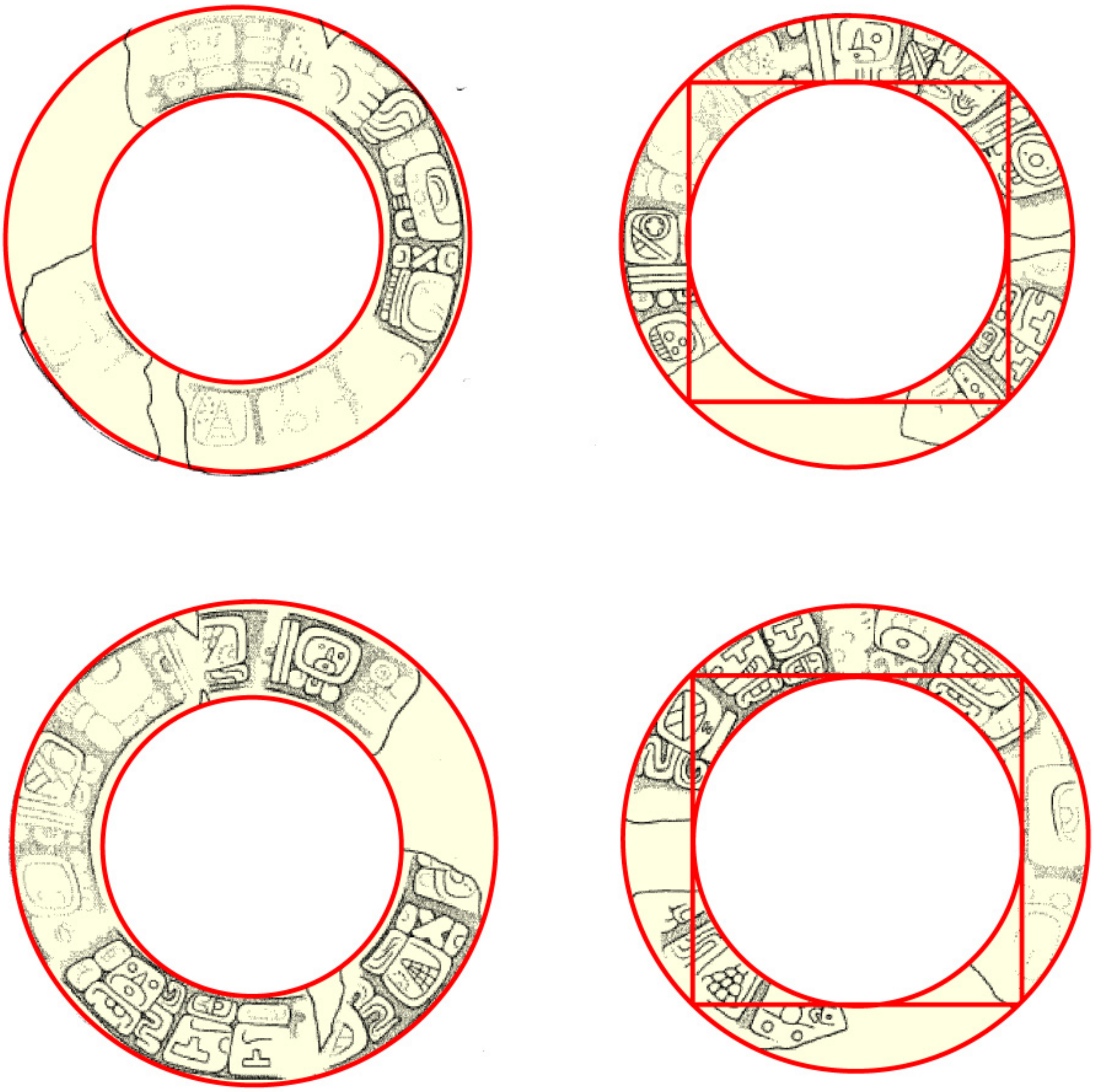
7. Navels, noses, and the temple region of the head are often foci of the geometry, though not as often noted in the compositions as are hands, feet, and brows.

8. Hieroglyphic writing is secondary to the rest of the composition. Glyph blocks are often included and accounted for in the geometrical compositions, but brief inscriptions may have been used to fill empty space. Names and titles are often crossed by principle lines of the geometry, and some appear to have been added after the artistic composition has been adapted to the geometrical composition.

9. Though much of the art analyzed so far focuses on one or more of the given set of Maya proportions, others are clearly attempts to unite the full set of proportions into a balanced and harmonious composition.

Figure 163

Ball Court Markers 1 and 2, Uxmal
Yucatan, Mexico
Measured Drawing by Ian Grahm



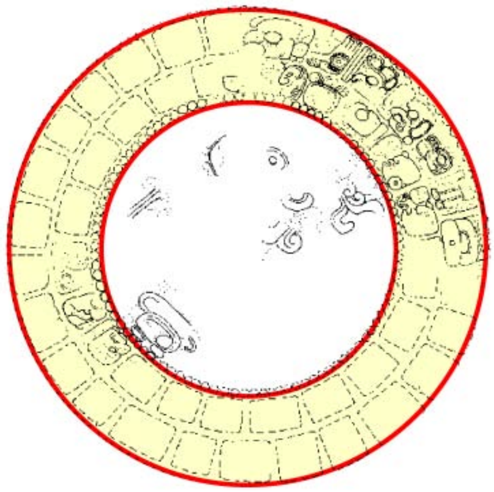
Uxmal. Ball Court Marker 1　　Uxmal. Ball Court Marker 2

The above are ball court markers from either side of the principle ball court a Uxmal. If the diameter of the inner circle of Ball Court Marker 1 is one, then the diameter of the outer circle is phi. If the diameter of the inner circle of Ball Court Marker 2 is one, then the diameter of the outer circle is square root of two.

Figure 164

Altar 7, Tikal and Monument AI-OP, Yaxchilan
Measured Drawing by W. R. Coe (a) and S. G. Morley (b)

a



Tikal Altar 7

b

Yaxchilan, Monument AI-OP

If the diameter of the inner circle of Tikal Altar 7 is one, then the diameter of the outer circle is phi.
If the diameter of the inner circle of Yaxchilan Monument AI-OP is one, then the diameter of the outer circle is four. If the diameter of the middle circle of Yaxchilan Monument AI-OP is one, then the diameter of the outer circle is phi.

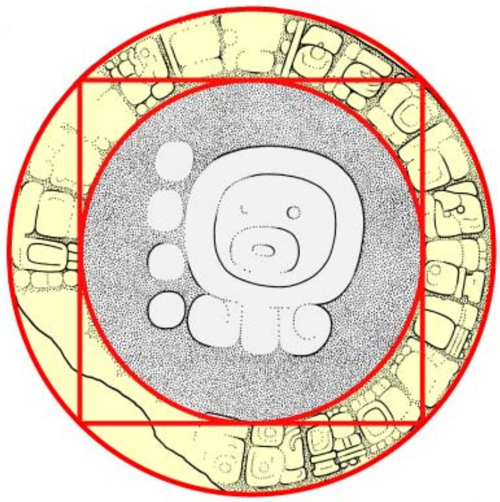
Figure 165

Monuments 69,11,and 34 and Altar 001, Tonina
Chiapas, Mexico
Measured Drawings by Ian Grahm

Tonina. Monument 69

Tonina Altar 001

Tonina. Monument 11

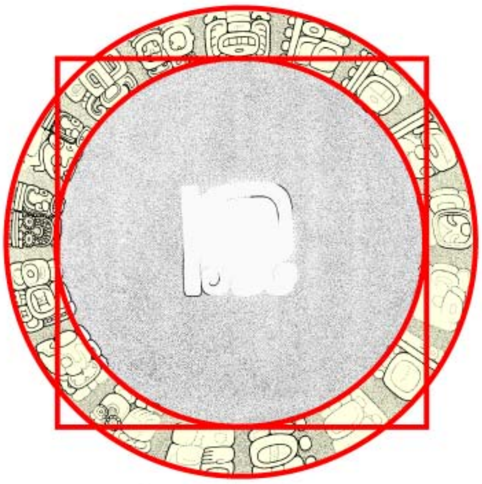


Tonina. Monument 34

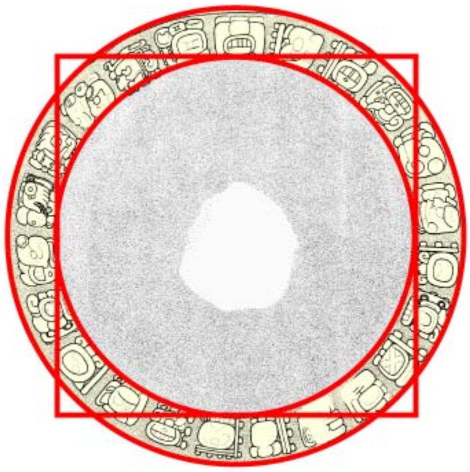
The inner circles are the diameter of the width of the squares inscribed in the outer circles. If the diameter of the inner circles is one, then the diameter of the outer circles is square root of two, as per Formula Circle-a.

Figure 166

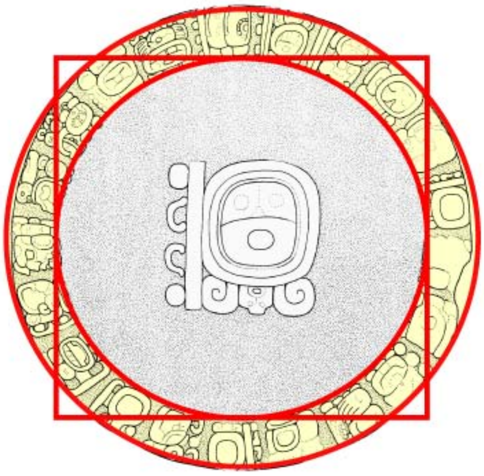
Monuments 110, 111, 139, and 137, Tonina
Chiapas, Mexico
Measured Drawings by Ian Grahm



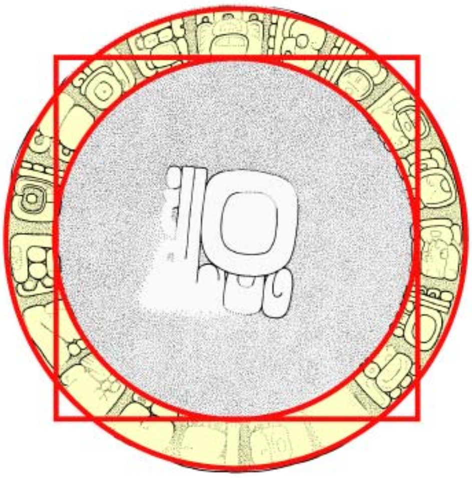
Tonina monument 110



Tonina monument 111



Tonina monument 139



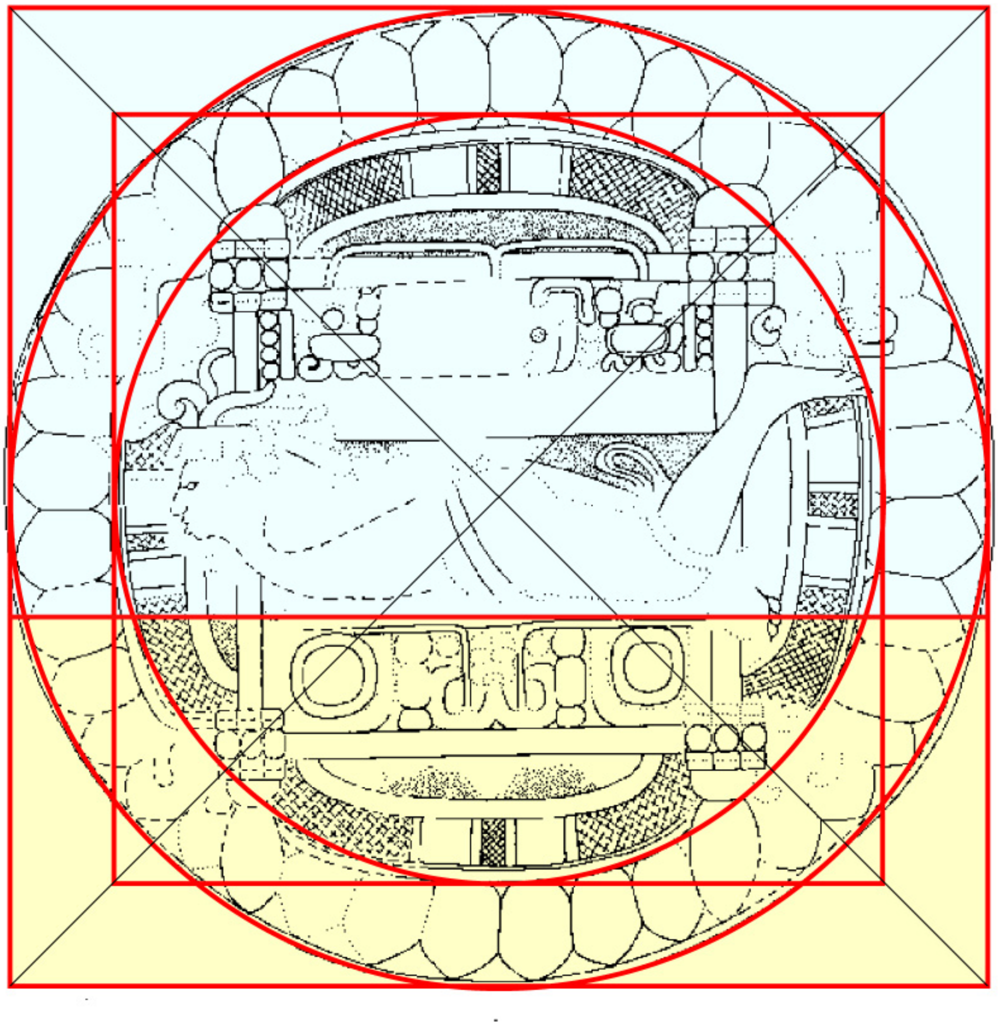
Tonina monument 137

If the diameter of the inner circles is one, then the diameter of the outer circles is square root of phi. The perimeters of the squares that inscribe the inner circles are virtually equal to the perimeters of the outer circles, as per Formula Circle-2.

Figure 167

Altar 10, Tikal,
Peten, Guatemala

Measured Drawing by W. R. Coe

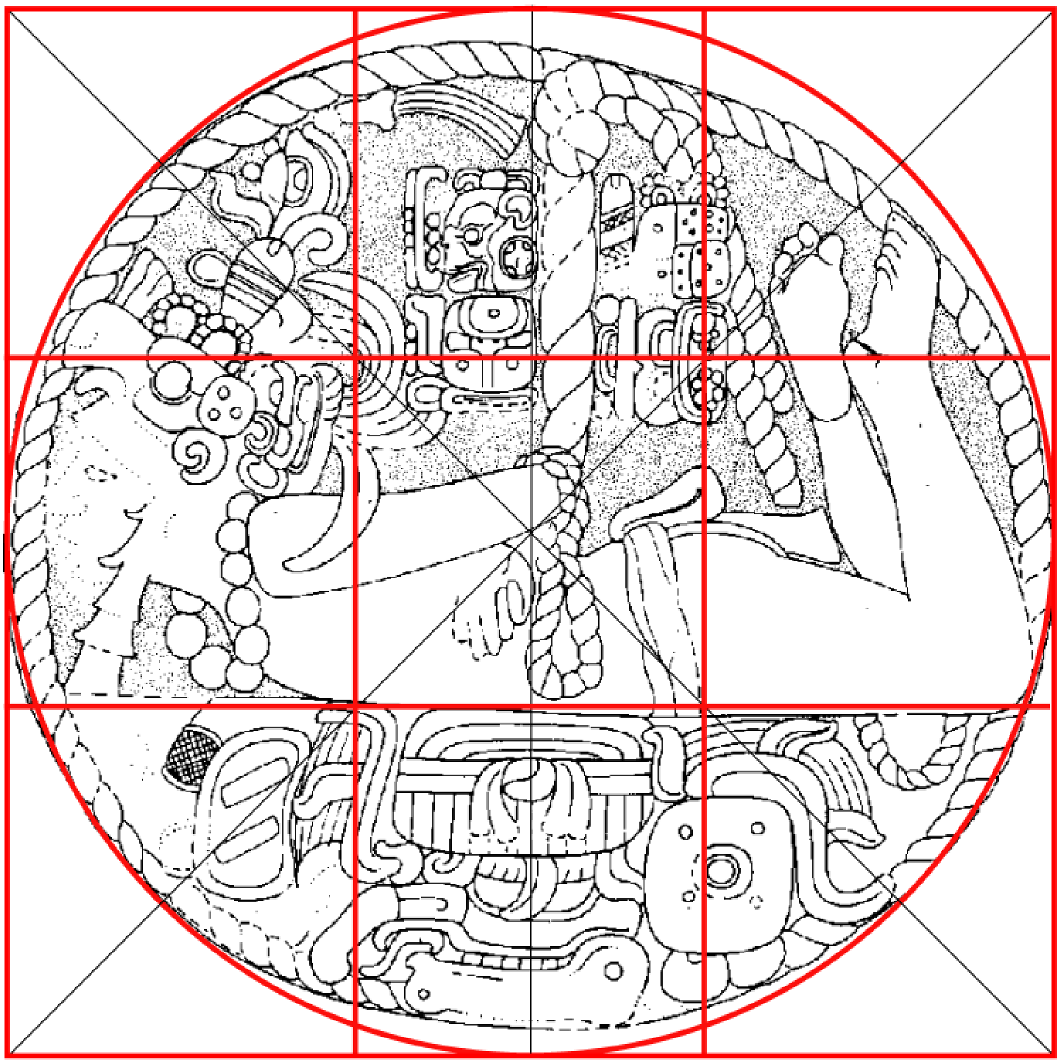


If the diameter of the inner circle is one, then the diameter of the outer circle is square root of phi. The diameter of the square that inscribes the inner circle is virtually equal to the diameter of the outer circle, as per Formula Circle-2

The square that inscribes the outer circle is subdivided into a phi rectangle (shaded blue) and a phi squared rectangle (shaded yellow), as per Formula Square-3.

Figure 168

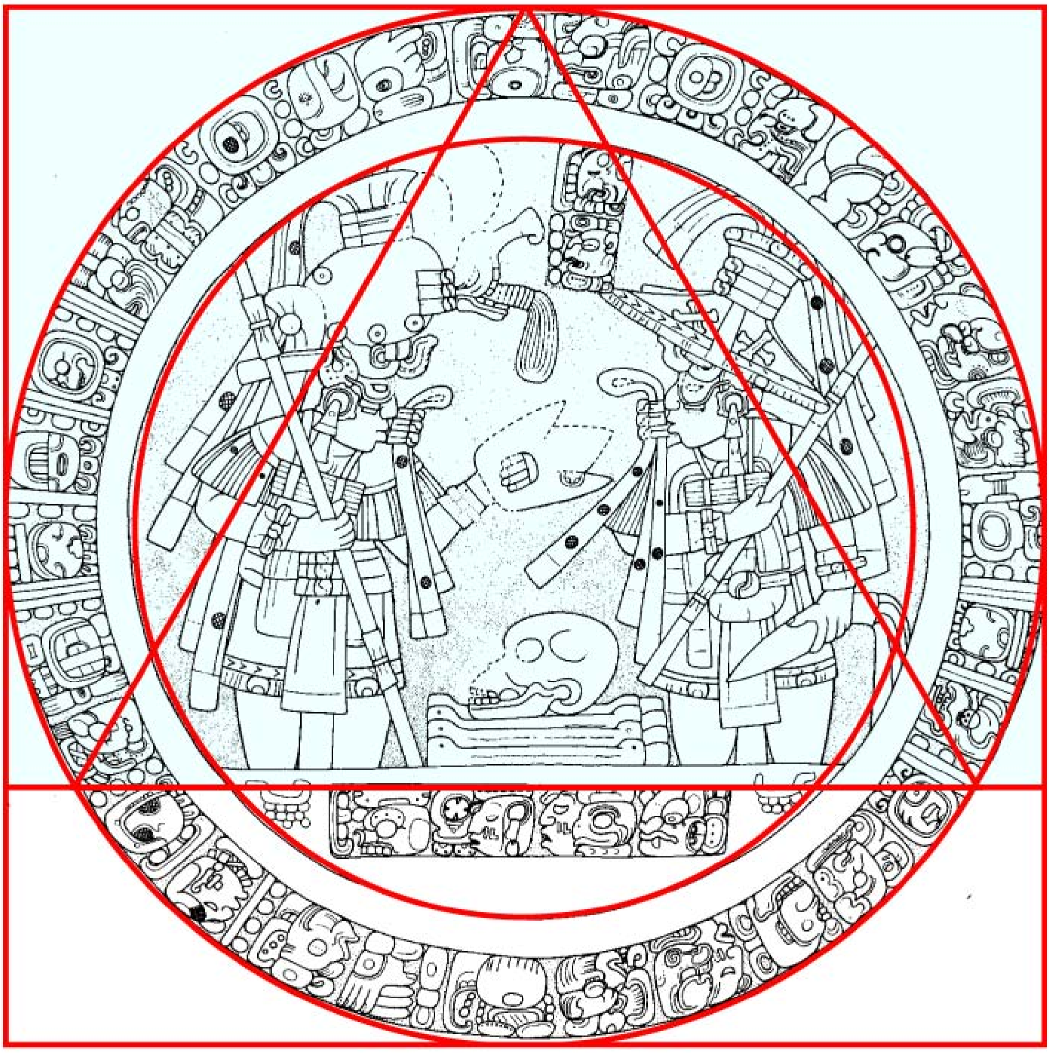
Altar 8, Tikal,
Peten, Guatemala
Measured Drawing by W. R. Coe



A circle is inscribed by a square that is trisected. Note the possibly intentional asymmetry of the upper portion of the circle.

Figure 169

Altar 5, Tikal,
Peten, Guatemala
Measured Drawing by W. R. Coe



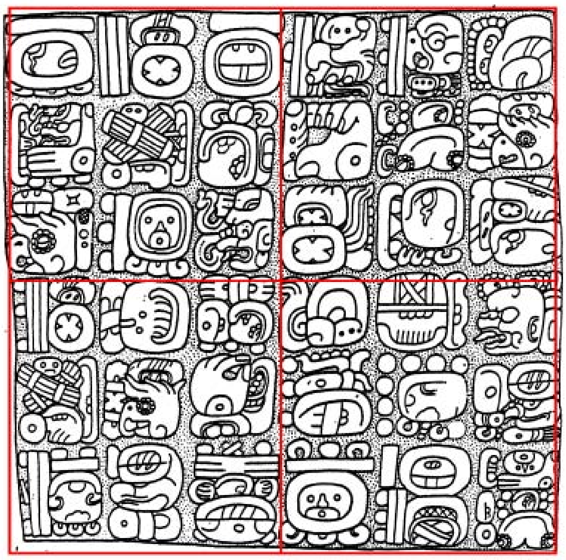
If the diameter of the inner circle is three, then the diameter of the outer circle is four.
An equilateral triangle is inscribed in the outer circle that is, in turn, inscribed within a square. A straight line across the base of the equilateral triangle to the sides of the square subdivides the square into a Pythagorean 3,4,5 rectangle (shaded blue) and a one to four rectangle (in white).

Figure 170

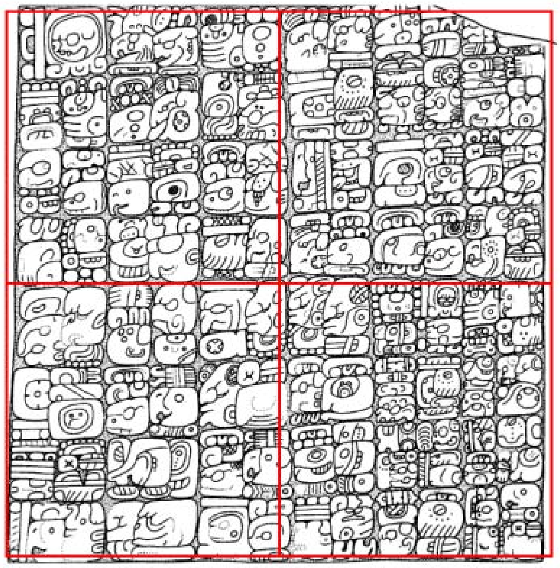
Square Hieroglyphic Text Blocks 1

Measured Drawings by Ian Grahm, Rubbing By Merle Greene Robertson

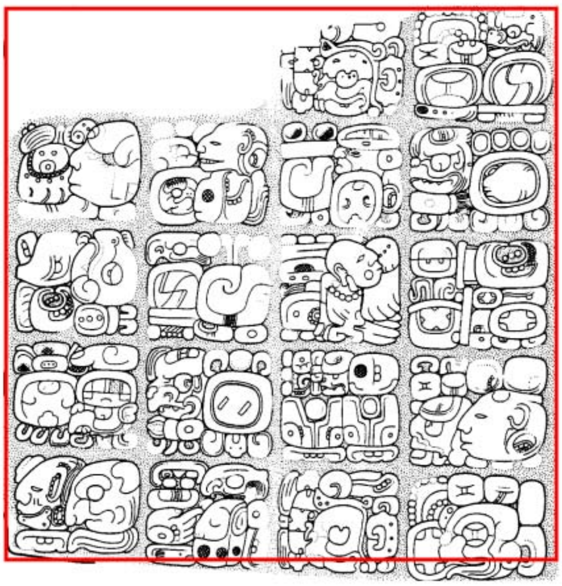
Altar Q, Copan



Lintel 11, Yaxchilan



Monument 141, Tonina



Lintel 1, Yula

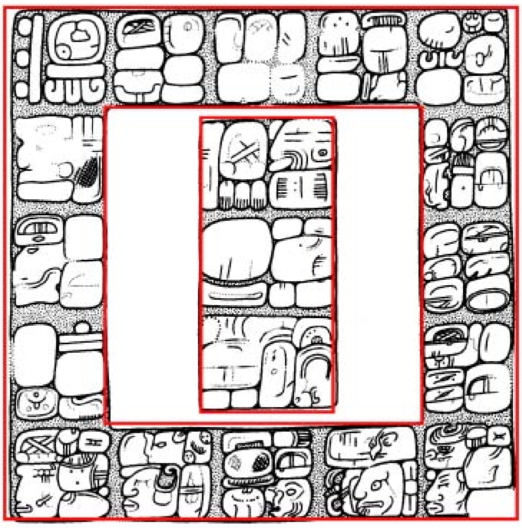
Hieroglyphic text blocks inscribed by squares.

Figure 171

Square Hieroglyphic Text Blocks 2

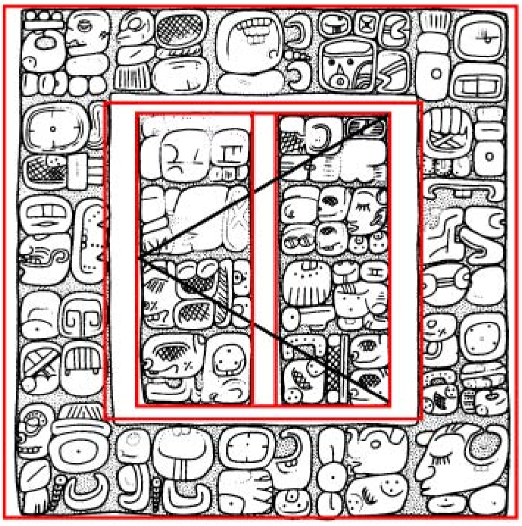
Measured Drawings by Ruth Krochock

Lintel 2

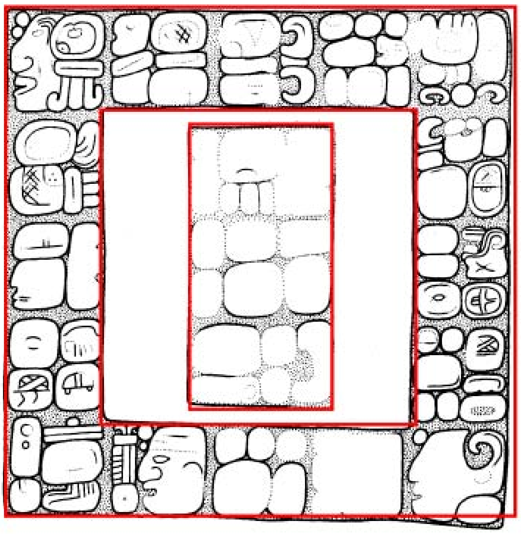


Lintel 3

Lintel 4



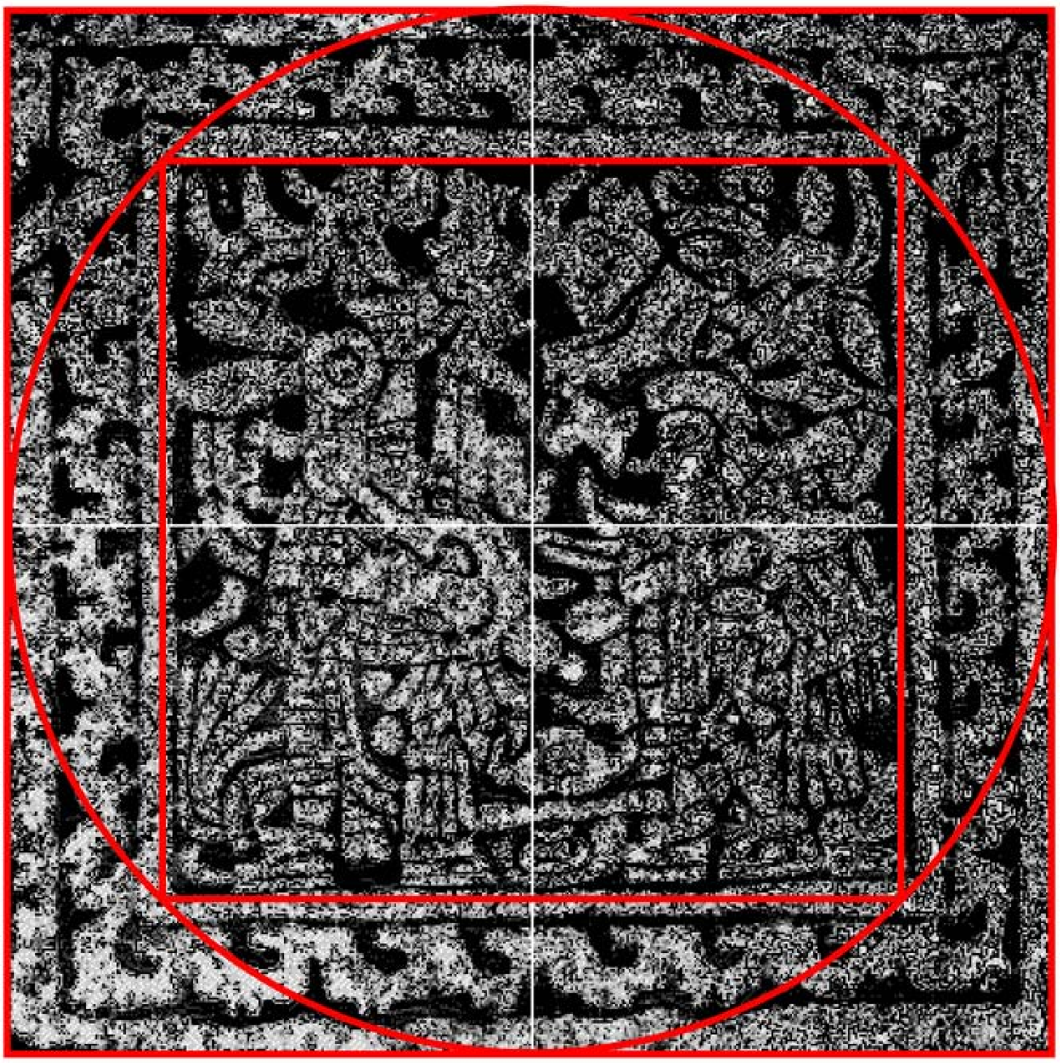
Lintel 5



For each of these lintels from the Las Monjas Temple at Chichen Itza, the ratio between the inner and outer squares is one to phi. The bar of hieroglyphics within the inner square of Lintel 2 is inscribed by a root five rectangle. The two bars of hieroglyphics within the inner square of Lintel 3 are inscribed by a square. The two bars of hieroglyphics within the inner square of Lintel 4 are inscribed by a two over root three rectangle (that inscribes an equilateral triangle) and the bars themselves are inscribed by two, two over root phi rectangles. The bar of hieroglyphics within the inner square of Lintel 5 is inscribed by a root four rectangle.

Figure 172

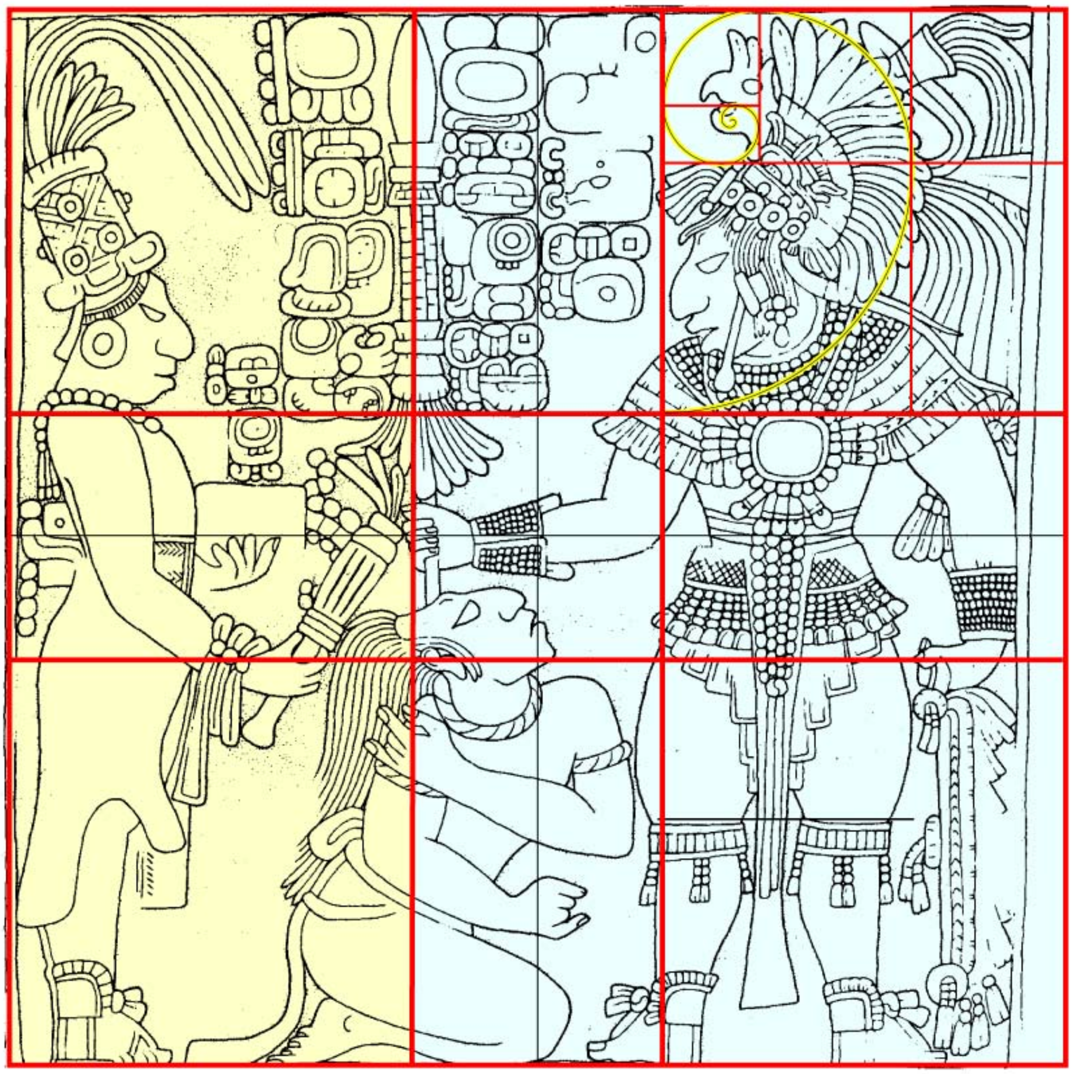
Monument M01,
Chichen Itza, Yucatan, Mexico
Rubbing by Merle Greene Robertson



Monument M01 is inscribed by a square that inscribes a circle that inscribes a square. The ratio of the inner square to the outer square is one to root two.

Figure 173

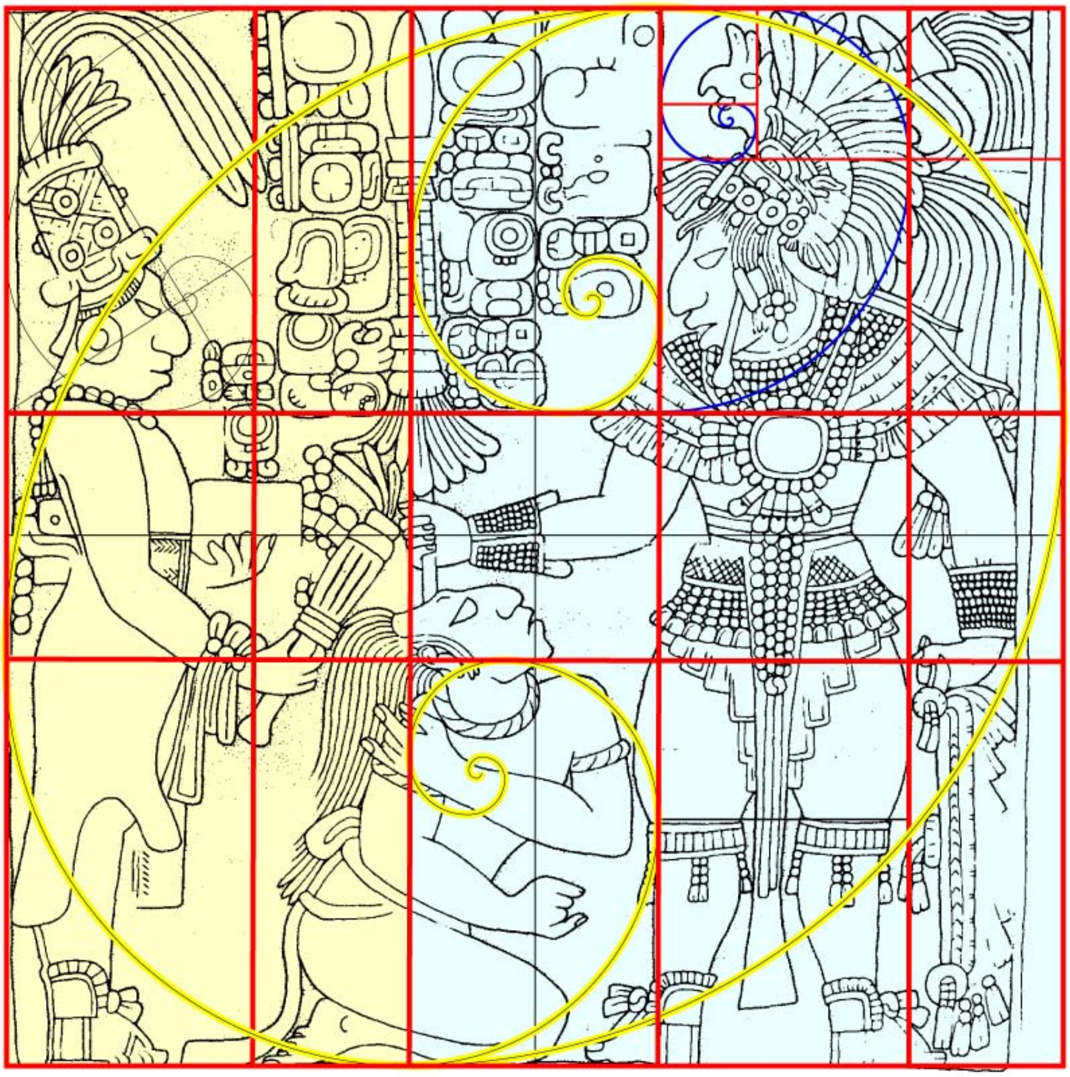
Yaxchilan area, Chiapas, Mexico
Measured Drawing by Ian Grahm



The La Pasadita Lintel is inscribed by a square that is quartered (fine black lines) and subdivided by a phi rectangle (shaded blue) and a phi squared rectangle (shaded yellow), and into smaller squares and phi rectangles (bold red lines), as per Formula Square-2. The square in the upper right corner is further subdivided into squares and phi rectangles, and an equiangular spiral is added that conforms to the curve of the quetzal bird headdress of the standing male figure and terminates in its mouth.
Note how the hands of the human figures, the position of the spear, and the blocks of hieroglyphic texts conform to this proposed geometrical diagram.

Figure 174

La Pasadita Lintel (b),
Yaxchilan area, Chiapas, Mexico
Measured Drawing by Ian Grahm



Here, the diagram that overlays the La Pasadita Lintel is further subdivided by vertical lines (a and b). These lines subdivide the corner squares of the previous diagram into phi and phi squared rectangles. Equiangular spirals are also added to the principal phi rectangles of the diagram. Note that the spiral in the blue shaded phi rectangle encapsulates the standing male figure and terminates on one of his royal titles. The second spiral encapsulates body of the captive and terminates at his wrist.

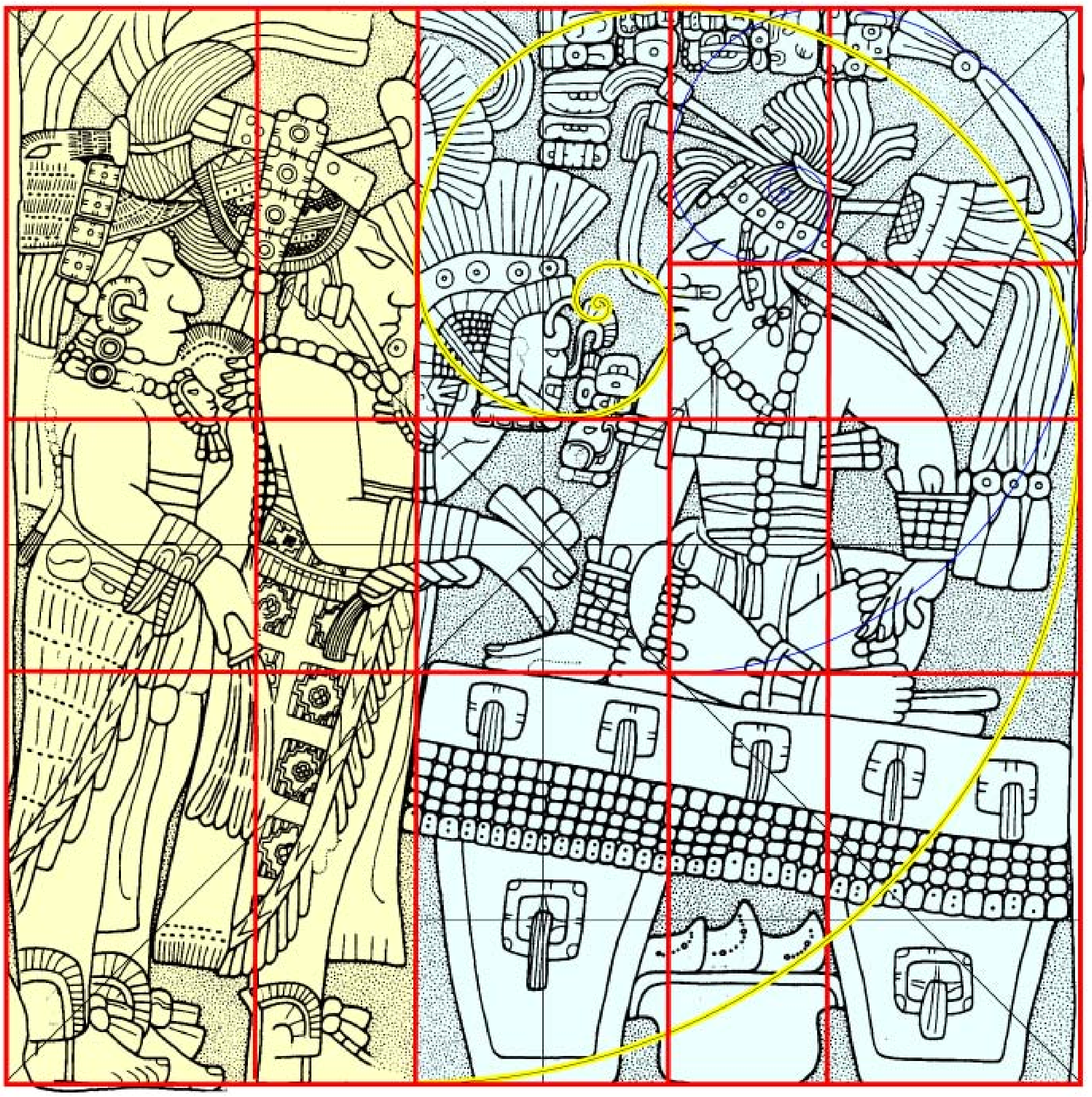
Figure 175

Lintel Mayers 6 (a),
Yaxchilan area, Chiapas, Mexico
Measured Drawing by Ian Grahm

Lintel Mayers 6 (a) is is inscribed by a square that is subdivided nearly identically via Formula Square-2, as was the La Pasadidta Lintel. The principle difference is that a vertical line has been moved to the inside of the squares at the upper and lower right-hand corners.
An equiangular spiral is added that appears to conform to the shape of the headdress of the seated figure.
Note the positions of the standing figures, the text block, the hands of each of the figures, the placement of the bench beneath the seated figure, and the position of the headdress being presented to the seated figure by the standing female figure relative to the suggested geometrical diagram.

Figure 176

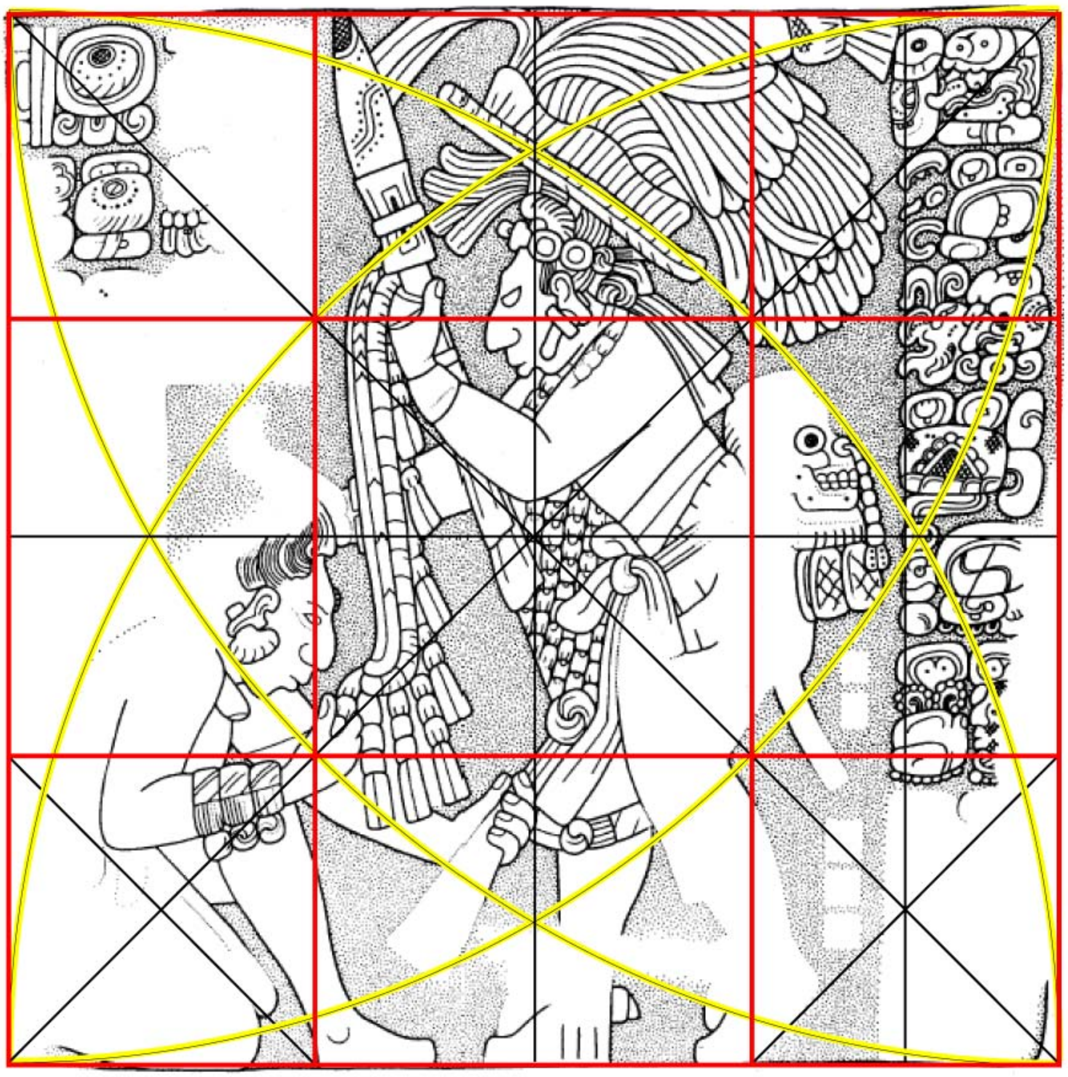
Lintel Mayers 6 (b),
Yaxchilan area, Chiapas, Mexico
Measured Drawing by Ian Grahm



Lintel Mayers 6 (b) shows an equiangular spiral within the principal phi rectangle (shaded blue) that appears to emanate, like a speech scroll, from the mouth of the seated figure, and touches the speech scroll emanating from the mouth of the head attached to the headdress.

Figure 177

Lintel, 45, Yaxchilan,
Chiapas, Mexico
Measured Drawing by Ian Grahm

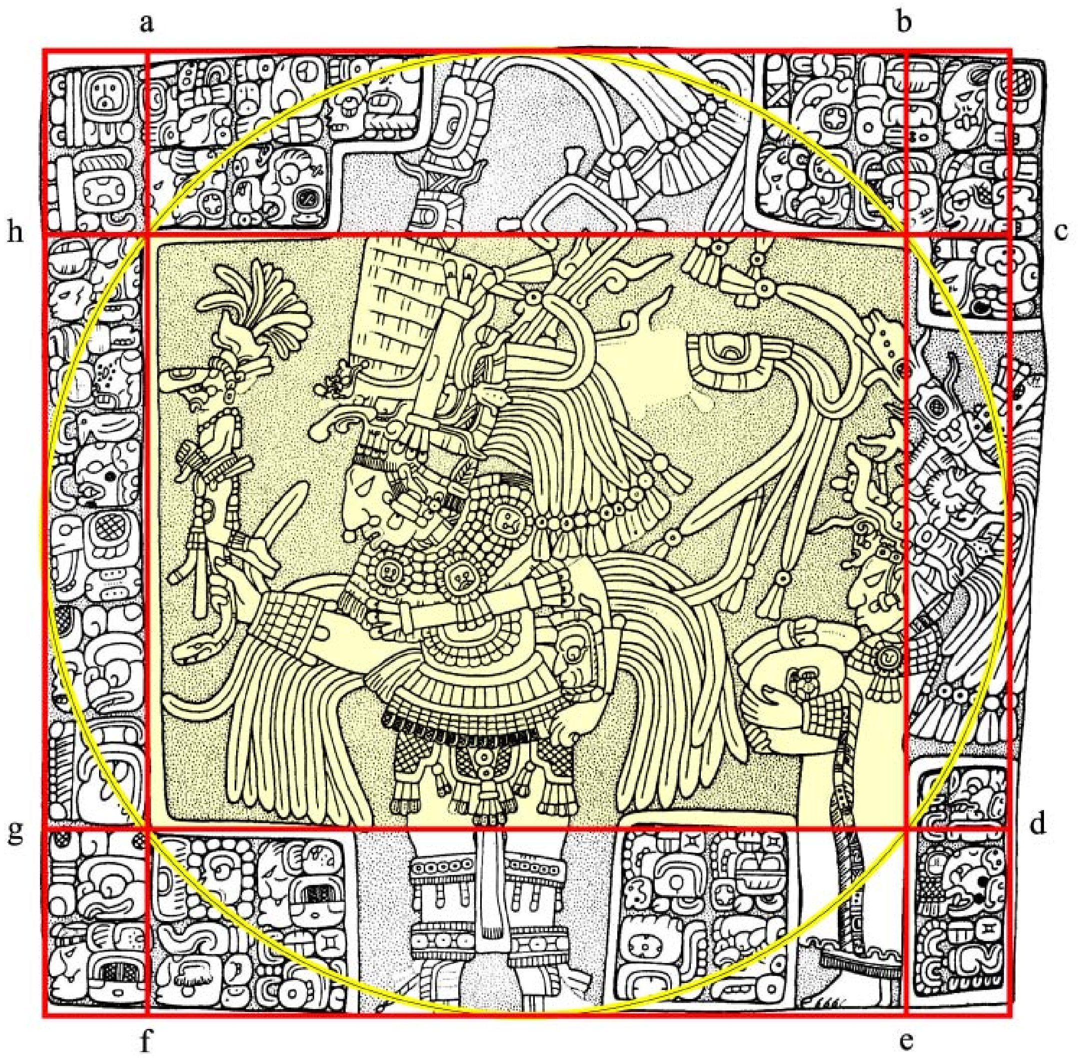


Lintel 45 is inscribed by a square that is dynamically subdivided into smaller squares and square root of two rectangles via Formula Square-1.

Note that both of the human figures are leaning forward as if to fit within the arcs (highlighted yellow lines) whose radii are the sides of the square. Note also the placement and the angles of the feet of the figures, the angle of the thighs of the seated figure, the placement of the hands of both figures and the position of the left arm of the standing figure relative to the suggested geometrical diagram.

Figure 178

Lintel 1, Yaxchilan,
Chiapas, Mexico
Measured Drawing by Ian Grahm



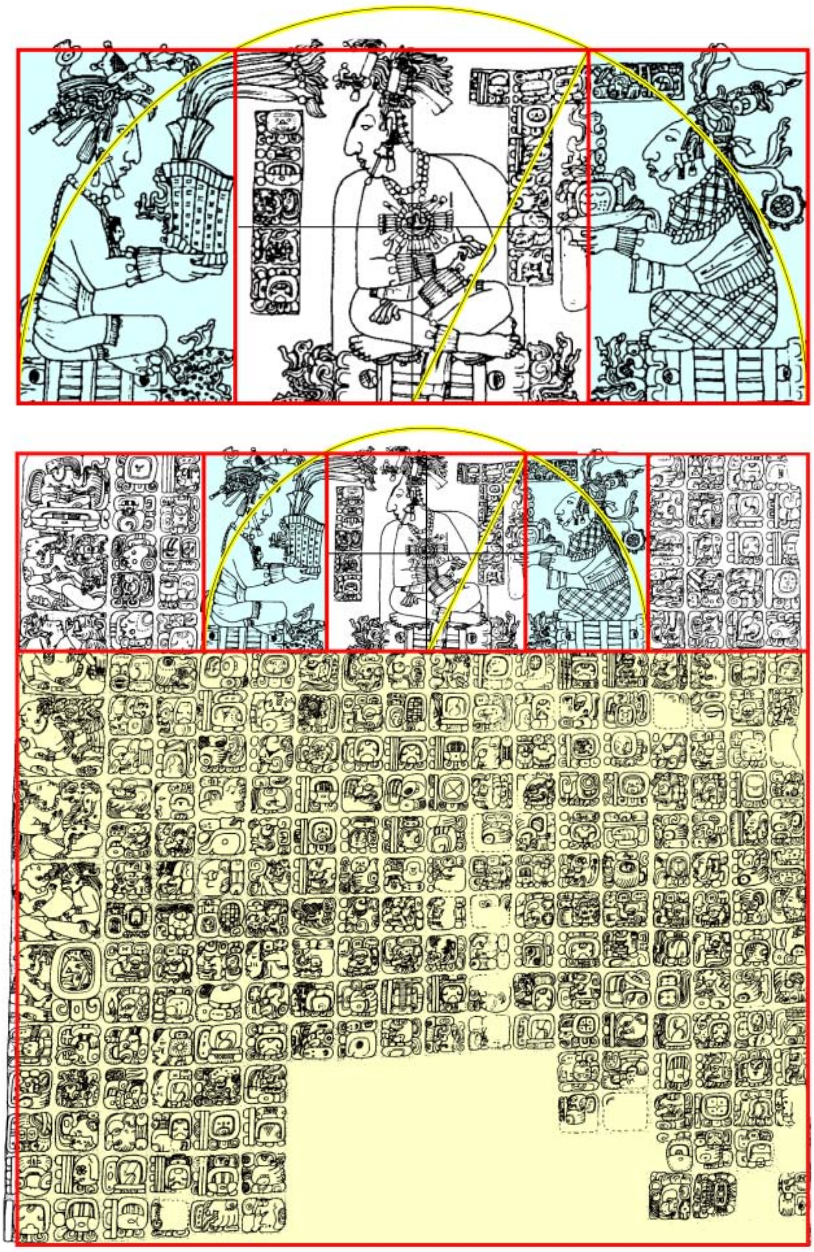
Lintel 1 is inscribed by a square that inscribes a circle that, in turn, inscribes a square root of phi rectangle (shaded yellow). Straight lines are drawn along the sides of the square root of phi rectangle that terminate at the sides of the square, as per Formula Square-4.

Rectangle h,g,c,d is a phi rectangle and rectangle a,b,f,e is a square root of phi rectangle that has virtually the same area as the yellow highlighted circle, again as per Formula Square-4.

Note the possibly intentional asymmetry of the artistic composition at the the upper right corner.

Figure 179

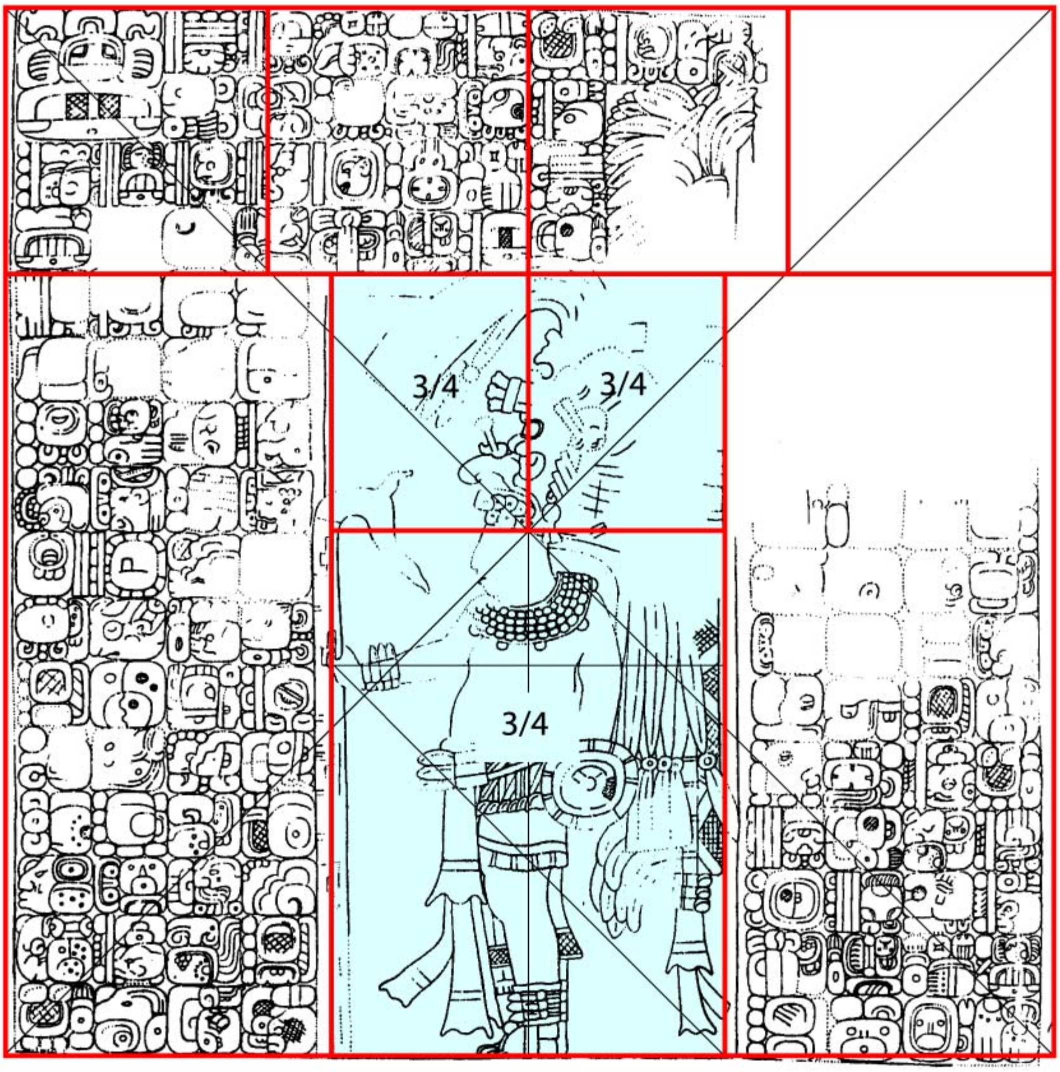
The Palace Tablet,
Palenque, Chiapas, Mexico
Measured Drawing by Merle Grene Robertson



The Palace Tablet is inscribed by a square that is subdivided by a Pythagorean 3,4,5 rectangle (shaded yellow) and a remaining one to four rectangle. The coronation scene embedded within the text is inscribed by a root five rectangle subdivided by a quartered square and two phi rectangles (shaded blue), as per Formula Square Root of Five-3.

Figure 180

Lintel 1,
El Cayo, Guatemala

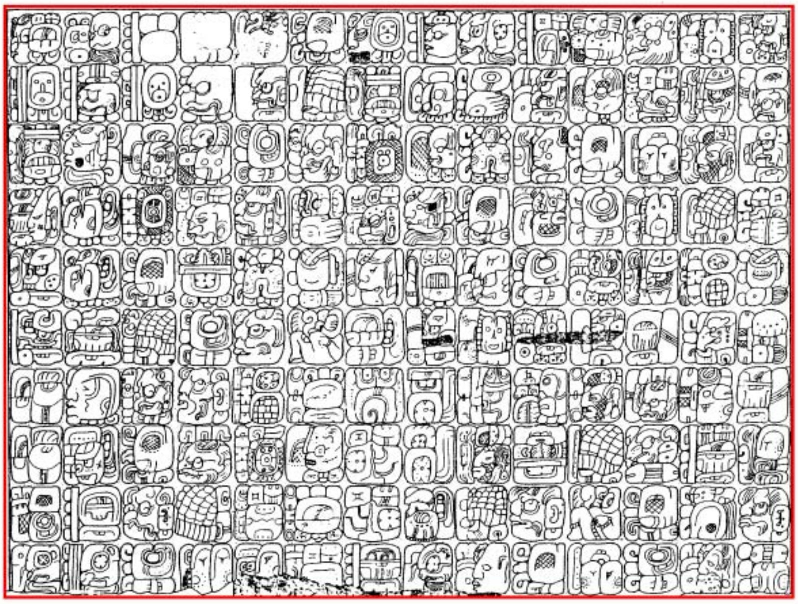


Like the Palace Tablet, Lintel 1 from El Cayo is inscribed by a square that is subdivided by a Pythagorean 3,4,5 rectangle (above the inset standing figure) and a remaining one to four rectangle. The standing figure imbedded within the text is inscribed by a root four rectangle (shaded blue) subdivided at the center of the exterior square by three Pythagorean 3.4.5 rectangles. Note that the left hand of the standing figure divides the root four rectangle in half.

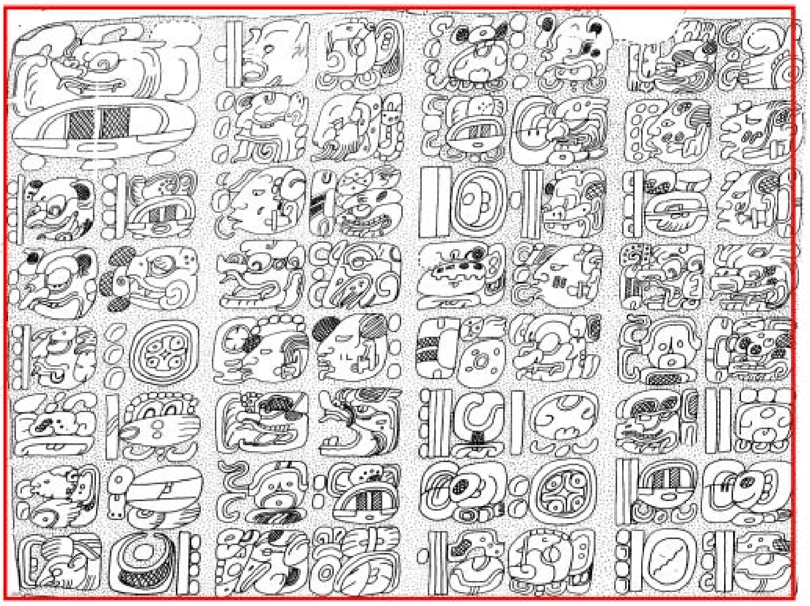
Figure 181

Pythagorean 3,4,5 Hieroglyphic Text Blocks 1
Measured Drawings by Merle Greene Robertson (a) and Ian Graham (b)

The Center Tablet of the Temple of the Inscriptions, Palenque



Panel 1, Calakmul

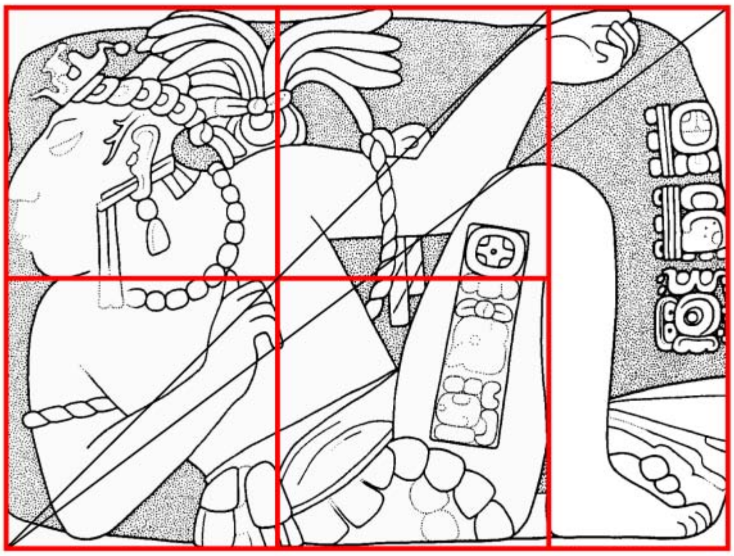


Hieroglyphic text blocks inscribed by Pythagorean 3,4,5 rectangles

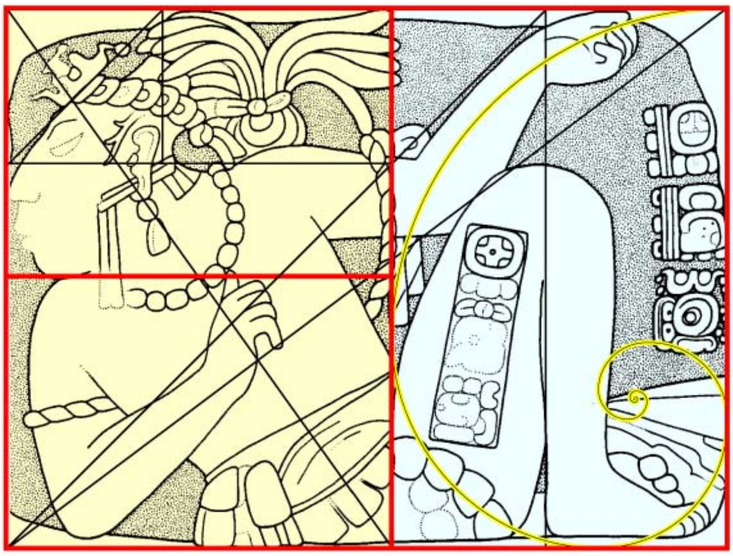
Figure 182

Monument 122, Tonina, Chiapas, Mexico
Measured Drawing by Ian Grahm

a



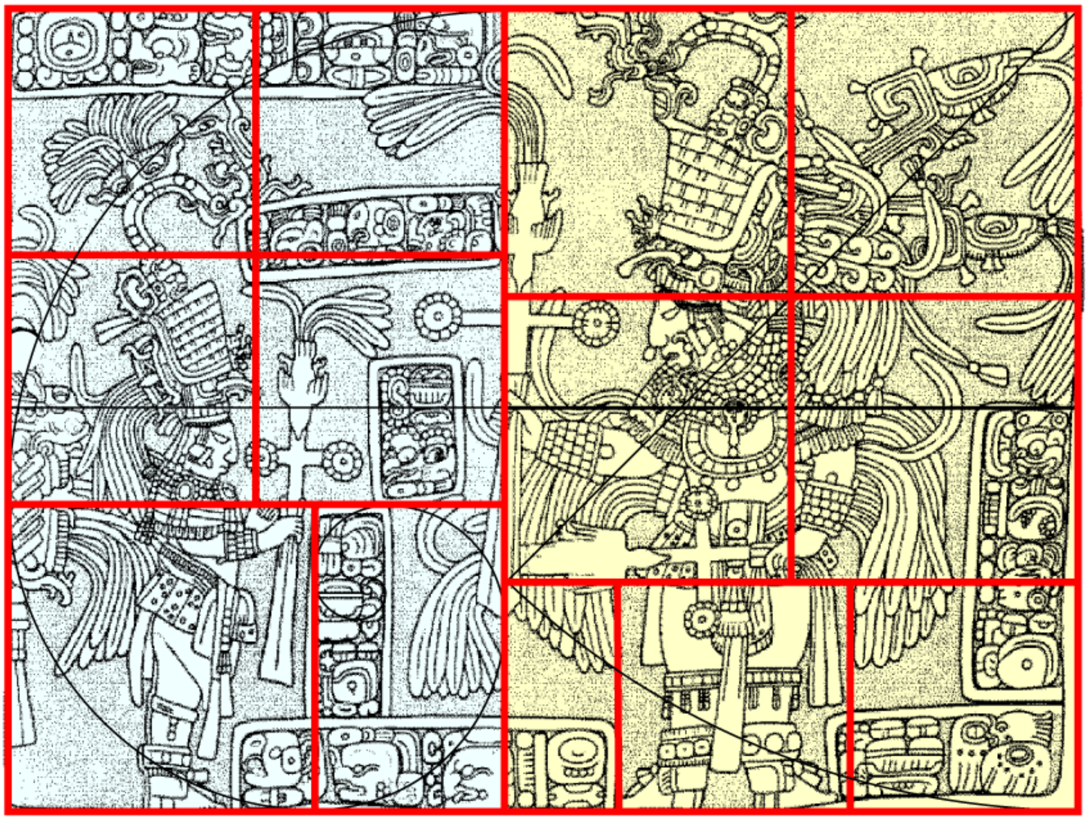
b



Monument 122 is inscribed by a Pythagorean 3.4.5 rectangle. In diagram a, this rectangle is subdivided into a quartered square (shaded grey) and a rectangle with a ratio of one to three (in white). The diagonals of the principle rectangle and the square are drawn in fine black lines.
In diagram b, the Pythagorean rectangle is divided into a phi rectangle (shaded blue) and square root of two rectangle (shaded yellow) via Formula Pythagorean 3.4.5-2.
Note how the posture and the positions of the hands of the reclining figure precisely conform to the proposed geometrical diagram.

Figure 183

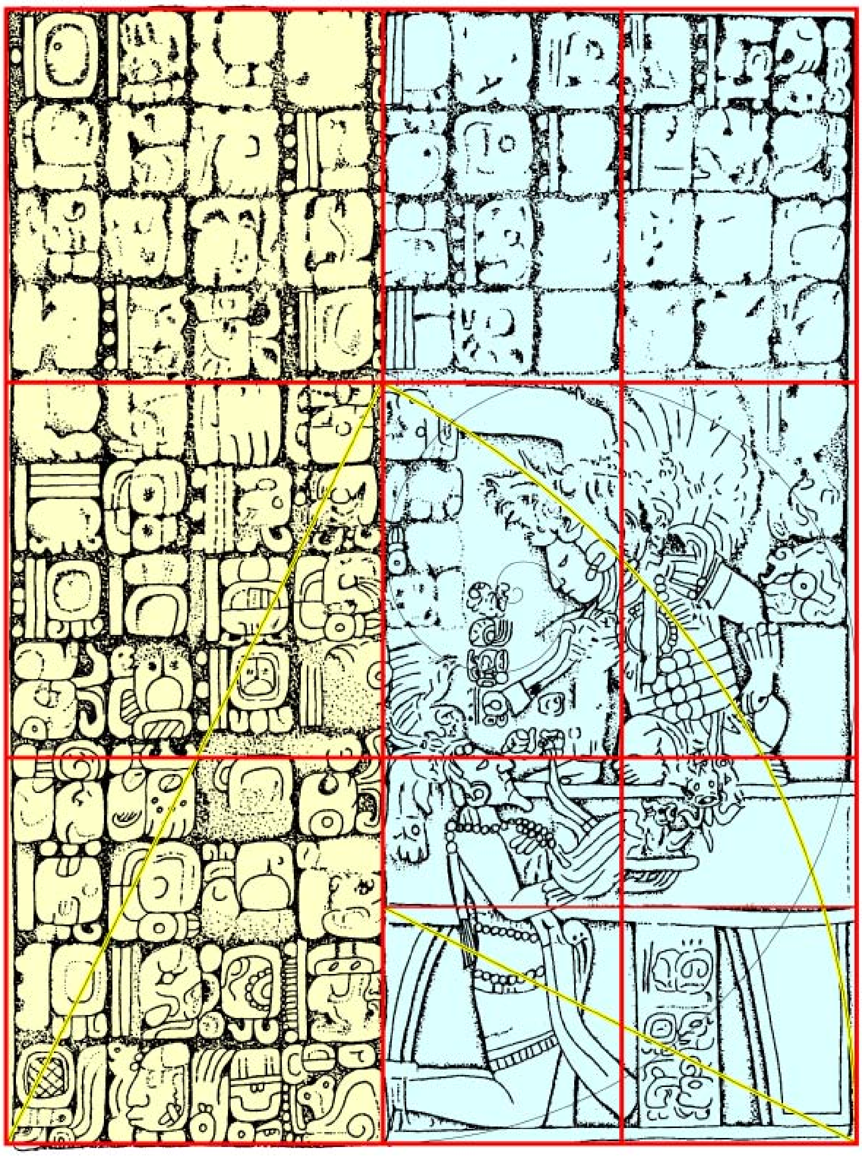
Lintel 2,
Yaxchilan, Chiapas, Mexico
Measured Drawing by Ian Grahm



Lintel 2 is inscribed by a Pythagorean 3,4,5 rectangle that is divided into a square root of two rectangle (shaded yellow) and a phi rectangle (shaded blue), as per Formula Pathagorean 3,4,5-2.

Figure 184

Lintel 2, Yaxchilan, Chiapas, Mexico
Measured Drawing by Ian Graham

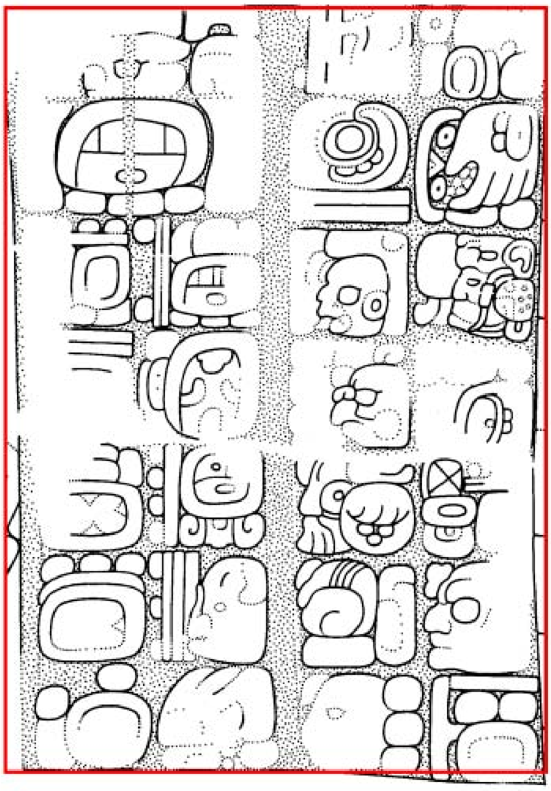


Lintel 2 is inscribed by a Pythagorean 3,4,5 rectangle that is divided into three root five rectangles, each of which is subdivided into squares (shaded yellow) and phi rectangles (shaded blue).
Note that the diagonal of the double square to the left of the seated and kneeling figures is the radius for the arc that defines the width of the phi rectangle that inscribes the iconography.

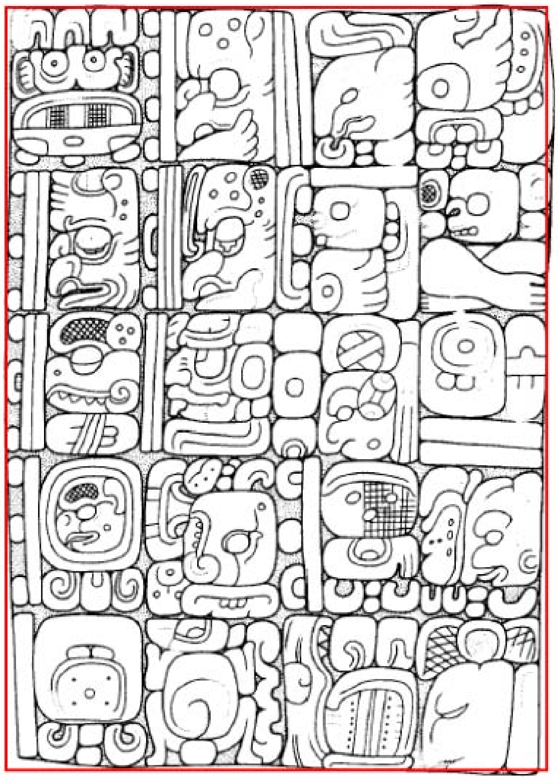
Figure 185

Square Root of Two Hieroglyphic Text Blocks
Measured Drawing by Ian Grahm

Monument 8-d, Tonina



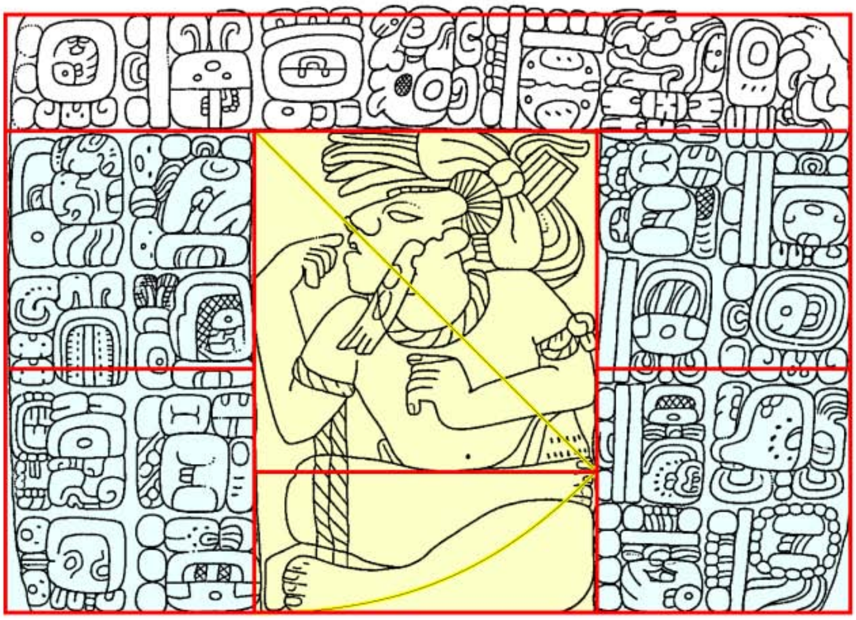
Lintel 29, Yaxchilan

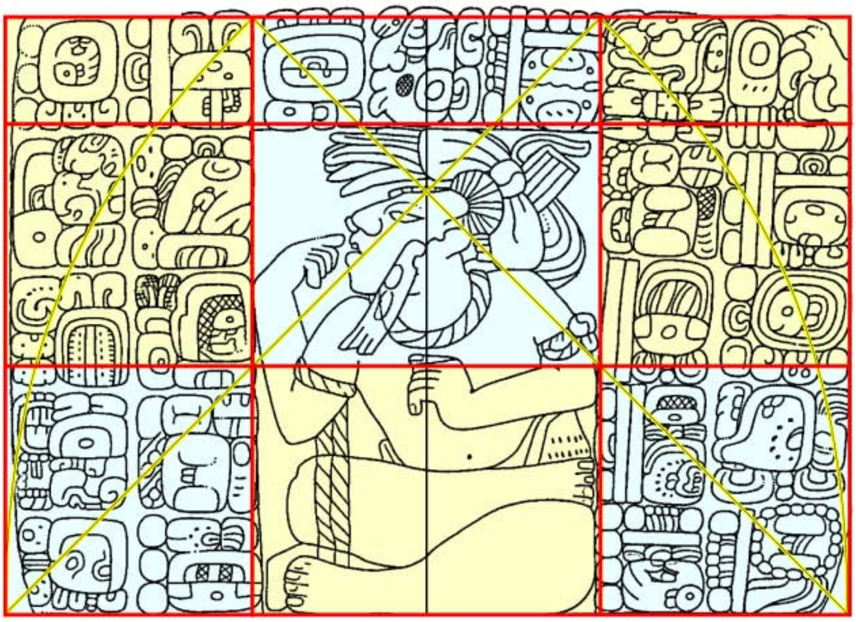


Hieroglyphic text blocks inscribed by root two rectangles

Figure 186

Panel n-3, Tonina, Chiapas, Mexico
Measured Drawing by Ian Grahm

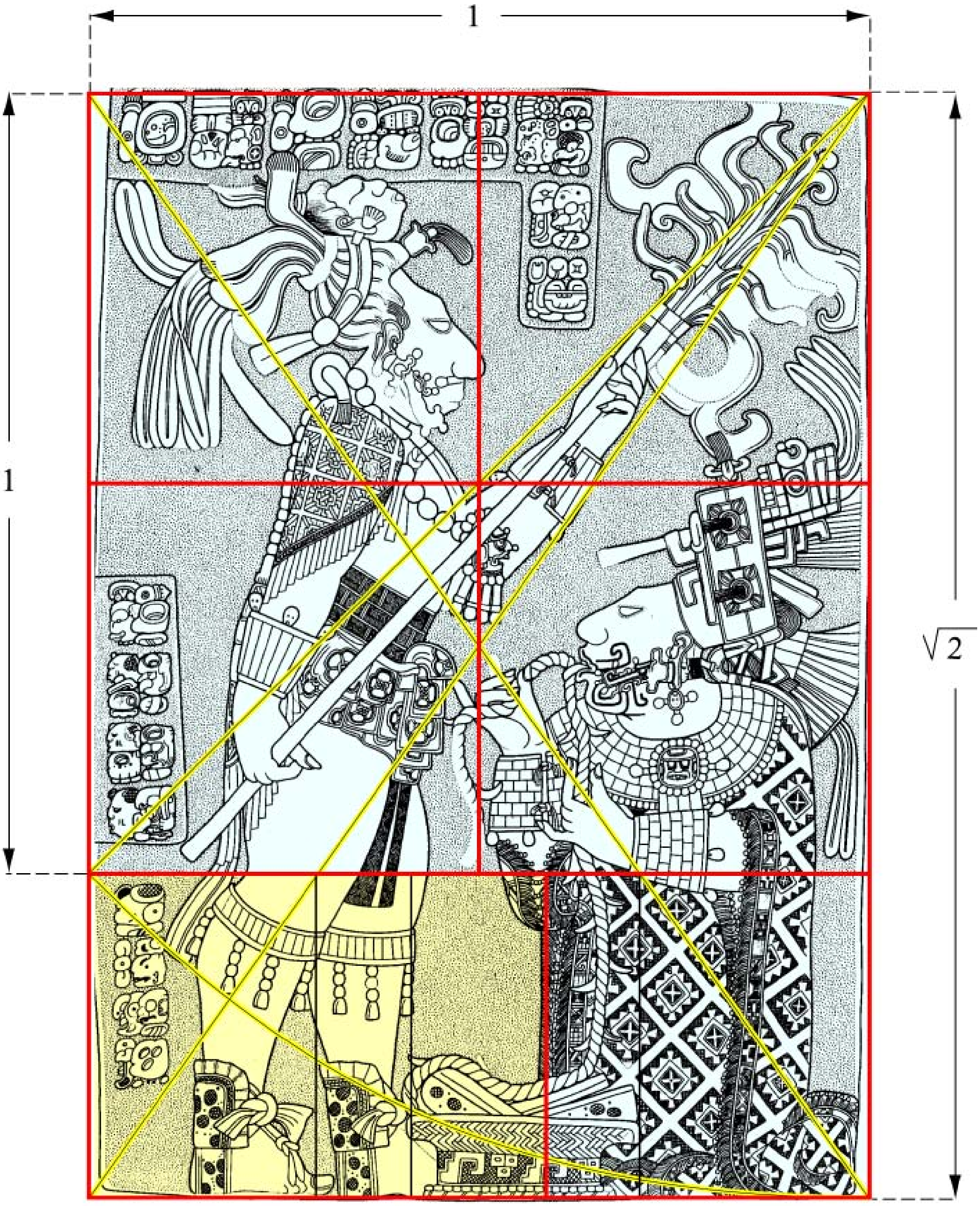




Panel n-3 from Tonina is inscribed by a root two rectangle that is subdivided into squares (shaded blue) and smaller root two rectangles (shaded yellow), as per Formula Square Root of Two-3.

Figure 187

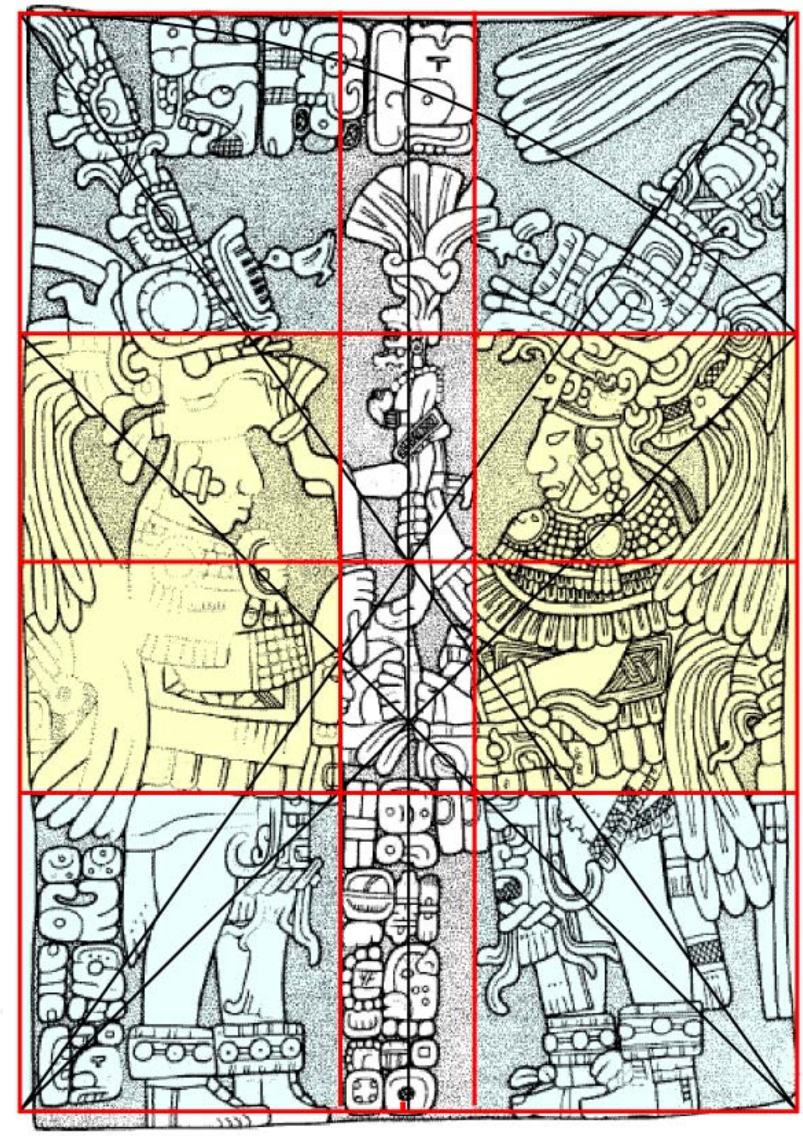
Lintel 24, Yaxchilan, Chiapas, Mexico
Measured Drawing by Ian Grahm



Lintel 24 from Yaxchilan is inscribed by a root two rectangle that is subdivided into squares (shaded blue) and smaller root two rectangles (shaded yellow), as per Formula Square Root of Two-3.

Figure 188

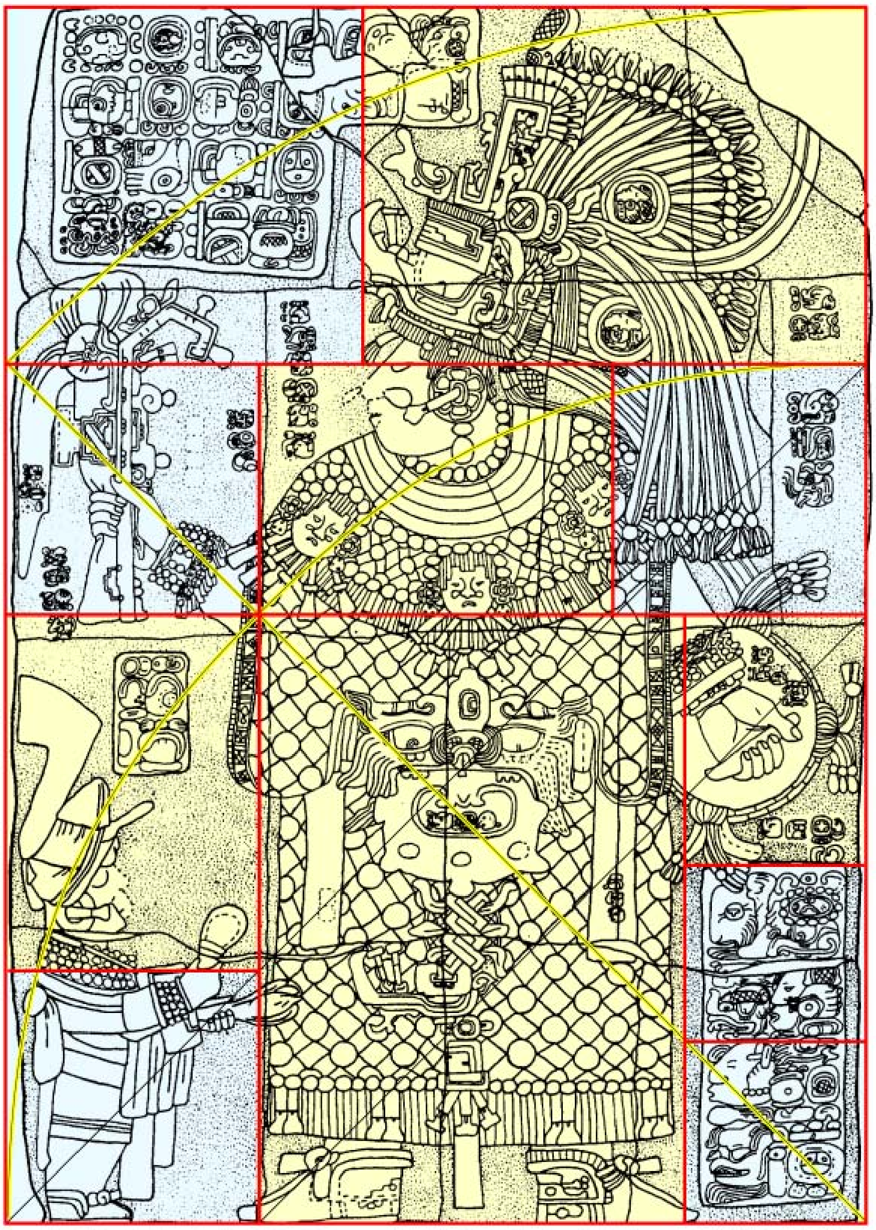
Lintel 58, Yaxchilan, Chiapas, Mexico
Measured Drawing by Ian Grahm



Lintel 58 from Yaxchilan is inscribed by a root two rectangle that is subdivided into squares (shaded blue) and smaller root two rectangles (shaded yellow), as per a variation of Formula Square Root of Two-3.
The unshaded rectangles added to the squares produce root two rectangle and added to the root two rectangles produce squares.

Figure 189

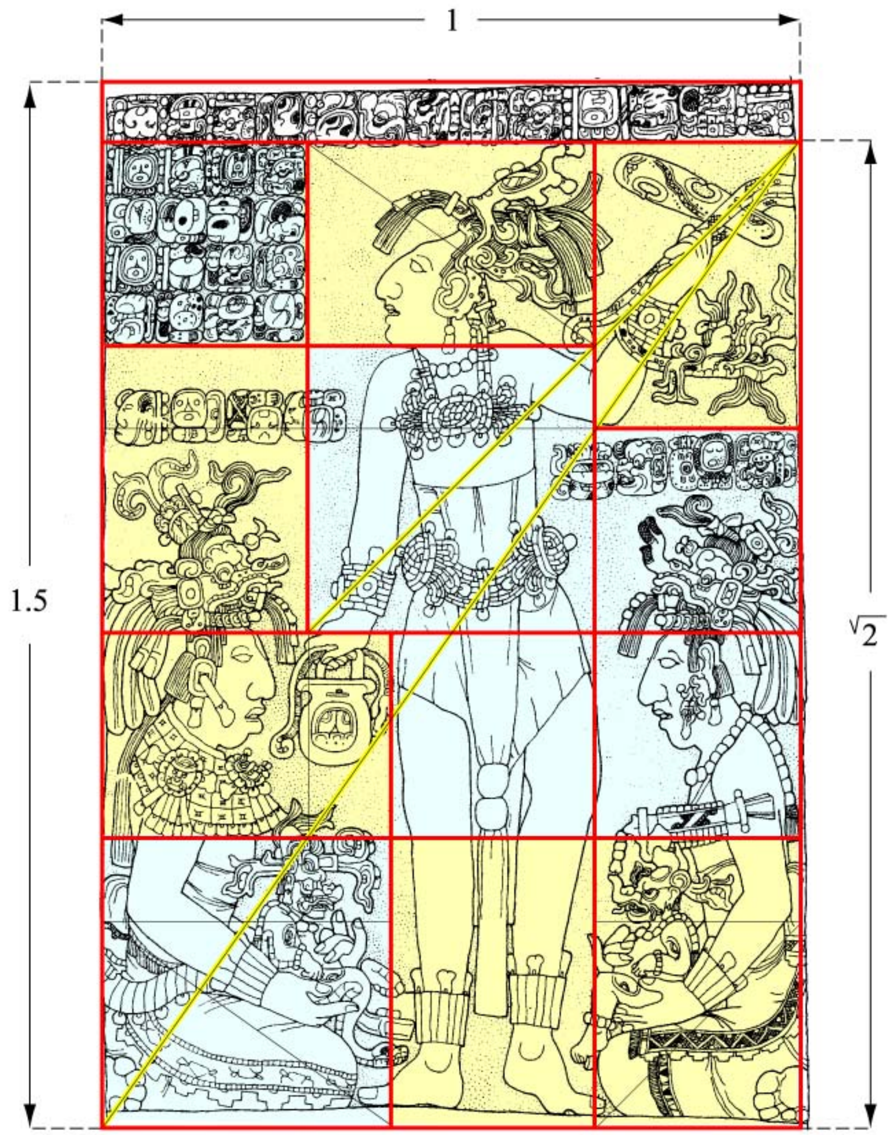
Stela 34, El Peru, Peten, Guatemala
Measured Drawing by Ian Grahm



Stela 34 from El Peru is inscribed by a root two rectangle that is subdivided into squares (shaded blue) and smaller root two rectangles (shaded yellow), as per a variation of Formula Square Root of Two-3.

Figure 190

Dumbarton Oaks Tablet, Palenque, Chiapas, Mexico
Measured Drawing by Merle Greene Robertson



The Dumbarto Oaks Table from Palenque is inscribed by a three over two rectangle, but the iconography, minus the band of hieroglyphs across the top of the panel, is inscribed by a root two rectangle that is subdivided into squares (shaded blue) and smaller root two rectangles (shaded yellow). as per a variation of Formula Square Root of Two-3.

Figure 191

Photograph of Dumbarton Oaks Tablet, Palenque, Chiapas, Mexico
Photograph by Merle Greene Robertson

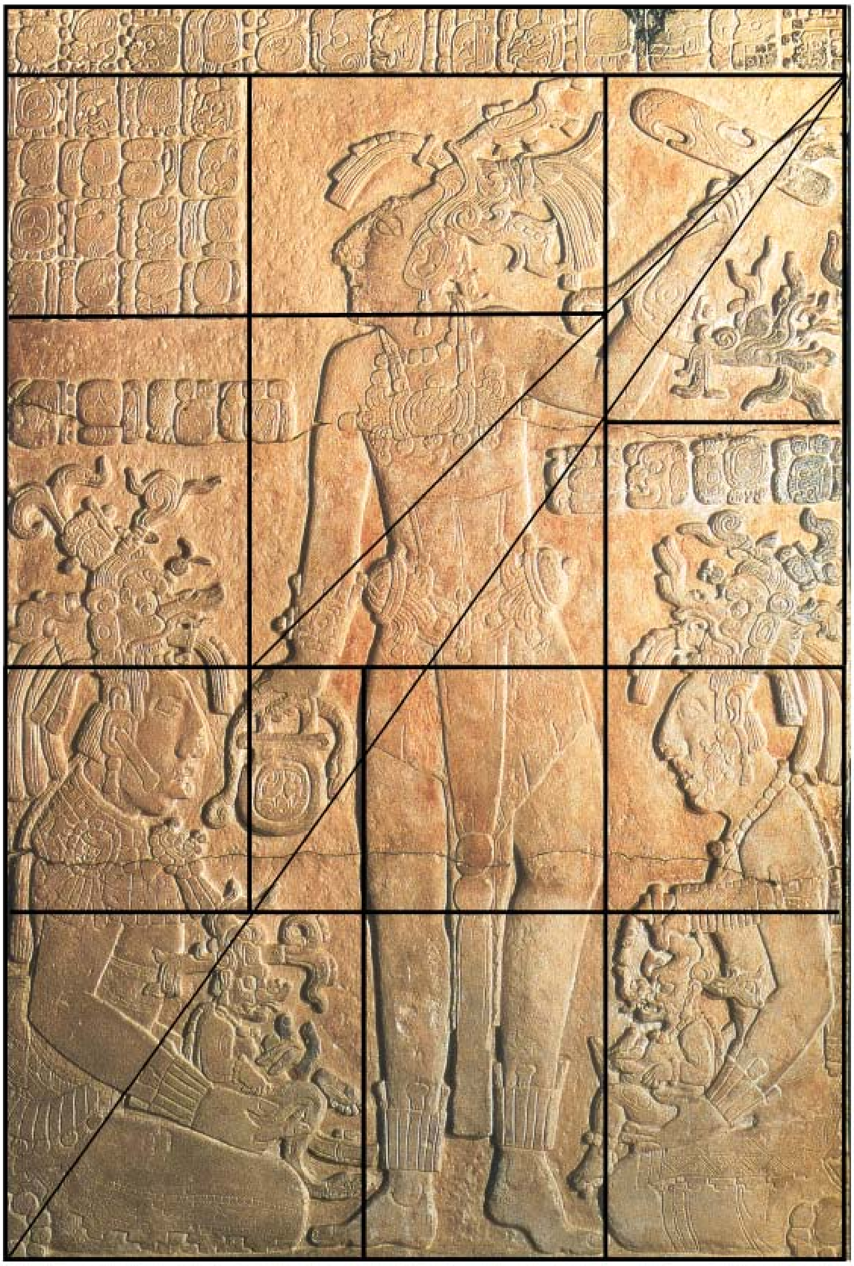
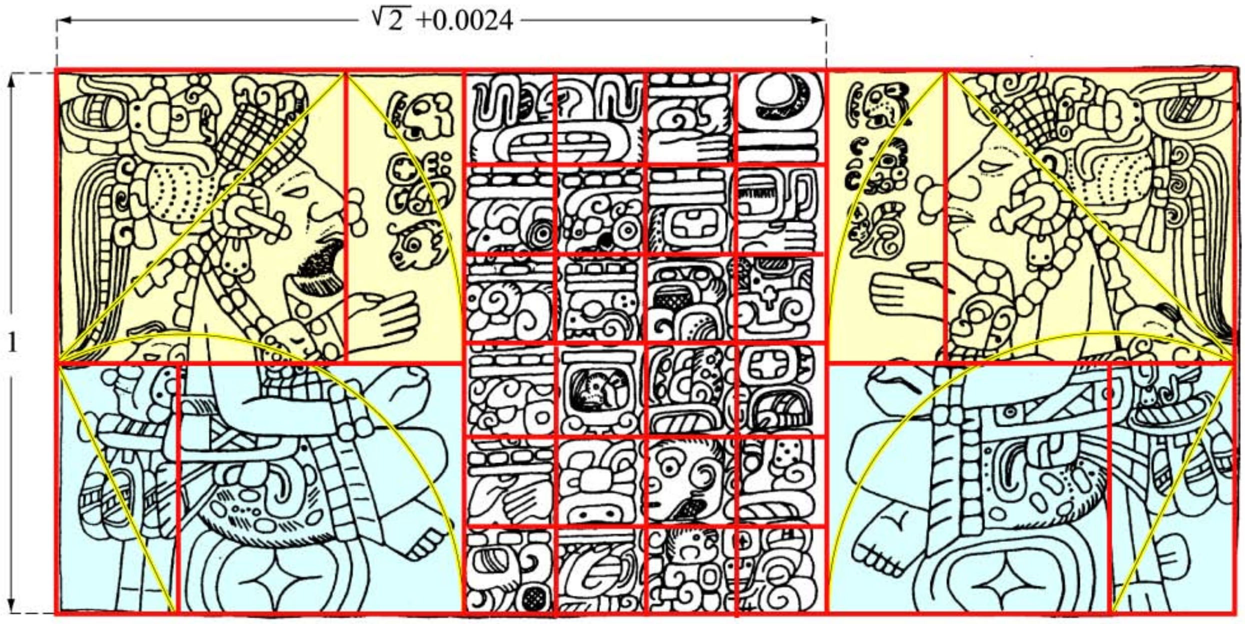


Figure 190 is a photograph of the Dumbarton Oaks Tablet with the same geometry as Figure 189 overlaid.

Figure 192

Lintel from the Bonampak Area,
Chiapas, Mexico



This lintel is a clear example of two formulae combined. The seated figures are inscribed by virtually perfect Pythagorean 3,4,5 rectangles subdivided by root two rectangles (shaded yellow) and phi rectangles (shaded blue), as per Formula Pythagorean 3,4,5-2. The text block, which may be viewed as two Pythagorean 3,4,5 rectangles divided into twenty-four equal sized squares, when added to either of the larger Pythagorean 3.4.5 rectangles, produces virtually perfect root two rectangles, as per Formula Square Root of Two-4.

Figure 193

Square Root of Three Hieroglyphic Text Blocks 1
Measured Drawings by Ian Grahm

Lintel 11, Yaxchilan

Lintel 32, Yaxchilan

Lintel 47, Yaxchilan

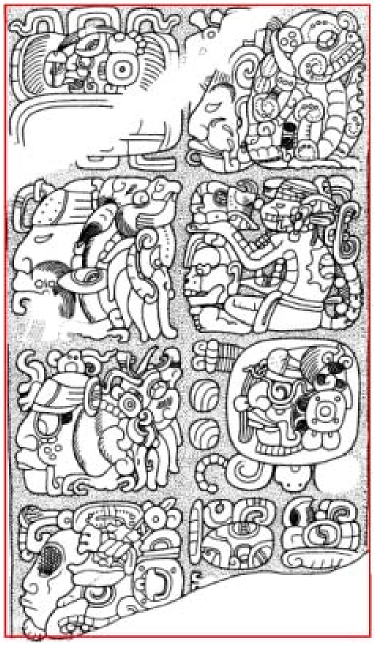
Lintel 60, Yaxchilan

Lintels`from Yaxchilan inscribed by square root of three rectangles

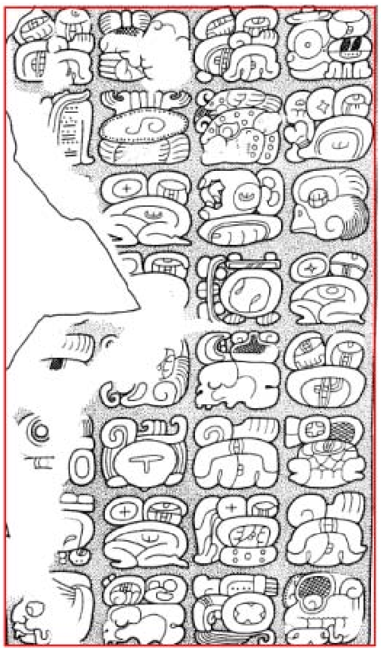
Figure 194

Square Root of Three Hieroglyphic Text Blocks 2
Measured Drawings by Ian Grahm

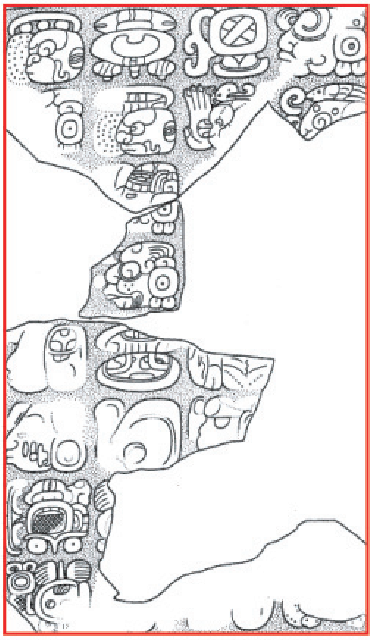
Lintel 48, Yaxchilan



Lintel 49, Yaxchilan



Lintel 34, Yaxchilan

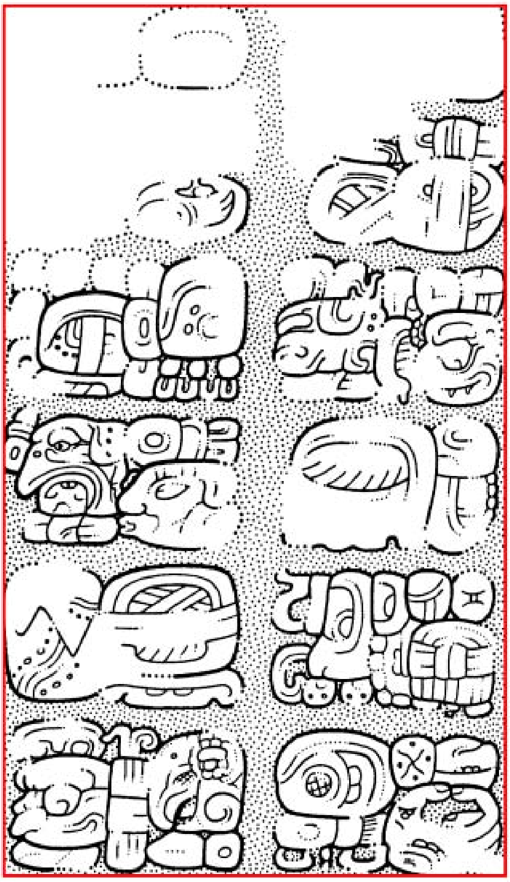


Lintels from Yaxchilan inscribed by square root of three rectangles

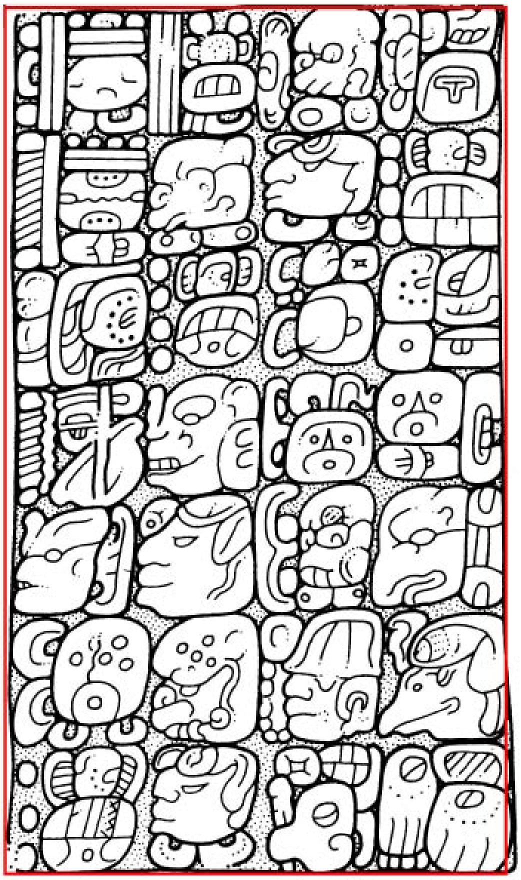
Figure 195

Square Root of Three Hieroglyphic Text Blocks 3

Panel 6-v, Dos Pilas



Provenience Unknown

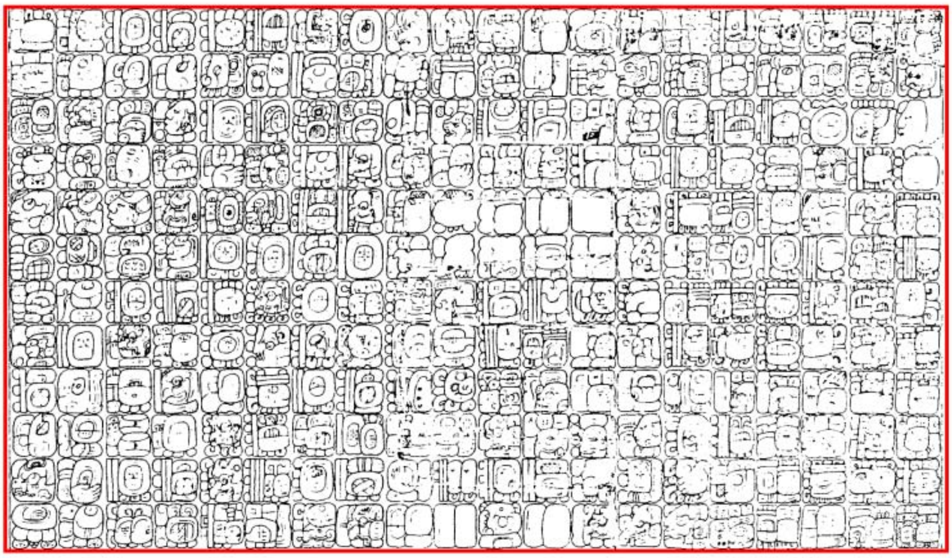


Hieroglyphic text blocks inscribed by square root of three rectangles

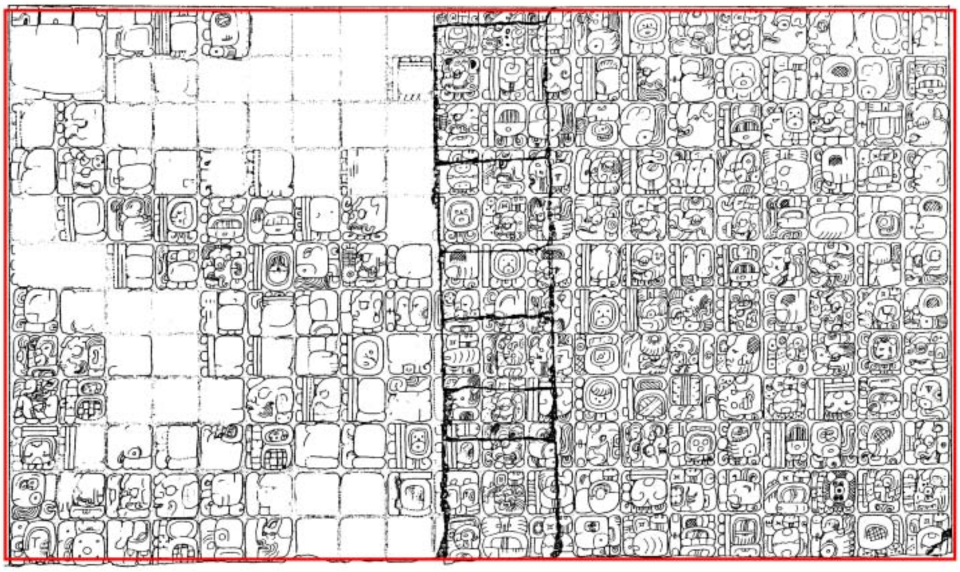
Figure 196

Square Root of Three Hieroglyphic Text Blocks 4
Measured Drawings by Merle Greene Robertson

West Tablet, Temple of the Inscriptions, Palenque



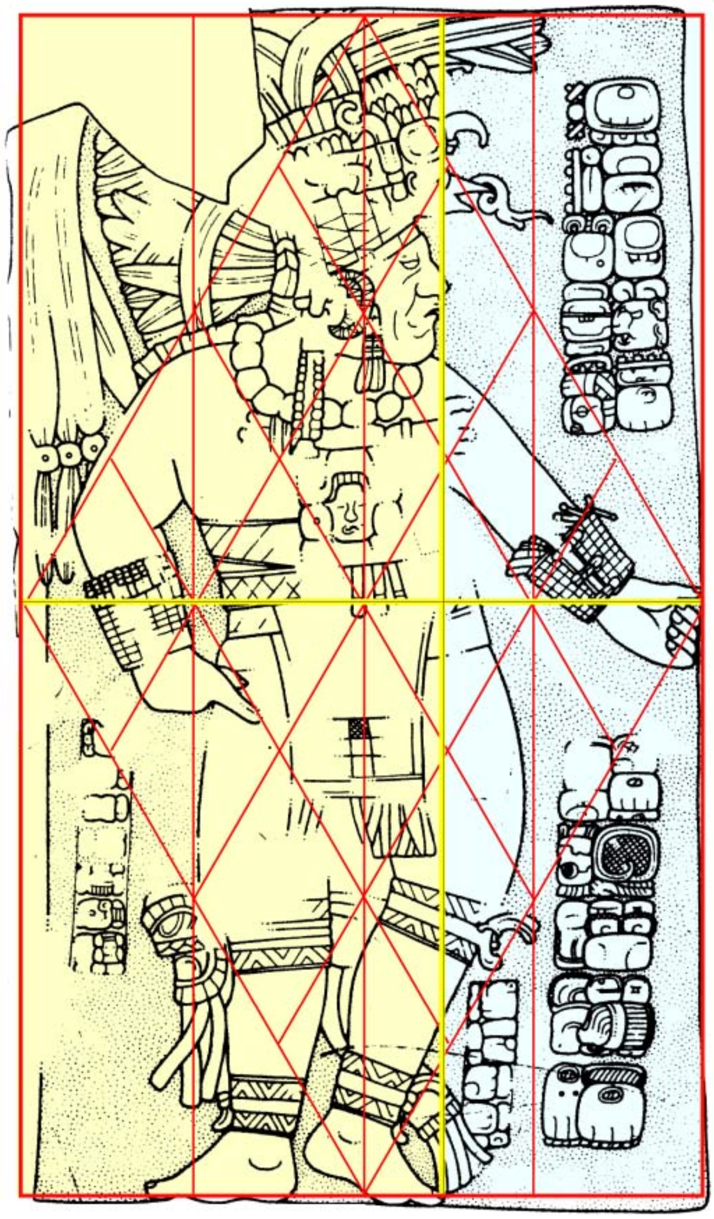
East Tablet, Temple of the Inscriptions, Palenque



Hieroglyphic text blocks inscribed by square root of three rectangles

Figure 197

Dancer, Coba, Quintana Roo, Mexico
Measured Drawing by Ian Graham

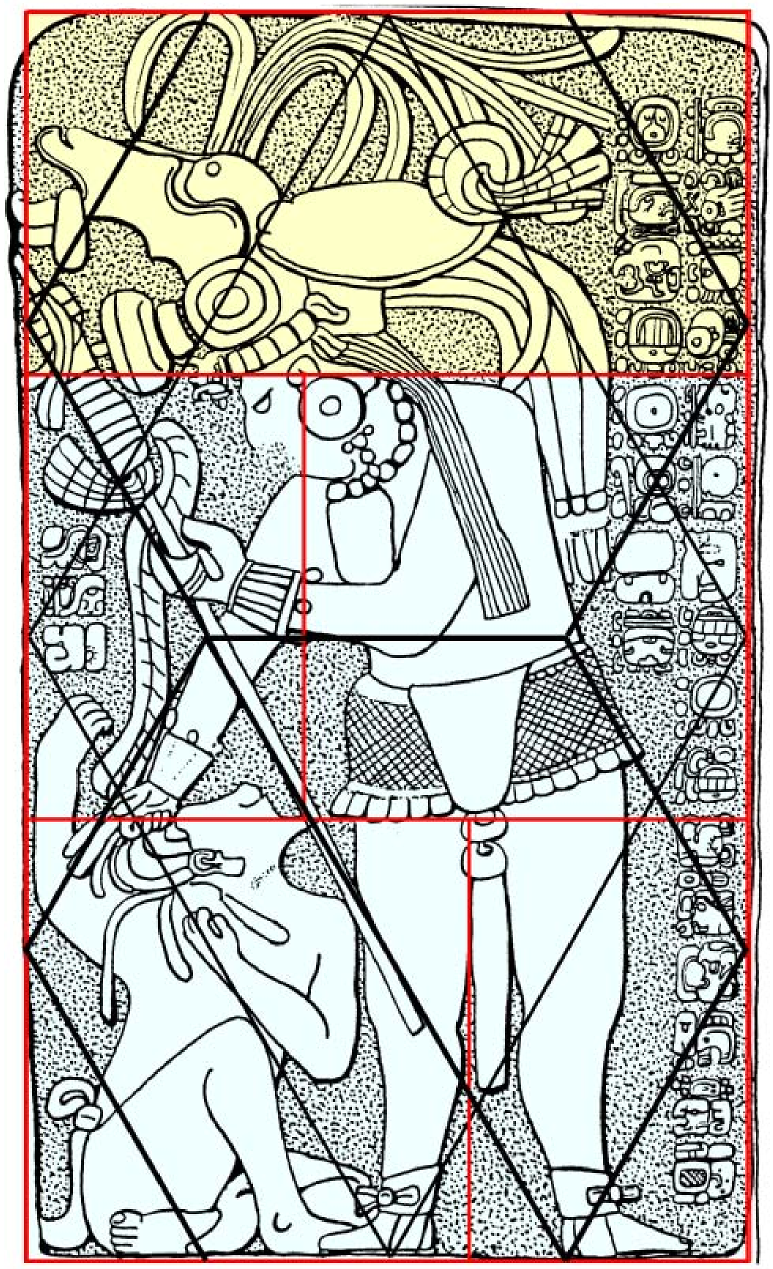


This stela from Coba is inscribed by a virtually perfect root three rectangle that is subdivided by root five rectangles (shaded blue) and root two rectangles (shaded yellow), as per Formula Square Root of Two-4.

Note these subdivisions relative to the positions of the left thumb and big toe of the dancing figure.

Figure 198

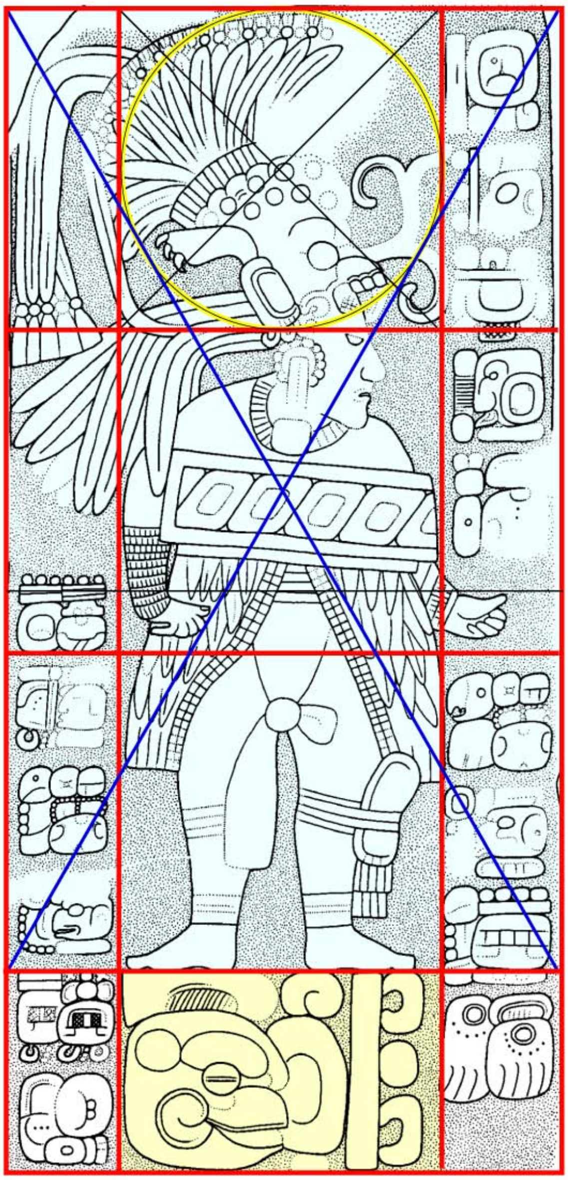
La Mar Stela 3
(Provenience Unknown)



La Mar Stela 3 is inscribed by a root three rectangle that is subdivided by a root four rectangle (shaded yellow) and two phi rectangles (shaded blue), as per the virtually perfect Formula Square Root of Three-3. Paired equilateral triangles and hexagons (overlaid in black lines) may have determined the placement of the hands and the angle of the spear.

Figure 199

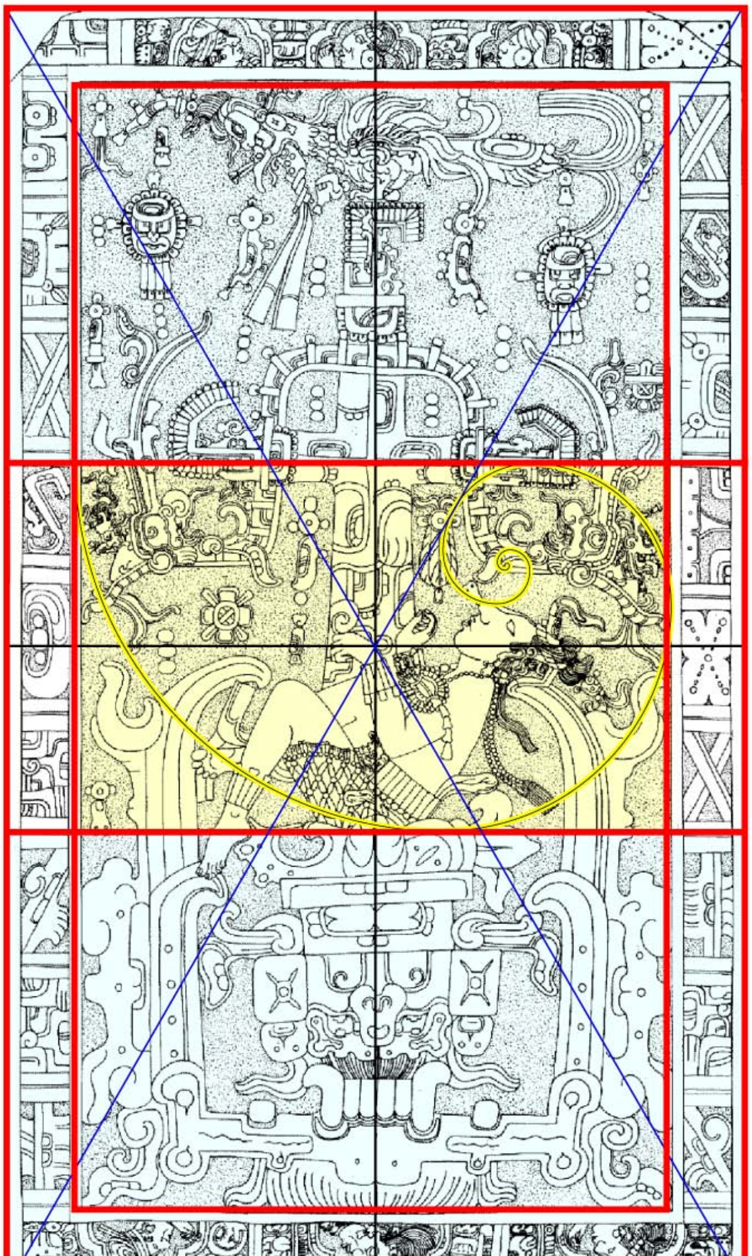
Stela 7, Seibal, Guatemala
Measured Drawing by Ian Graham



The standing figure in Stela 7 from Seibal is inscribed by a root three rectangle that is subdivided into three smaller root three rectangles, as per Formula Square Root of Three-1. At the center of these subdivisions are three squares that separate the standing figure from the hieroglyphic texts. The large hieroglyph beneath the feet of the standing figure is inscribed by a phi rectangle (shaded yellow).

Figure 200

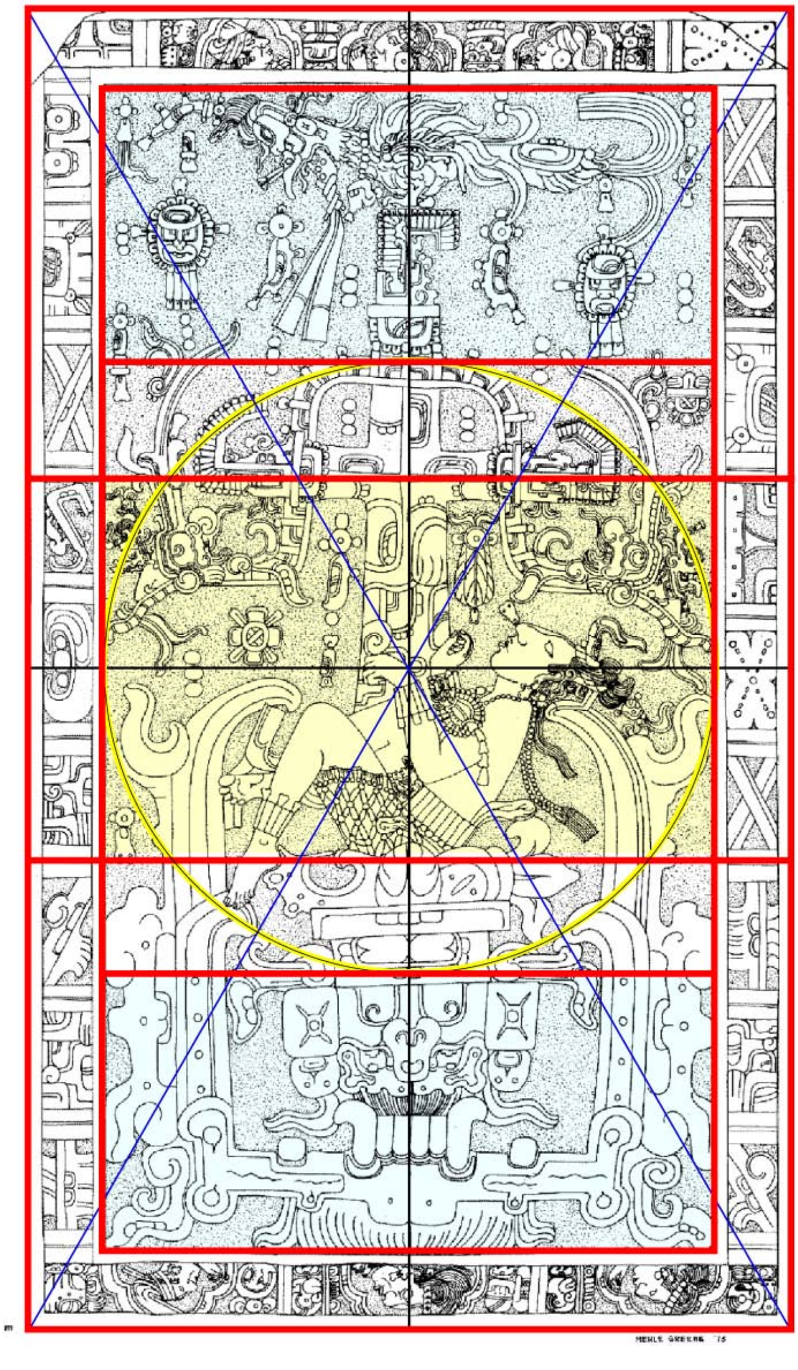
Sarcophagus Lid (a), Temple of the Inscriptions, Palenque, Chiapas, Mexico
Measured Drawing by Merle Greene Robertson



The Sarcophagus Lid from the Temple of the Inscriptions at Palenque is inscribed by a root three rectangle that is subdivided by a root four rectangle (white) and two phi rectangles (shaded blue), as per the virtually perfect Formula Square Root of Three-3. Where the root four rectangle cuts across the inside of the sky band it defines a phi rectangle (shaded yellow).

Figure 201

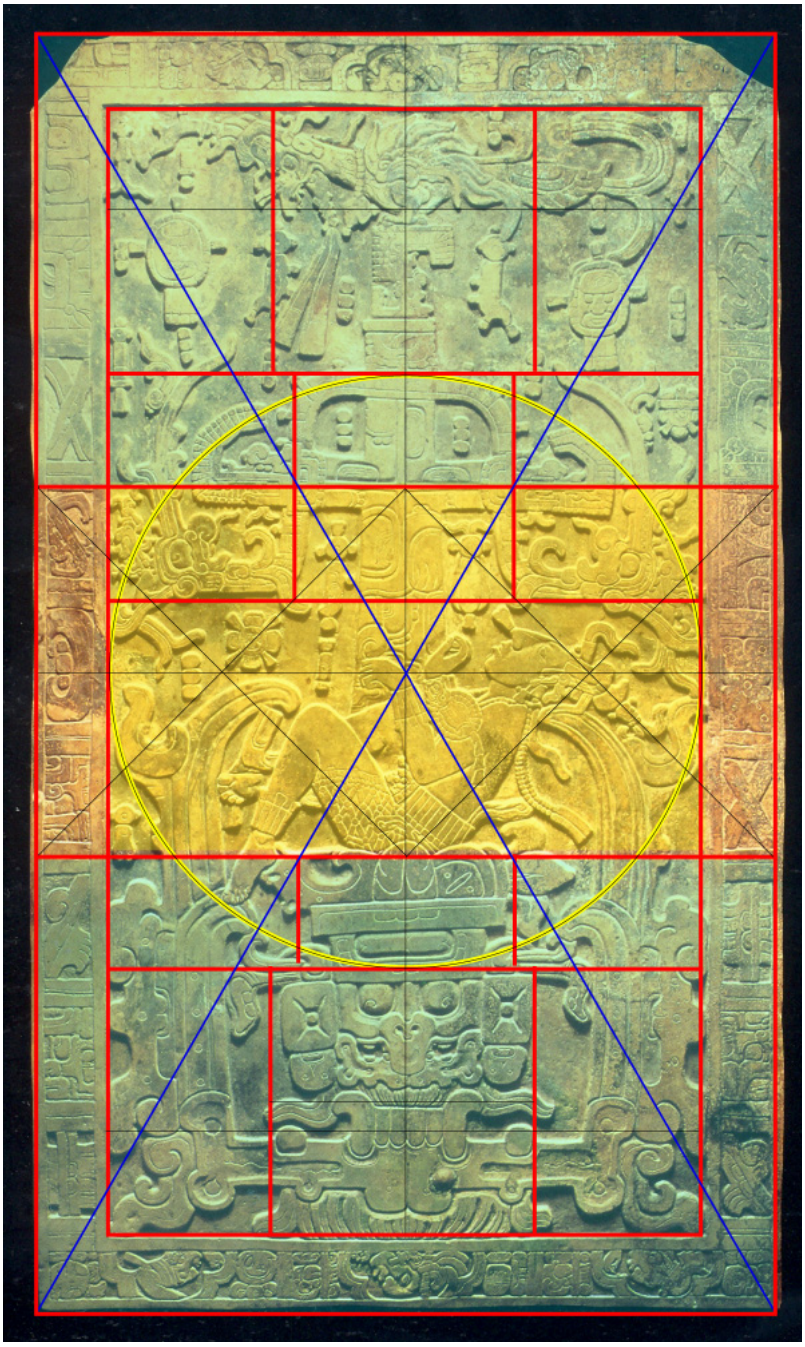
Sarcophagus Lid (b), Temple of the Inscriptions, Palenque, Chiapas, Mexico
Measured Drawing by Merle Greene Robertson



The rectangle inside of the sky band is inscribed by a square (white) and two root five rectangles (shaded blue). The central square is inscribed by a circle (shaded yellow).

Figure 202

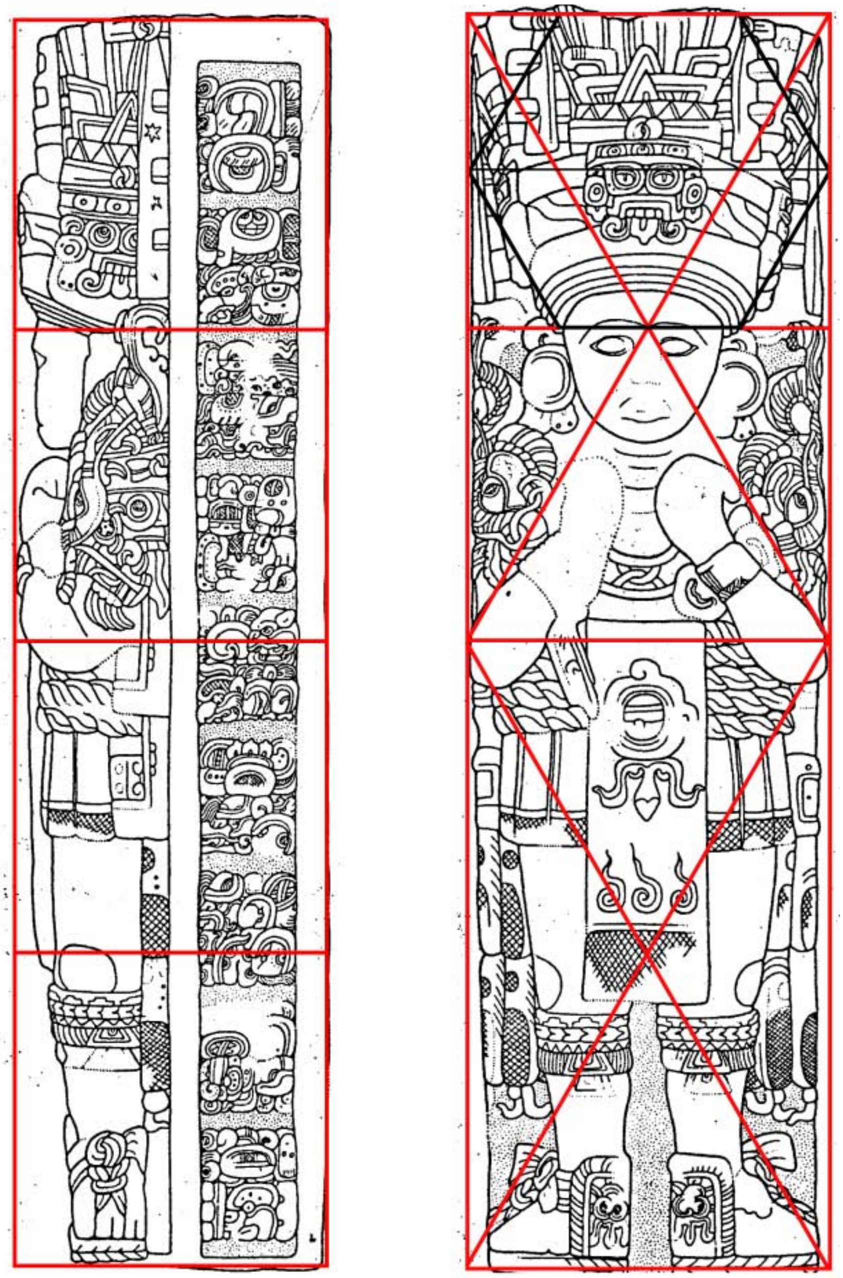
Sarcophagus Lid (c), Temple of the Inscriptions, Palenque, Chiapas, Mexico
Photograph by Merle Greene Robertson



This is a photograph of the Sarcophagus Lid with the suggested geometry overlaid. The rectangle inside of the sky band is subdivided into squares and phi rectangles.

Figure 203

Stela 6, Copan, Honduras
Measured Drawing by Barbara Fash

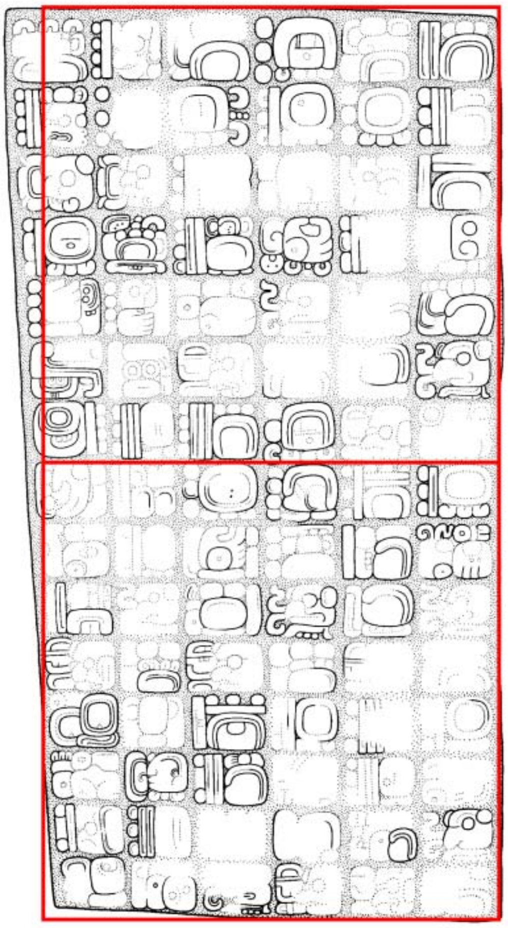


The side view of Stela 6 from Copan is inscribed by two root four rectangles subdivided by four equal sized squares.
The front view of the stela is inscribed by two root three rectangles. The uppermost root three rectangle is divided in two at the brow of the standing figure, and a hexagon inscribes the headdress.

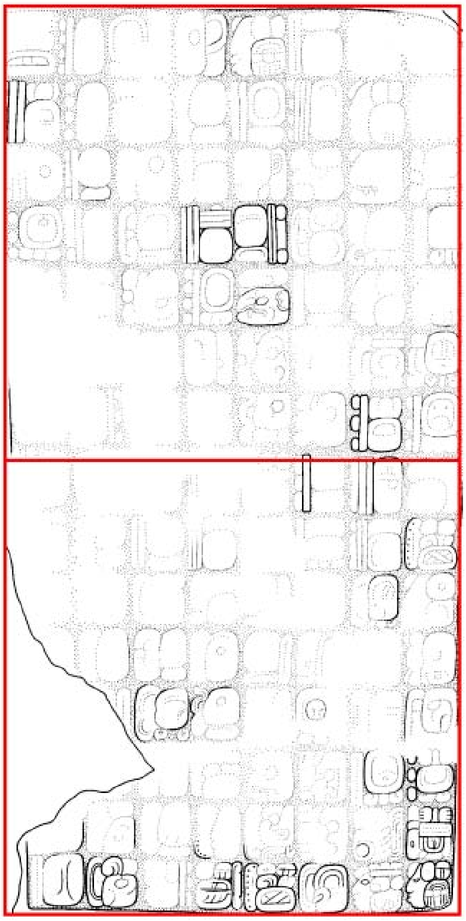
Figure 204

Square Root of Four Hieroglyphic Text Blocks 1
Measured Drawings by Ian Grahm

Stela 30-b, Naranjo



Stela 31-b, Naranjo

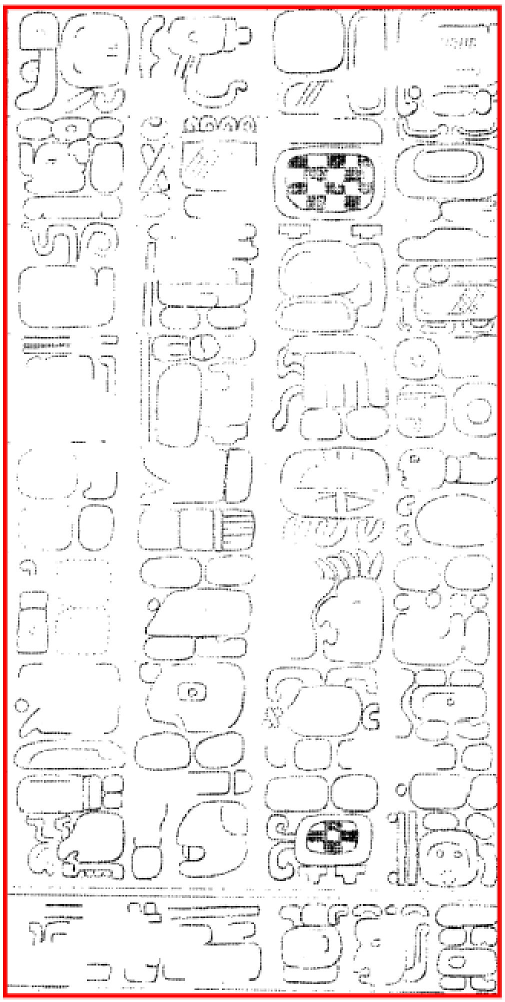


Hieroglyphic text blocks inscribed by root four rectangles. Note the possibly intentional asymmetry of the upper left corner of Stela 30-b.

Figure 205

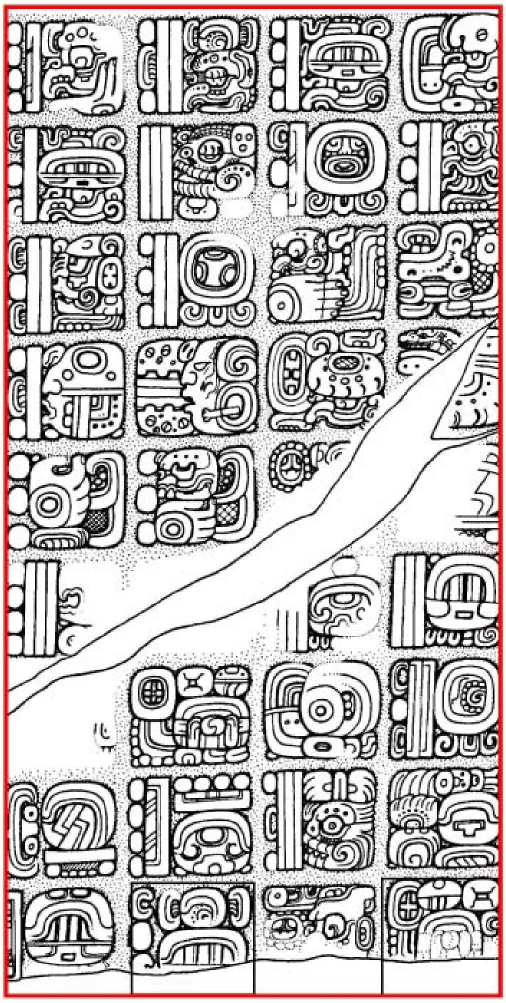
Square Root of Four Hieroglyphic Text Blocks 2
Measured Drawings by Ruth Krochoc (a) and Ian Graham (b)

Stela A, Chichen Itza



a

Stela 8, Dos Pilas



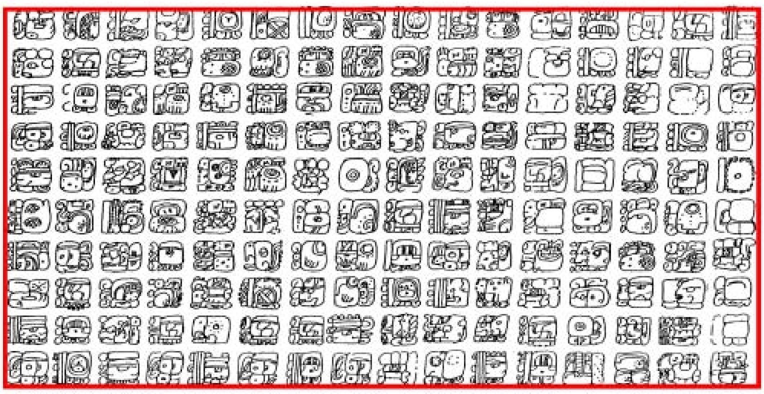
b

Hieroglyphic text blocks inscribed by root four rectangles

Figure 206

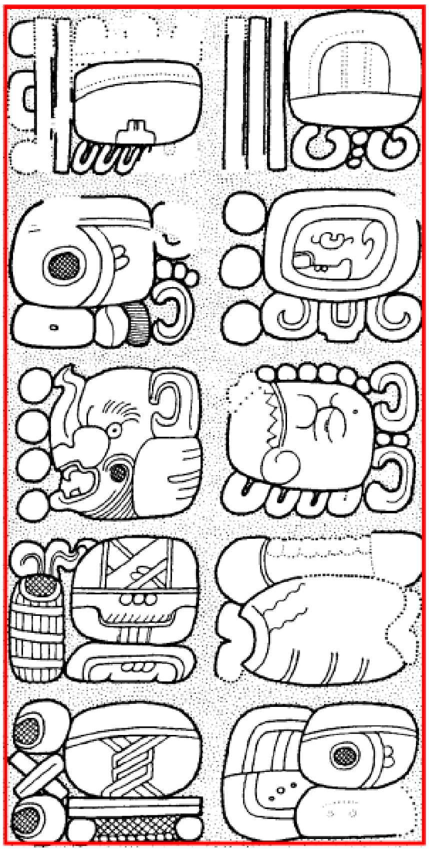
Square Root of Four Hieroglyphic Text Blocks 3
Measured Drawing by Nikolai Grube (b)

Panel 1,Cancuen



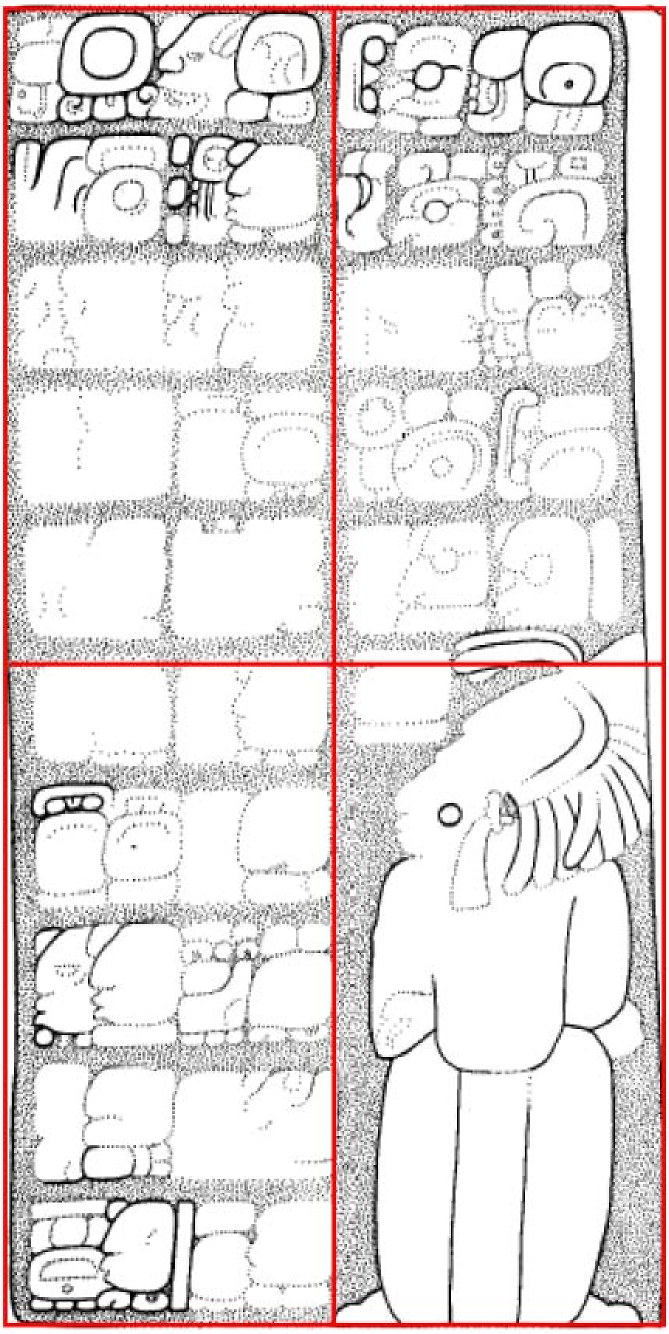
Provenience Unknown

b



Hieroglyphic text blocks inscribed by root four rectangles

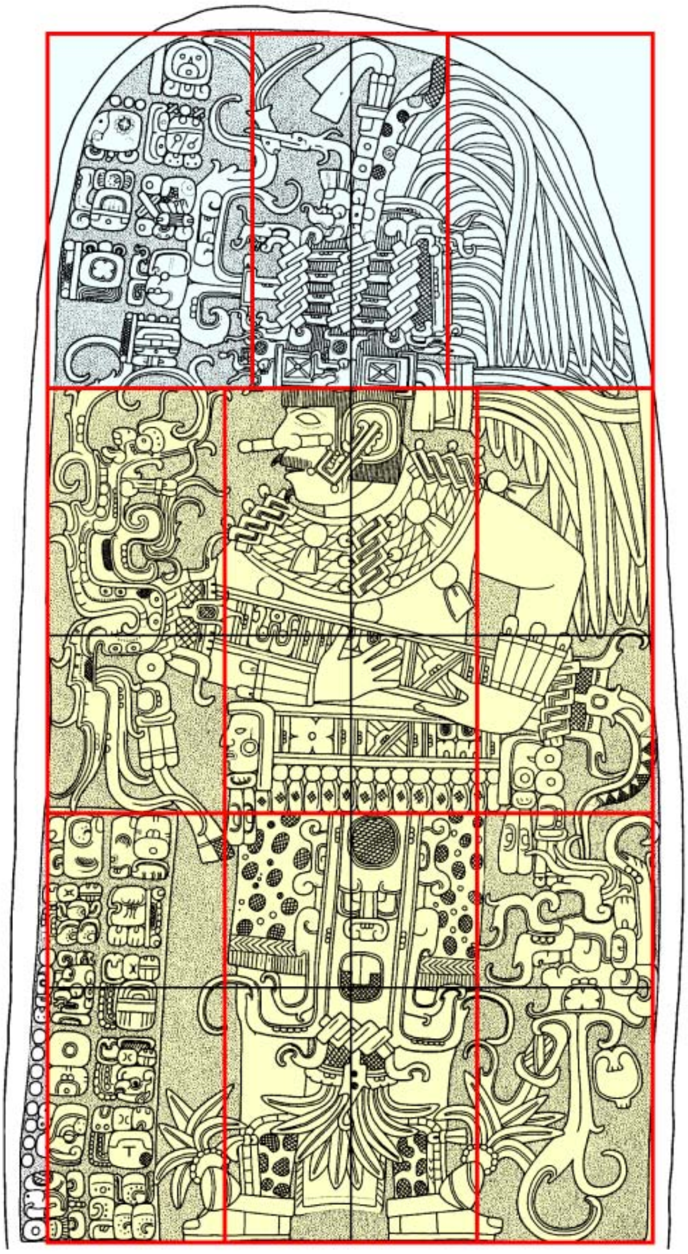
Figure 207
Hieroglyphic Stair 03-2, Yaxchilan, Chiapas, Mexico
Measured Drawing by Ian Grahm



This Hieroglyphic Stair from Yaxchilan is inscribed by a root four rectangle that is quartered. The upper right-hand corner may be an example of intentional asymmetry.

Figure 208

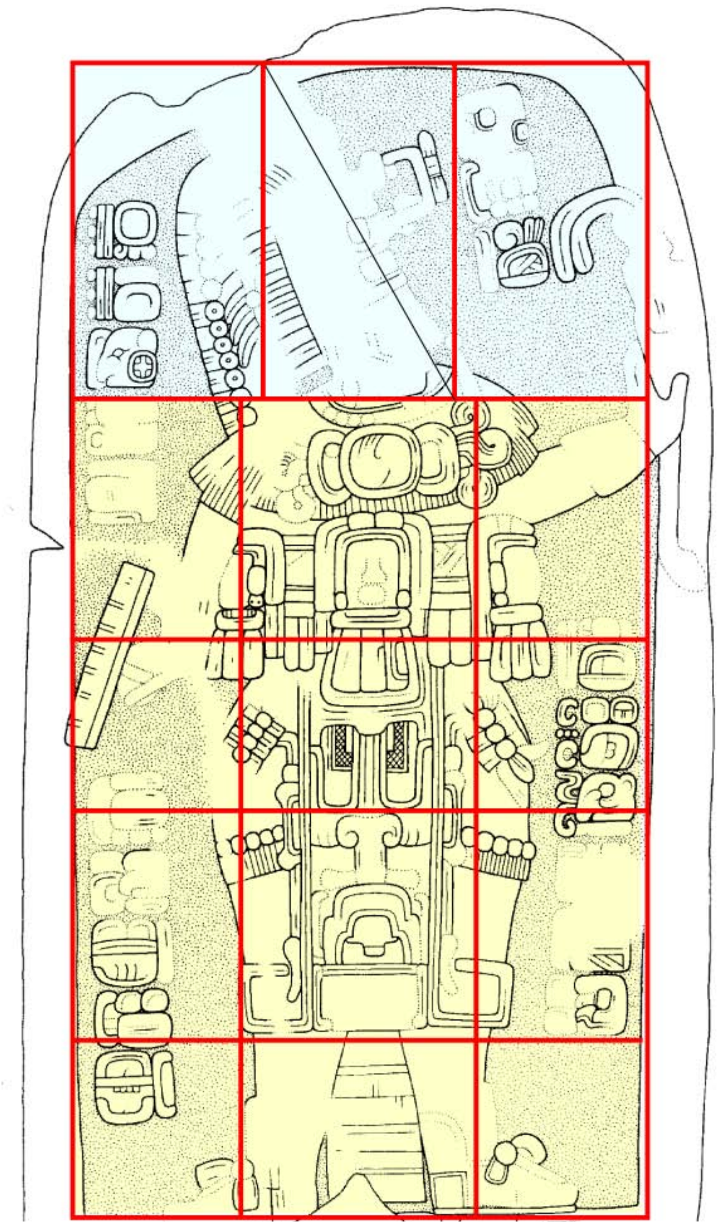
Stela 10, Seibal, Guatemala
Measured Drawing by Ian Grahm



Stela 10 from Seibal is inscribed by a root four rectangle that is subdivided by root two and root three rectangles, as per the virtually perfect Formula Square Root of Four-4. The root three rectangle (shaded blue) is subdivided into three smaller root three rectangles, and the root two rectangle (shaded yellow) is subdivided into squares and smaller root two rectangles, as per Formula Square Root of Two-3

Figure 209

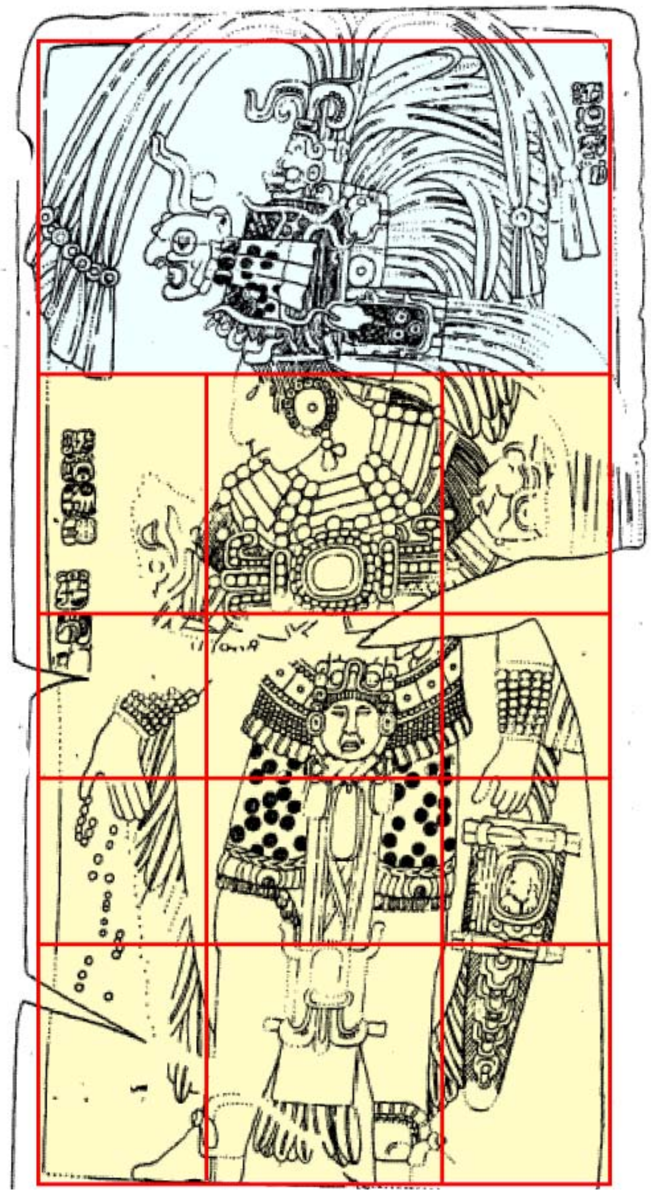
Stela 28-a, Naranjo, Belize
Measured Drawing by Ian Grahm



As in Stela 10 from Seibal, Stela 28-a from Naranjo is inscribed by a root four rectangle that is subdivided by root two and root three rectangles, as per the virtually perfect Formula Square Root of Four-4. The root three rectangle (shaded blue) is subdivided into three smaller root three rectangles, and the root two rectangle (shaded yellow) is subdivided into squares and smaller root two rectangles, as per Formula Square Root of Two-3

Figure 210

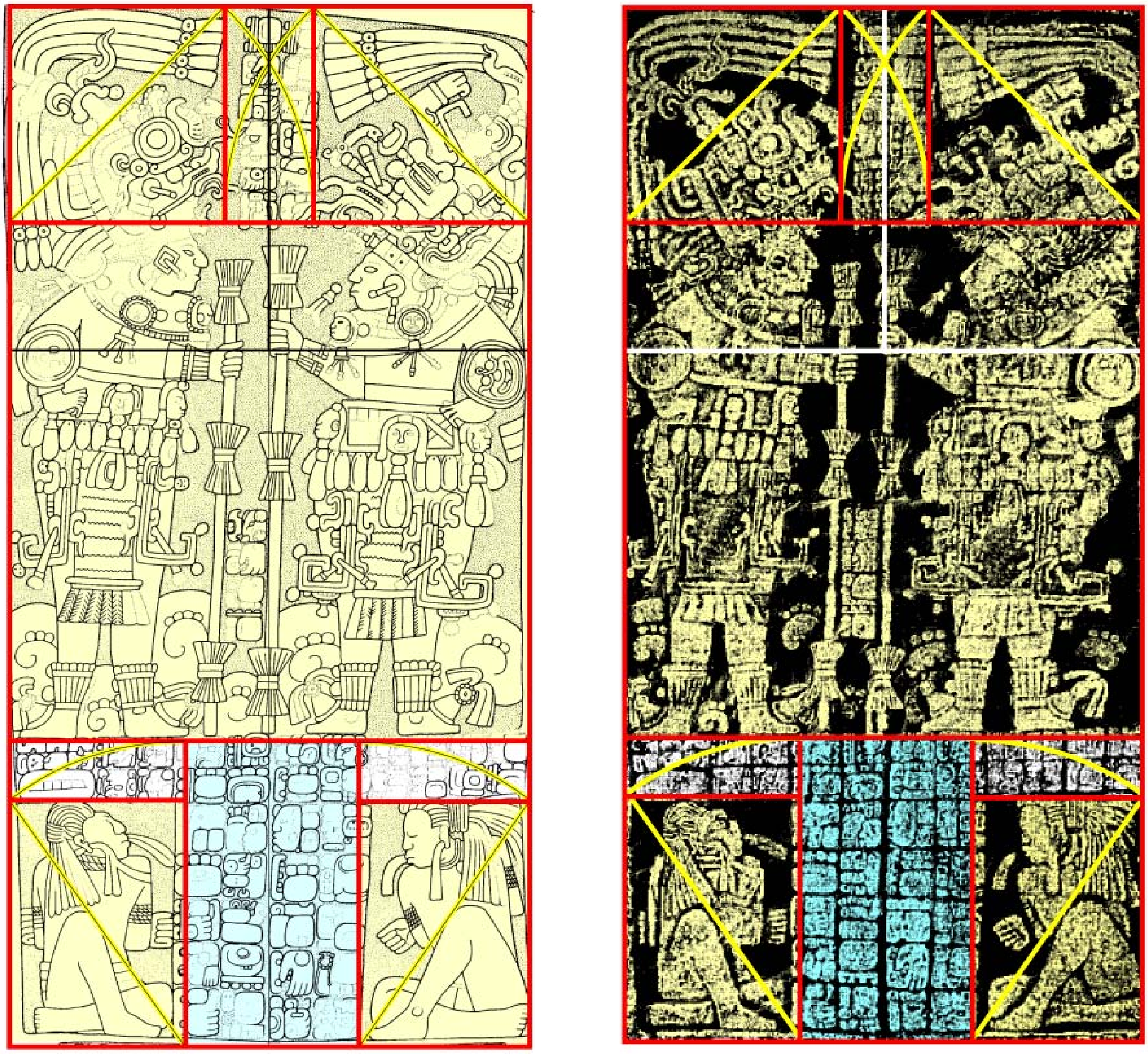
Stela 13, Piedras Negras, Guatemala
Measured Drawing by John Montgomery



Identically to Stela 10 from Seibal and Stela 28-a from Naranjo, Stela 13 from Piedras Negras is inscribed by a root four rectangle that is subdivided by root two and root three rectangles, as per the virtually perfect Formula Square Root of Four-4. The root three rectangle is shaded blue. The root two rectangle (shaded yellow) is subdivided into squares and smaller root two rectangles, as per Formula Square Root of Two-3

Figure 211

Stela 1, Ixcun, Yucatan, Mexico
Measured Drawing by Ian Grahm, Rubbing by Merle Greene Robertson



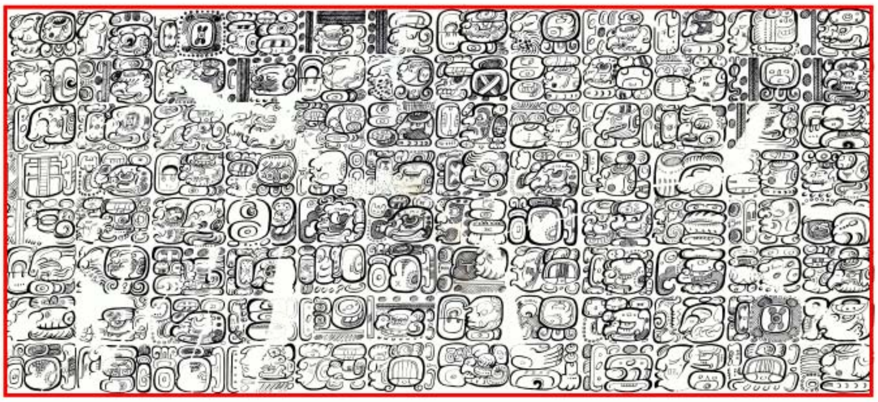
Identically to Figures 220-222, the rubbing and the drawing of Stela 1 from Ixcun are inscribed by a root four rectangle that is subdivided by root two and root three rectangles, as per the virtually perfect Formula Square Root of Four-4. The root three rectangles are subdivided into three smaller root three rectangles, two of which are subdivided by root two rectangles that inscribe the captives. The root two rectangles (shaded yellow) are subdivided into squares and smaller root two rectangles, as per Formula Square Root of Two-3 The white and black lines subdivide the root two rectangles into three Pythagorean 3,4,5 rectangles, as per Formula Square Root of Two-4.

Figure 212

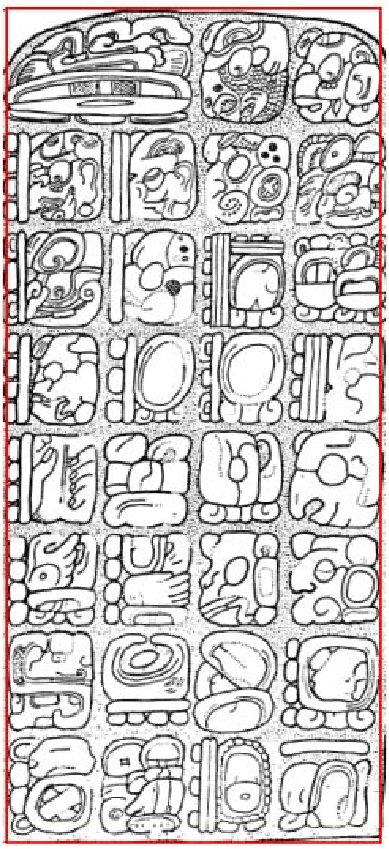
Square Root of Five Hieroglyphic Text Blocks
Measured Drawings by Merly Greene Robertson (a) and John Montgomery

Tablet of the Ninety-Six Glyphs, Palenque

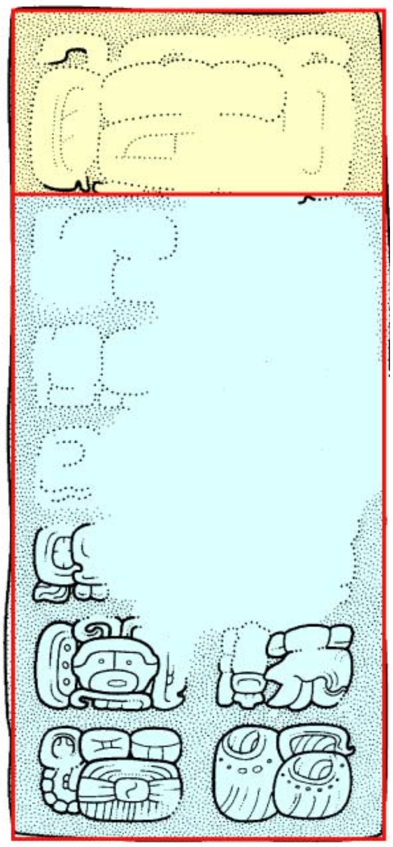
a



Panel 36, Piedras Negras



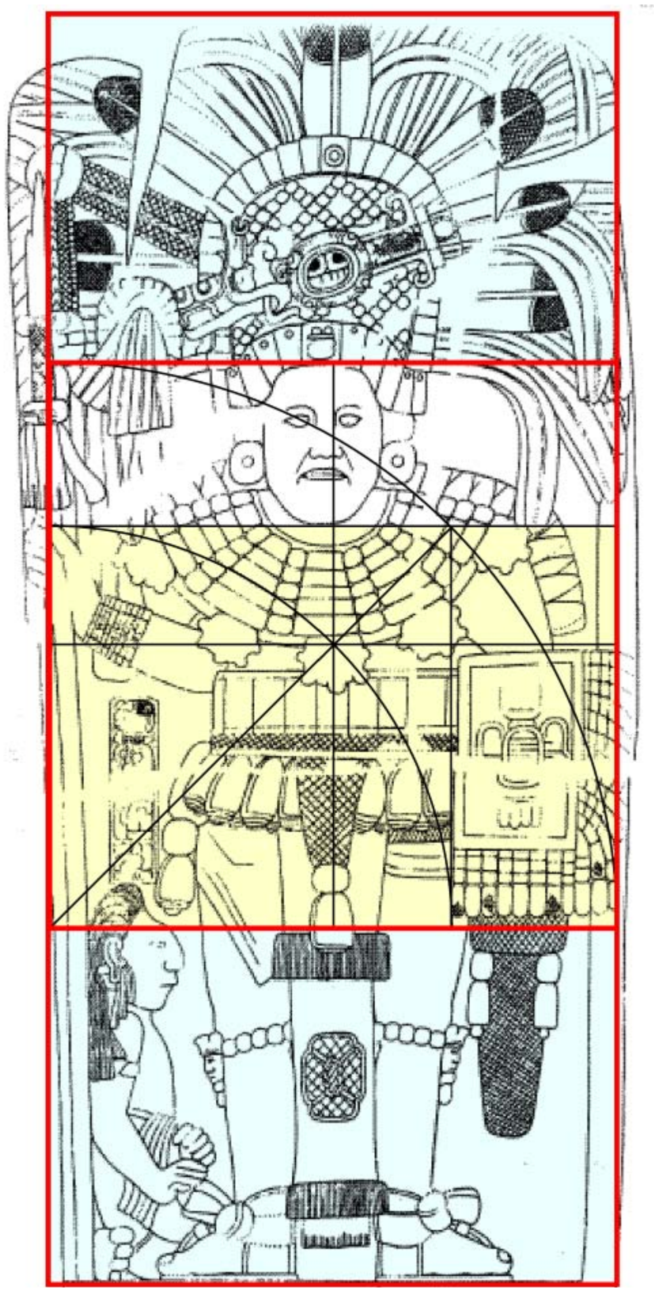
Panel 6, Piedras Negras



Hieroglyphic text blocks inscribed by a root five rectangles. Note that the root five rectangle that inscribes Panel 6 from Piedras Negras is subdivided by a root three rectangle (shaded blue) and a root four rectangle (shaded yellow).

Figure 213

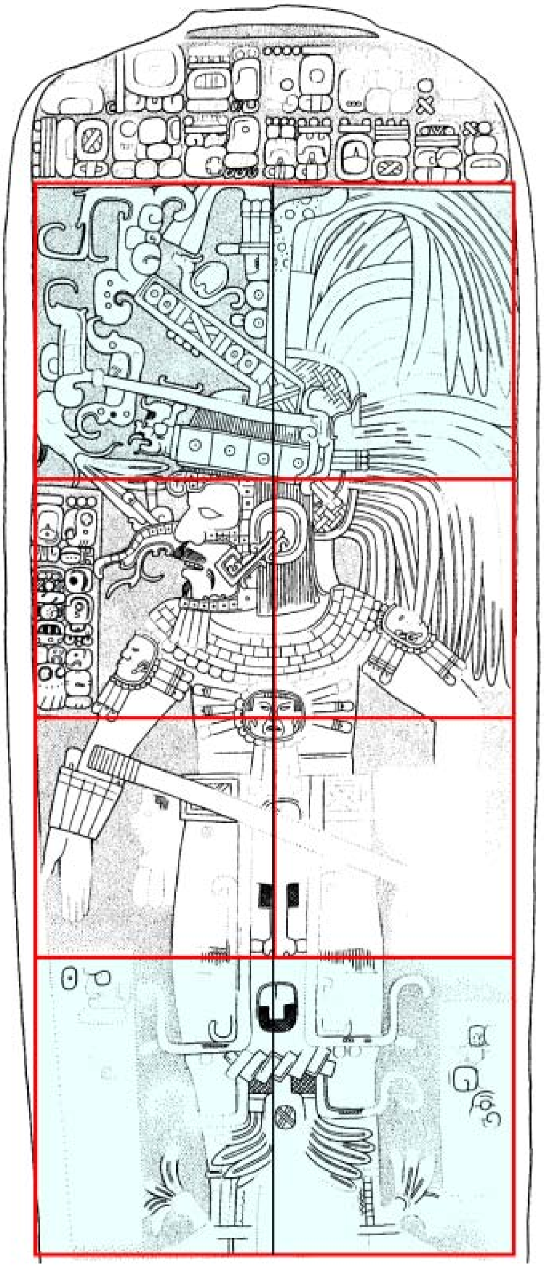
Stela 35, Piedras Negras, Guatemala
Measured Drawing by John Montgomery



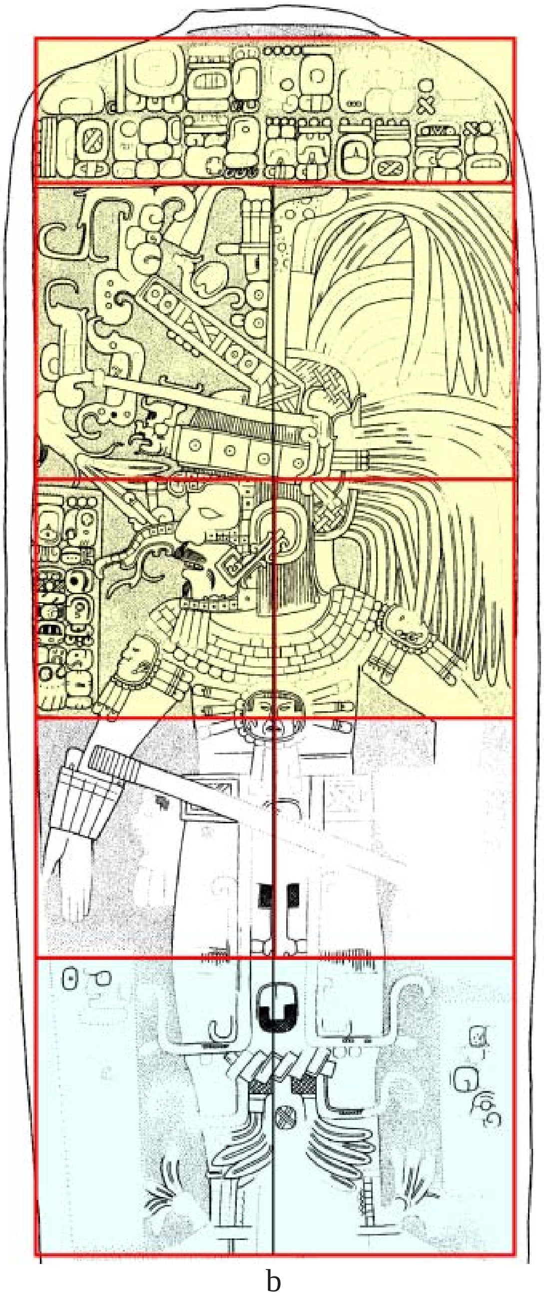
Stela 35 from Piedras Negras is inscribed by a root five rectangle that is subdivided by a square and two phi rectangles, as per Formula Square Root of Five-3. The phi rectangles are shaded blue and the remaining square is subdivided by a root two rectangle (shaded yellow).
The upper left side may be an example of intentional asymmetry.

Figure 214

Stela 11, Seibal, Peten, Guatemala
Measured Drawing by Ian Grahm



a

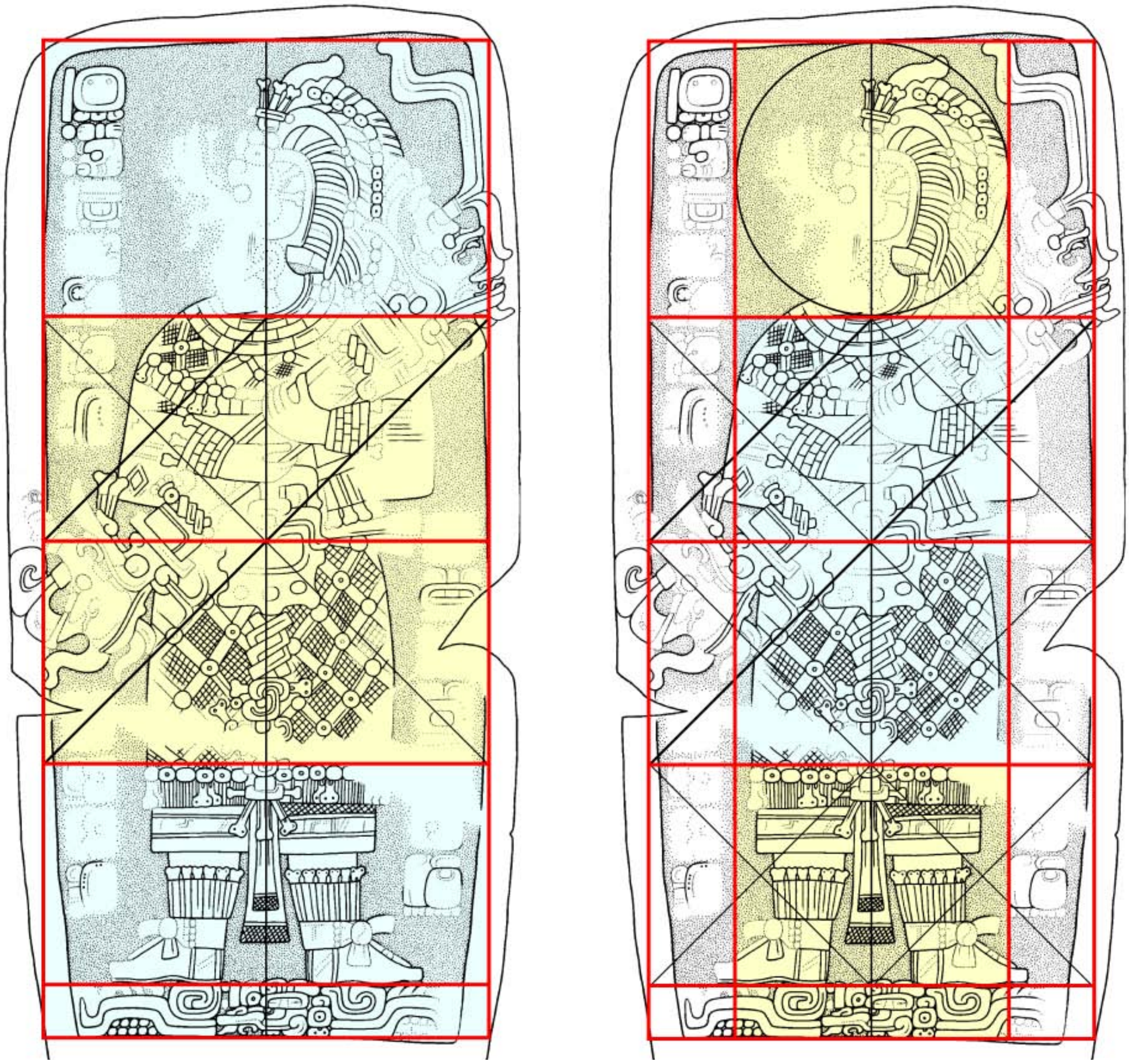


b

Stela 11 (a) from Seibal is inscribed by a root five rectangle that is subdivided by a square (white) and two phi rectangles (shaded blue), as per Formula Square Root of Five-3.
Stela 11 (b) includes the hieroglyphic text block. From the top of the stela to the center of the root five rectangle is a root two rectangle (shaded yellow).

Figure 215

Stela 31, Naranjo, Belize
Measured Drawing by Ian Grahm

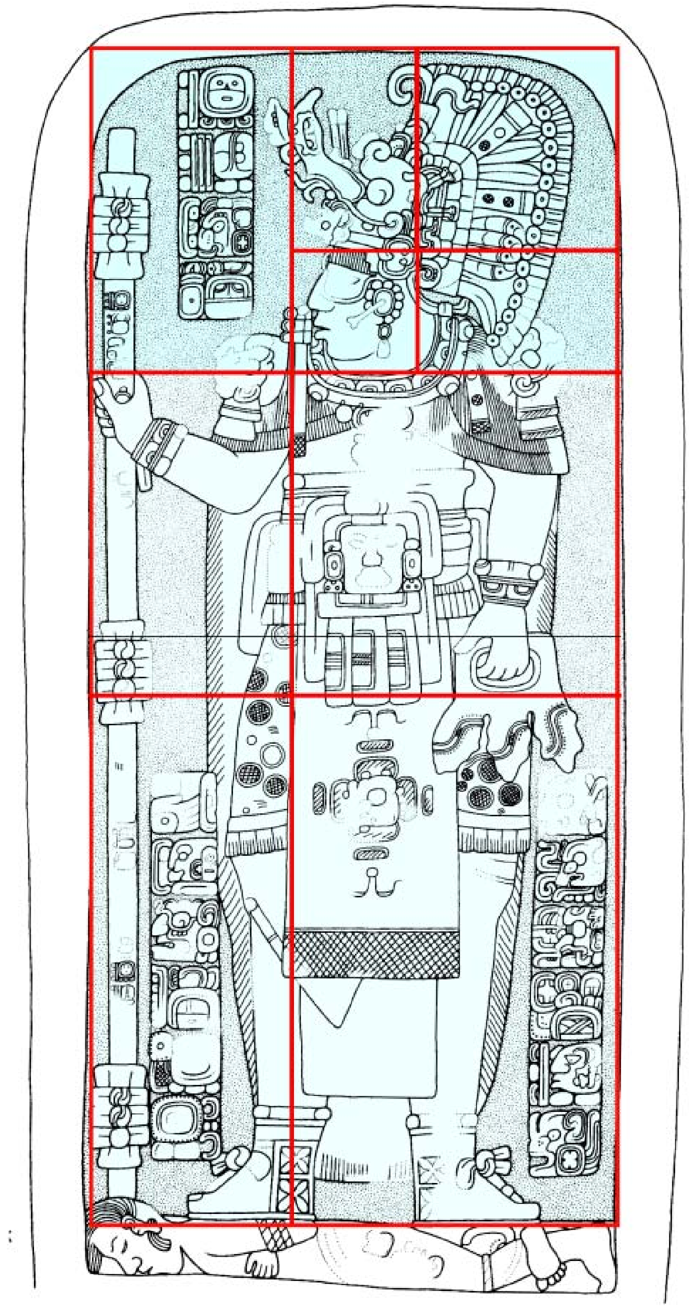


a b

Stela 31 (a) from Seibal is inscribed by a root five rectangle that is subdivided by a square (shaded yellow) and two phi rectangles (shaded blue), as per Formula Square Root of Five-3.
In Stela 31 (b), two squares (shaded yellow) and a phi rectangle (shaded blue) isolate the standing figure from the hieroglyphic texts.
All of the diagonal black lines are forty-five degrees.

Figure 216

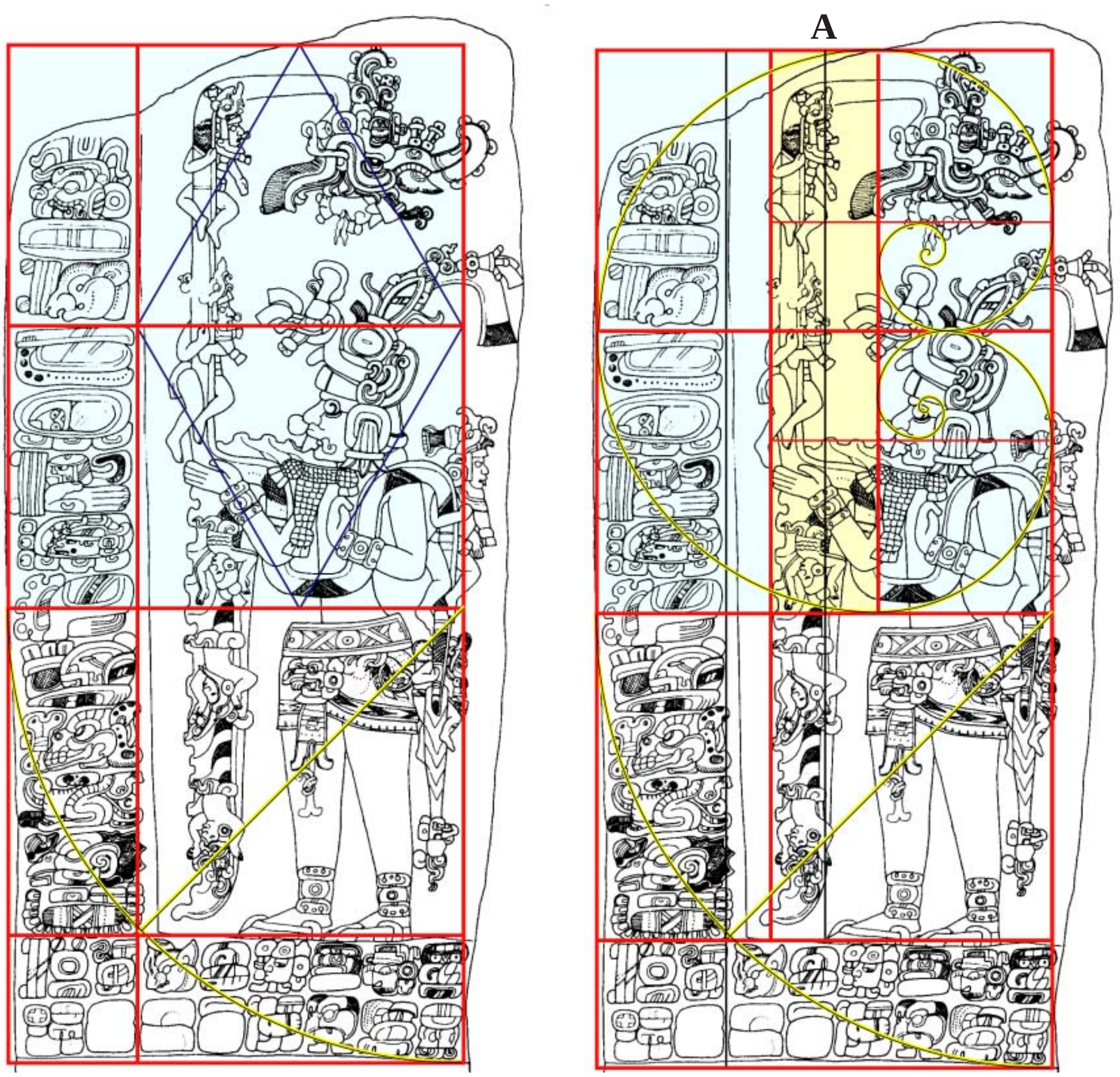
Stela 30-a, Naranjo, Belize
Measured Drawing by Ian Grahm



Stela 30-a from Naranjo is inscribed by a root five rectangle that is subdivided by a vertical phi rectangle (light blue) and horizontal phi rectangle (dark blue), as per Formula Square Root of Five-3. These rectangles are further subdivided into squares and smaller phi rectangles. The fine black line is the horizontal center of the root five rectangle. Note the placement of the hands of the standing figure.

Figure 217

Hauberg Stela, Kaminaljuyu, Guatemala



a b

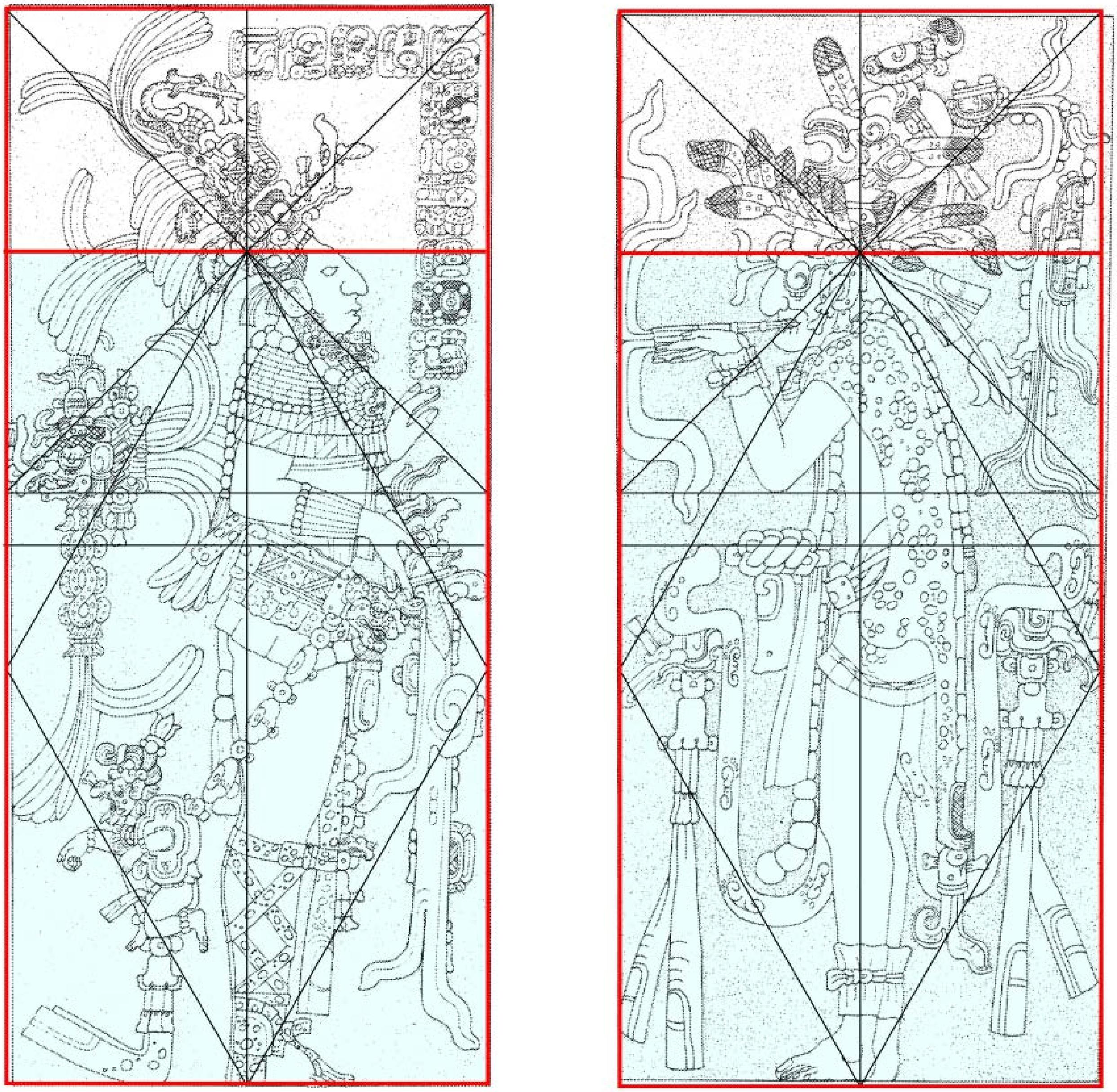
The Hauberg Stela is inscribed by a root five rectangle that is subdivided by a square (white) and two phi rectangles (shaded blue), as per Formula Square Root of Five-3. The square is subdivided by the hieroglyphic text into a smaller square whose diagonal is equal to the length of the sides of the larger square by a ratio of one to root two.

In Figure 216 a, the hieroglyphic text divides the phi rectangles by a root three rectangle that inscribes two equilateral triangles (blue lines).

In Figure 216 b, the phi rectangles are separated by phi squared rectangles (shaded yellow), and the equiangular phi spirals terminate in the spiral eye of the standing figure and the bifurcated tongue of the human face emerging from the serpent. Black line (A) is the horizontal center of the root five rectangle.

Figure 218

East and West Jambs, Temple of the Cross, Palenque, Chiapas,Mexico
Measured Drawings by Merle Greene Robertson

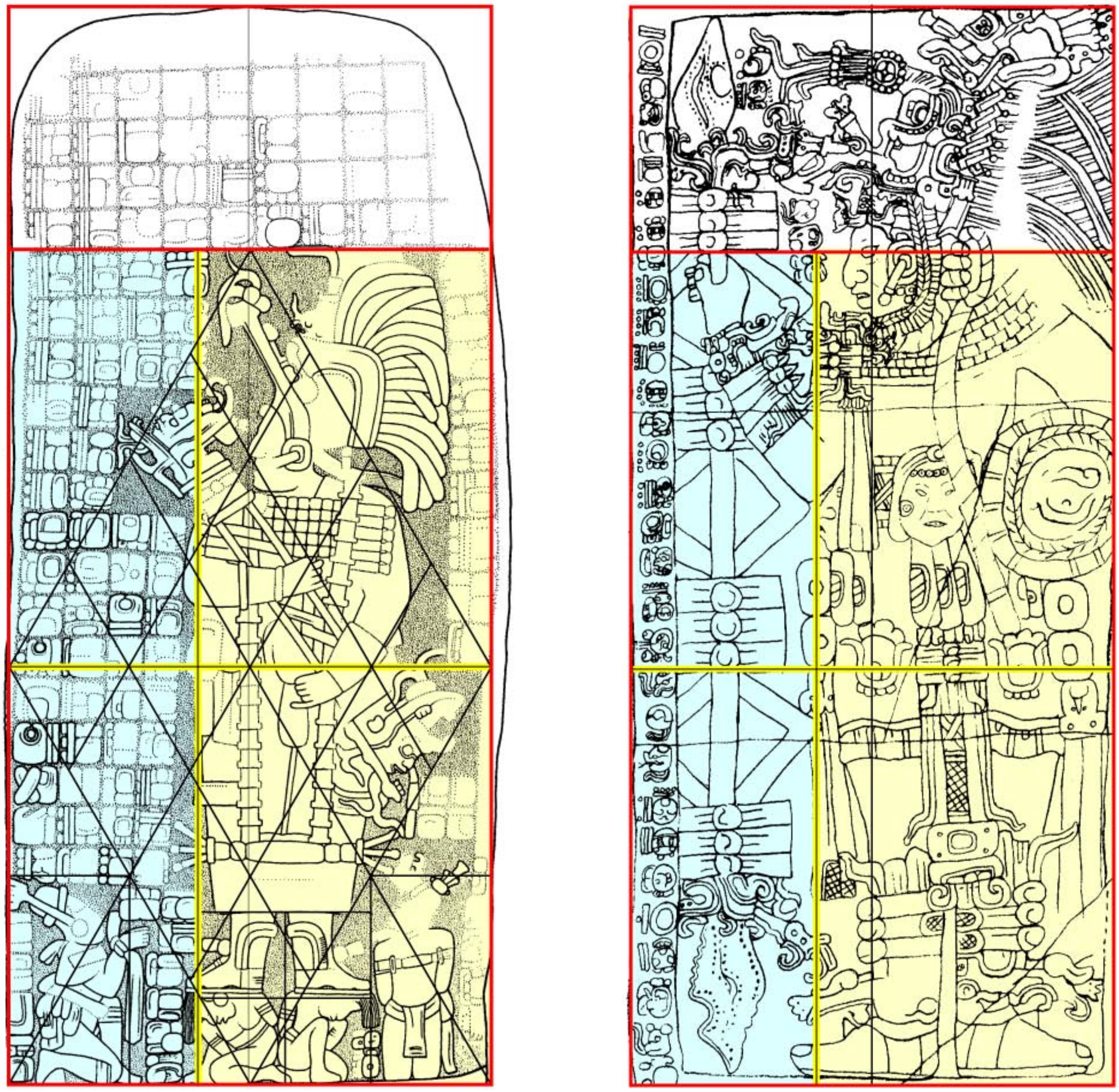


a b

Both the East (b) and West (a) Jambs are inscribed by virtually perfect root five rectangles that are subdivided at the brows of the standing figures by root three rectangles (shaded blue) and root four rectangles (in white), as per Formula Square Root of Five-4.

Figure 219

Stela 1, Coba, Quintana Roo, Mexico (a) and an Unprovenienced Stele (b)
Measured drawings by Ian Grahm (a) and Christian Prager (b)



a b

Both of these stelae are inscribed by virtually perfect root five rectangles that are subdivided by root four rectangles (white) and root three rectangles (shaded yellow and blue), as per Formula Square Root of Five-4. The root two rectangles are further subdivided by root two rectangles (shaded yellow) and root five rectangles (shaded blue), as per Formula Square Root of Three-4.

Figure 220

Jamb 1, Structure 1A1, Kaba, Yucatan, Mexico
Rubbings by Merle Greene Robertson

a b

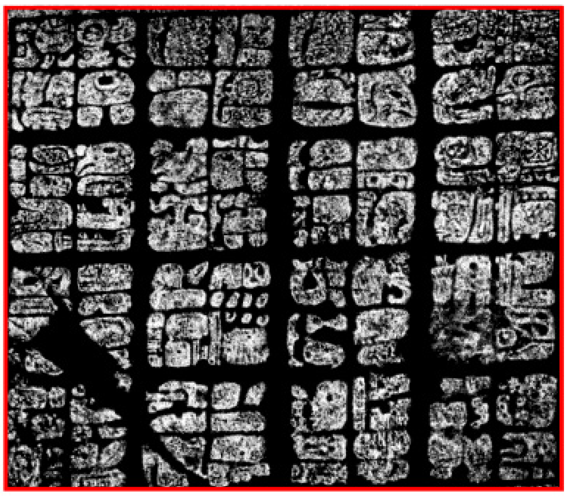
Both Jamb 1 (a) from Kaba and an unprovenienced stela (b) are inscribed by virtually perfect root five rectangles that are subdivided by root four rectangles (white) and root three rectangles (shaded yellow and blue), as per Formula Square Root of Five-4. The root two rectangles are further subdivided by root two rectangles (shaded yellow) and root five rectangles (shaded blue), as per Formula Square Root of Three-4.

Figure 221

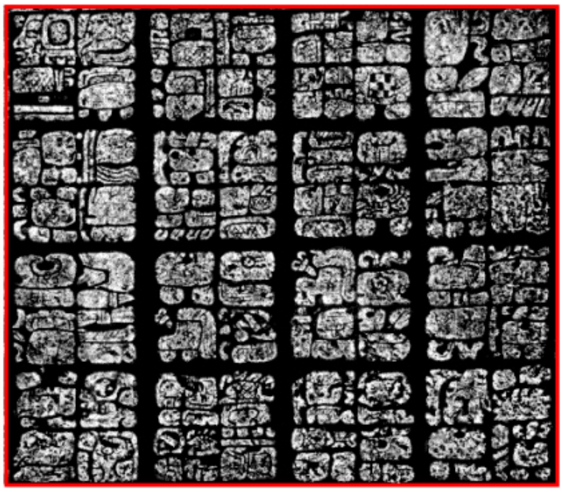
Two Over Square Root of Three Hieroglyphic Text Blocks 1
Rubbings by Merle Greene Robertson

Lintel 1, Temple of the Four Monuments, Chichen Itza

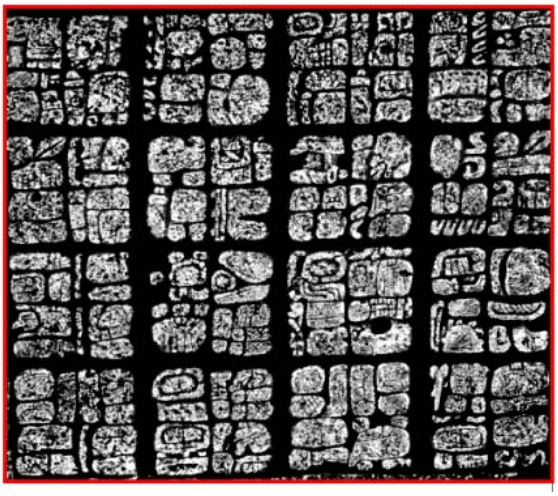
Lintel 2, Temple of the Four Monuments, Chichen Itza



Lintel 3, Temple of the Four Monuments, Chichen Itza



Lintel 4, Temple of the Four Monuments, Chichen Itza

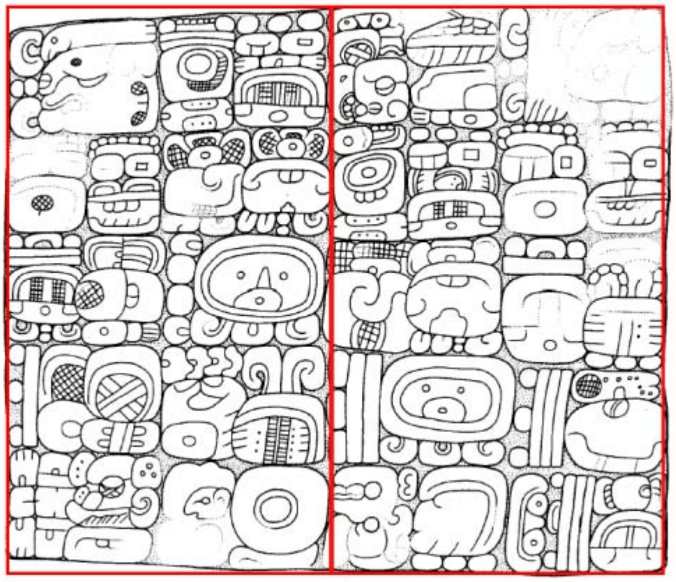


Hieroglyphic text blocks inscribed by two over square root of three rectangles

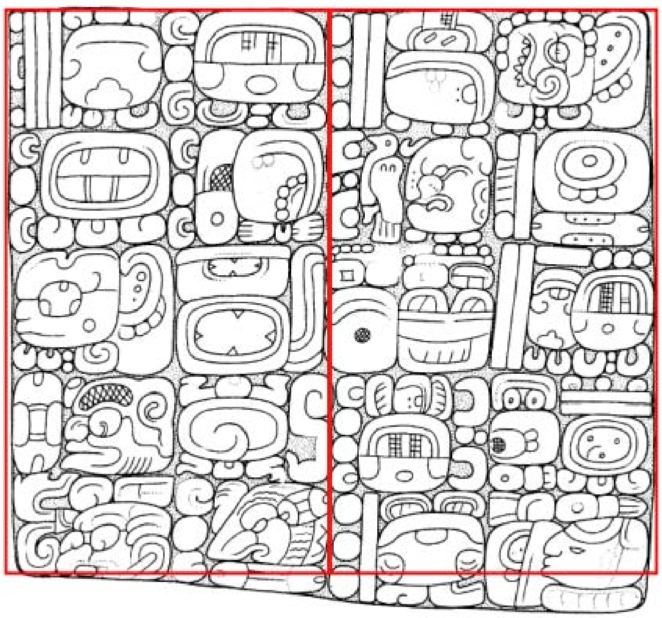
Figure 222

Two Over Square Root of Three Hieroglyphic Text Blocks 2
Measured Drawings by Ian Grahm

Lintel 30, Yaxchilan



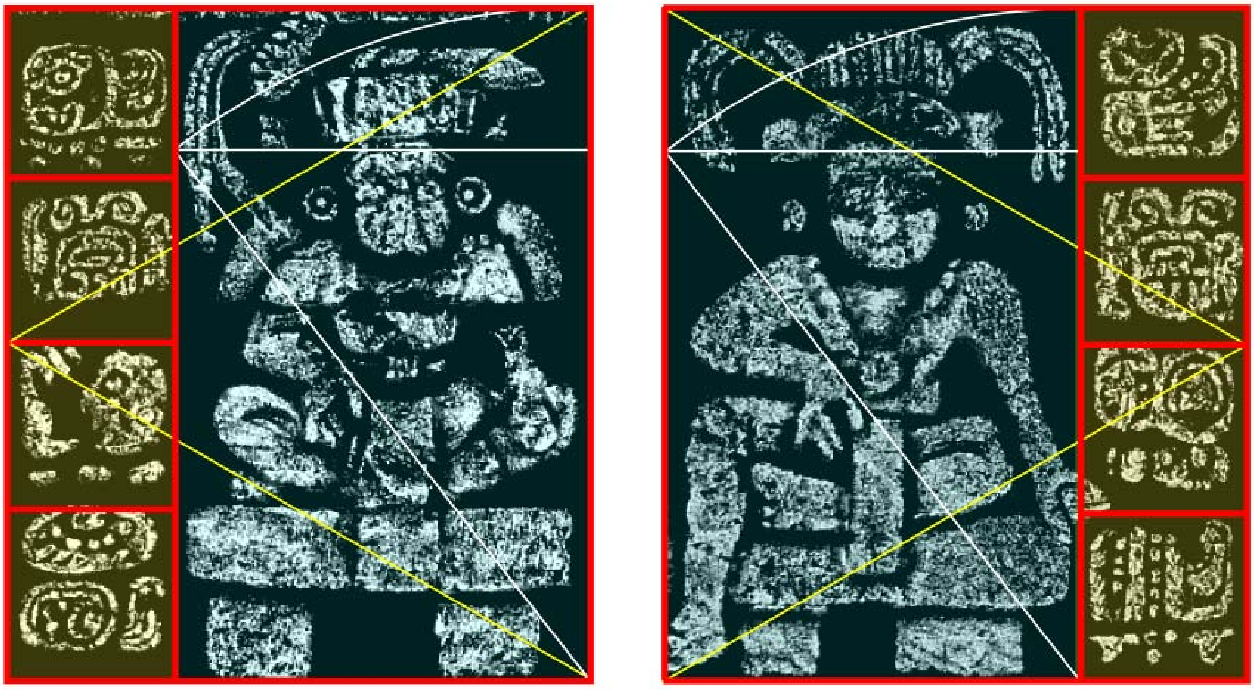
Lintel 31, Yaxchilan



Hieroglyphic text blocks inscribed by two over square root of three rectangles

Figure 223

Panels T112 East and West, Xcalumkin,
Yucatan, Mexico
Rubbings by Merle Greene Robertson

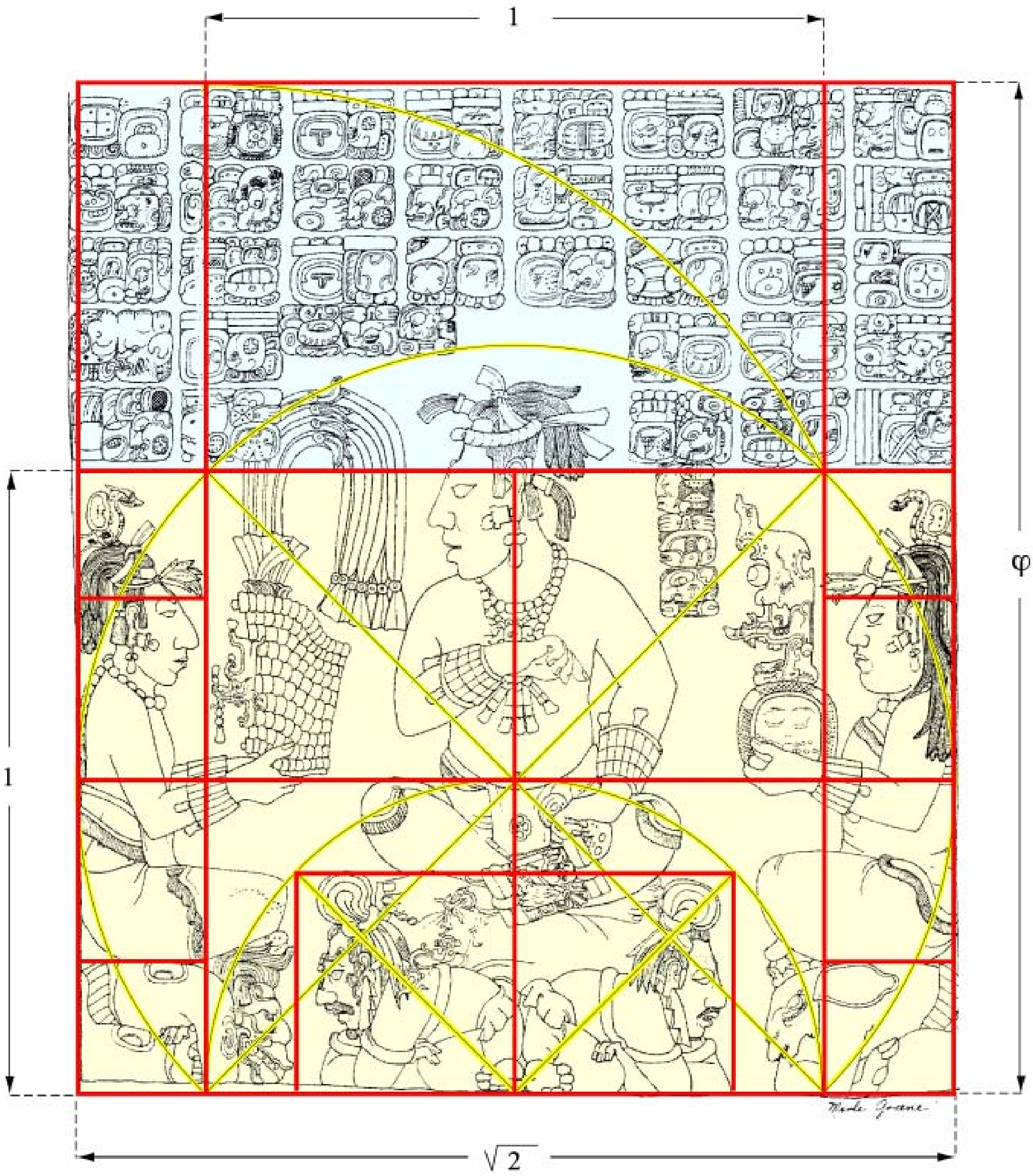


a b

The East Panel (a) and the West Panel (b) from Xcalumkin are inscribed by two over root three rectangles that inscribe equilateral triangles (yellow lines) and are subdivided by phi rectangles (shaded blue) and four squares (shaded yellow), as per Formula Two Over Square Root of Three-4. The phi rectangles that inscribe the seated figures are subdivided by a root phi rectangle (white lines) at the brows, whose diagonals are the radii for the arcs that determine the length of the phi rectangles.

Figure 224

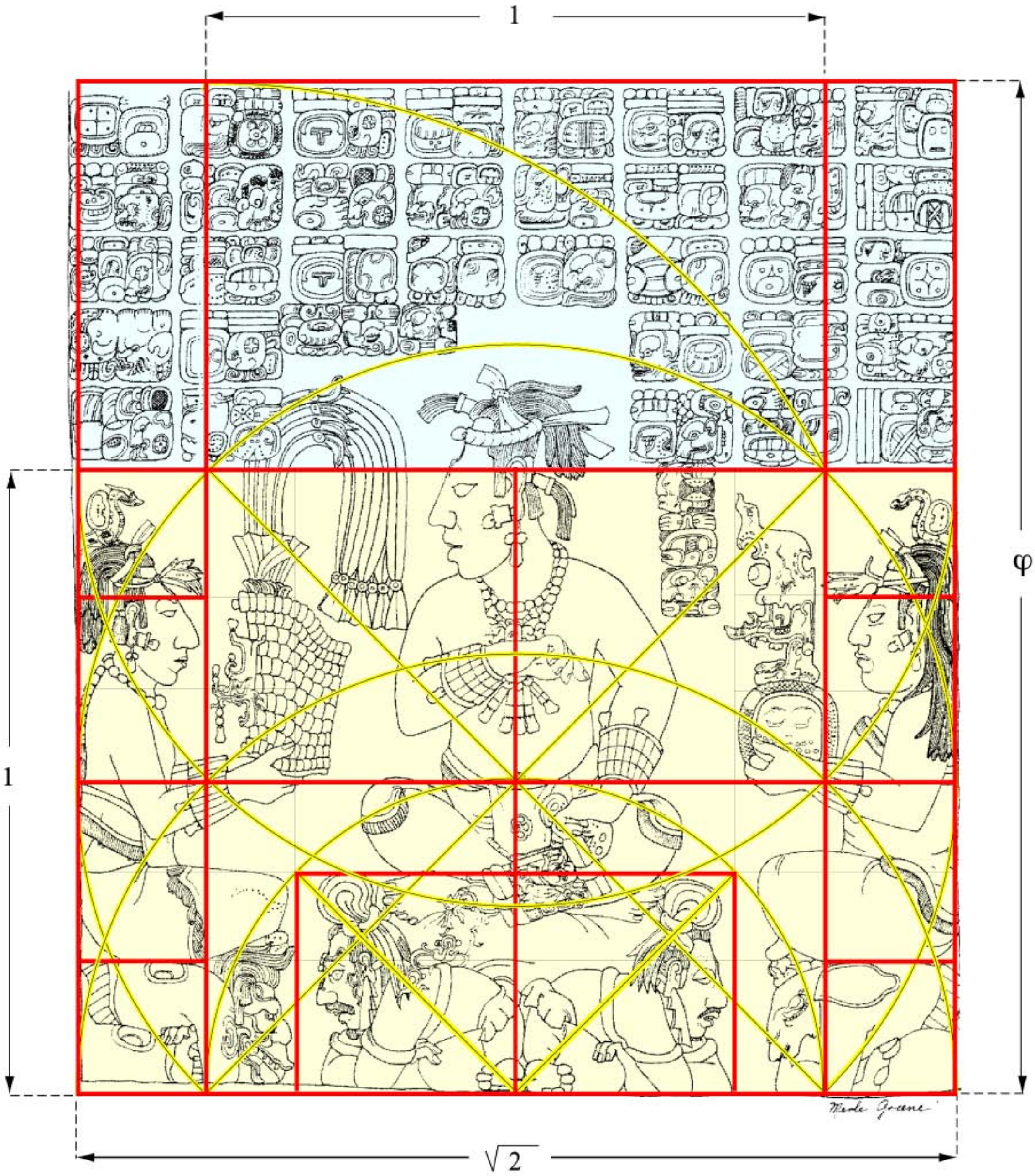
Tablet of the Slaves (a), Palenque, Chiapas, Mexico
Measured Drawing by Merle Greene Robertson



The Tablet of the Slaves is inscribed by a virtually perfect two over root three rectangle that is subdivided at the base of the text block and the brow of the central seated figure by a root five rectangle (shaded blue) and a root two rectangle (shaded yellow), as per Formula Two Over Square Root of Three-5. The root two rectangle is further subdivided by squares and smaller root two rectangles via a variation of Formula Square Root of Two-3.

Figure 225

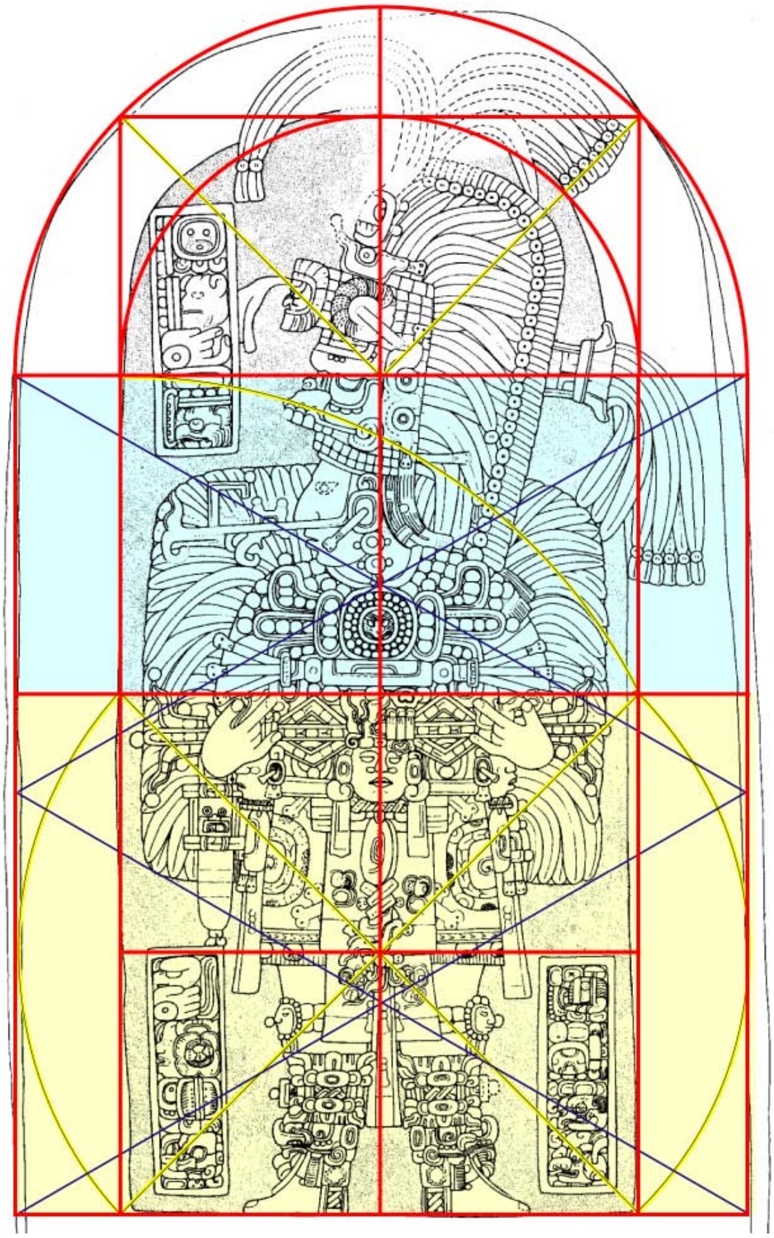
Tablet of the Slaves (b), Palenque, Chiapas, Mexico
Measured Drawing by Merle Greene Robertson



In the Tablet of the Slaves b, the root two rectangle (shaded yellow) is further subdivided by additional square and root two subdivisions (arcs and fine black lines).

Figure 226

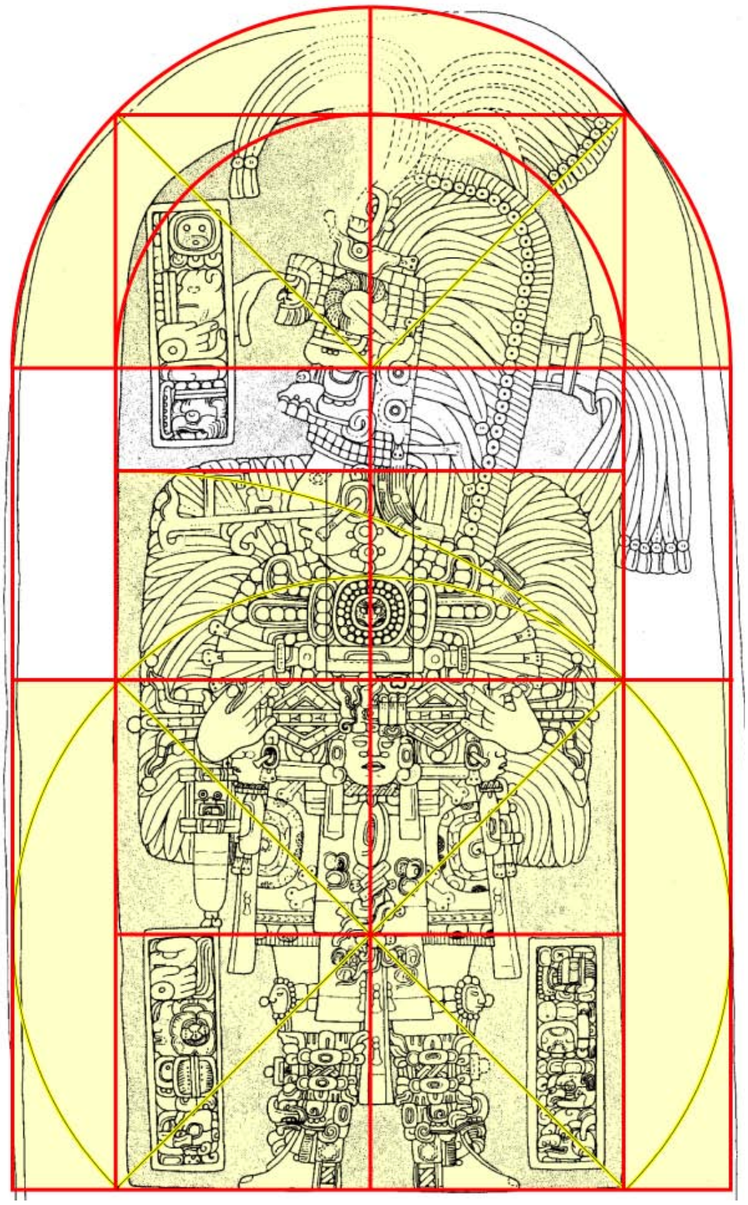
Stela 16 (a), Tikal, Peten, Guatemala
Measured Drawing by W. R. Coe



Like the Tablet of the Slaves, Tikal's Stela 16, minus the rounded half circle at the top, is inscribed by a virtually perfect two over root three rectangle that inscribes equilateral triangles (blue lines) and is subdivided by a root five rectangle (shaded blue) and a root two rectangle (shaded yellow). The diagrams on the following pages demonstrate how all of the noted Maya proportions are concisely incorporated into the layout of this stela.

Figure 227

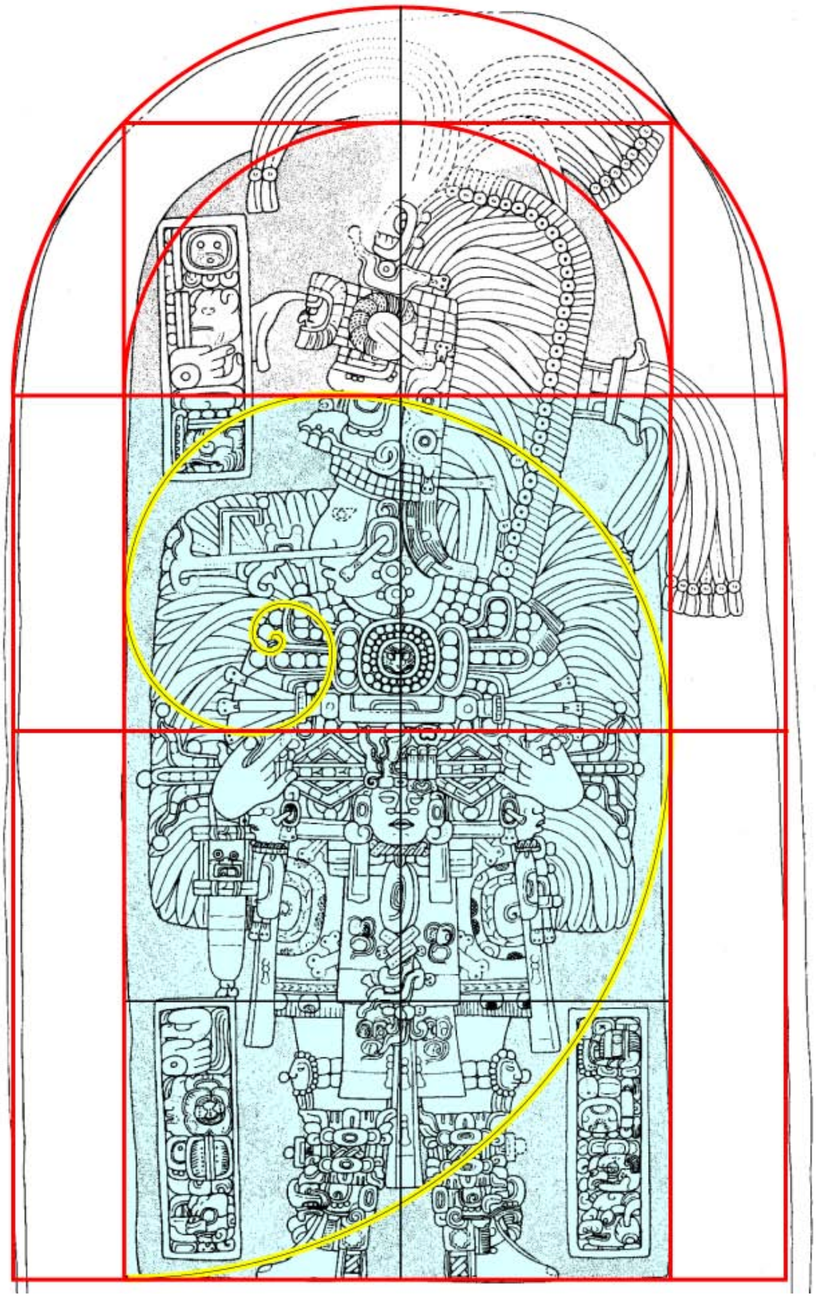
Stela 16 (b), Tikal, Peten, Guatemala
Measured Drawing by W. R. Coe



All of the yellow highlighted areas represent root two proportions. The interior carved portion of the stela (minus the border), from the fingertips to the bottom of the feet, is inscribed by a quartered square. From the center of this square to its corners are the radii for the arc that defines the width of the border area and produces a horizontal root two rectangle. An equal sized vertical root two rectangle terminates at the brow of the standing figure and is derived by swinging an arc from the diagonal of the quartered square. The half circles at the top of the stela also display root two proportions. The diagonals (highlighted yellow lines) of double squares that inscribe the inner half circle are the radii used to create the outer half circle.

Figure 228

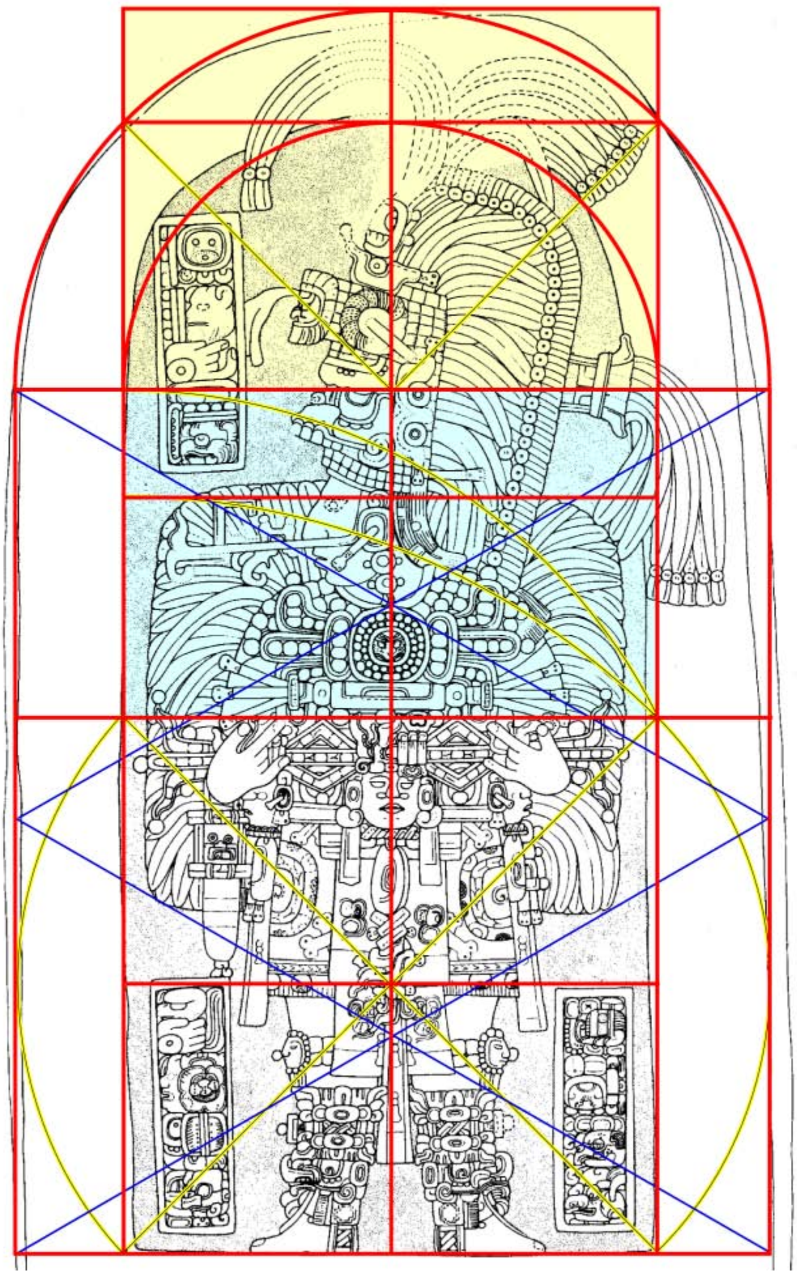
Stela 16 (c), Tikal, Peten, Guatemala
Measured Drawing by W. R. Coe



The same quartered square used to produce the root two rectangles in the diagram above may be used to create the phi rectangle shown here (shaded blue).
Note that the phi rectangle terminates at the brow of the monster headdress worn by the standing figure.

Figure 229

Stela 16 (d), Tikal, Peten, Guatemala
Measured Drawing by W. R. Coe



Here, all of the suggested geometry of Stela 16 from Tikal are shown, and the virtually perfect formula for a Pythagorean 3,4,5 rectangle is highlighted. The phi rectangle (shaded blue) and the root two rectangle (shaded yellow) together inscribe a virtually perfect Pythagorean 3,4,5 rectangle, as per Formula Pythagorean 3,4,5-2

Figure 230

Phi Hieroglyphic Text Blocks 1

Lintel 12, Yaxchilan

Lintel 21,Yaxchilan

Lintel 37,Yaxchilan

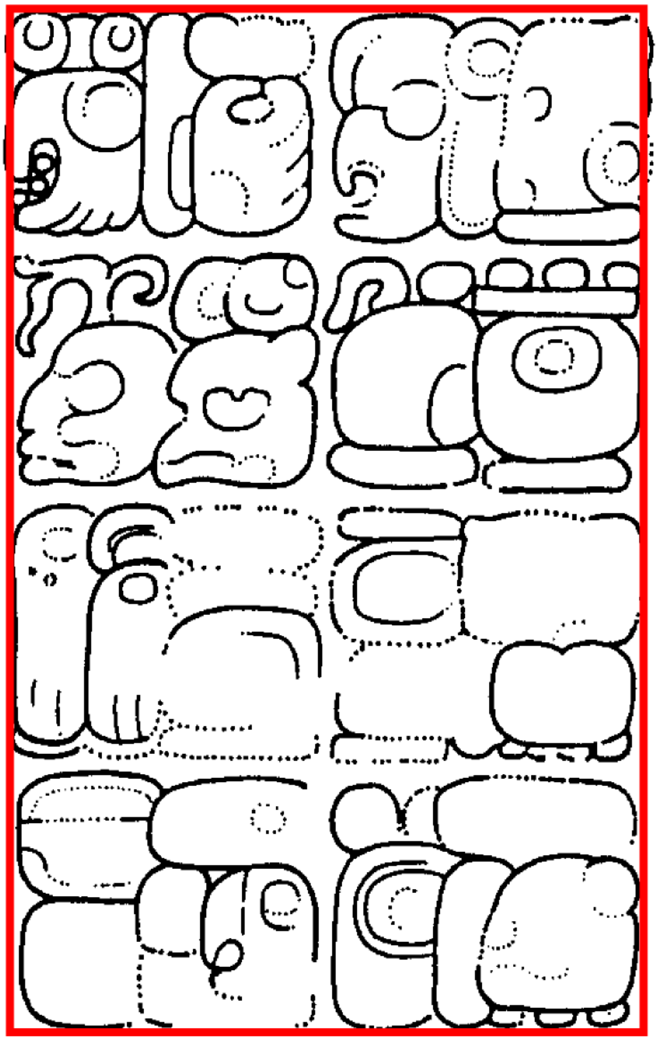
Initial Series Tablet, Chichen Itza

Hieroglyphic text blocks inscribed by phi rectangles

Figure 231

Phi Hieroglyphic Text Blocks 2

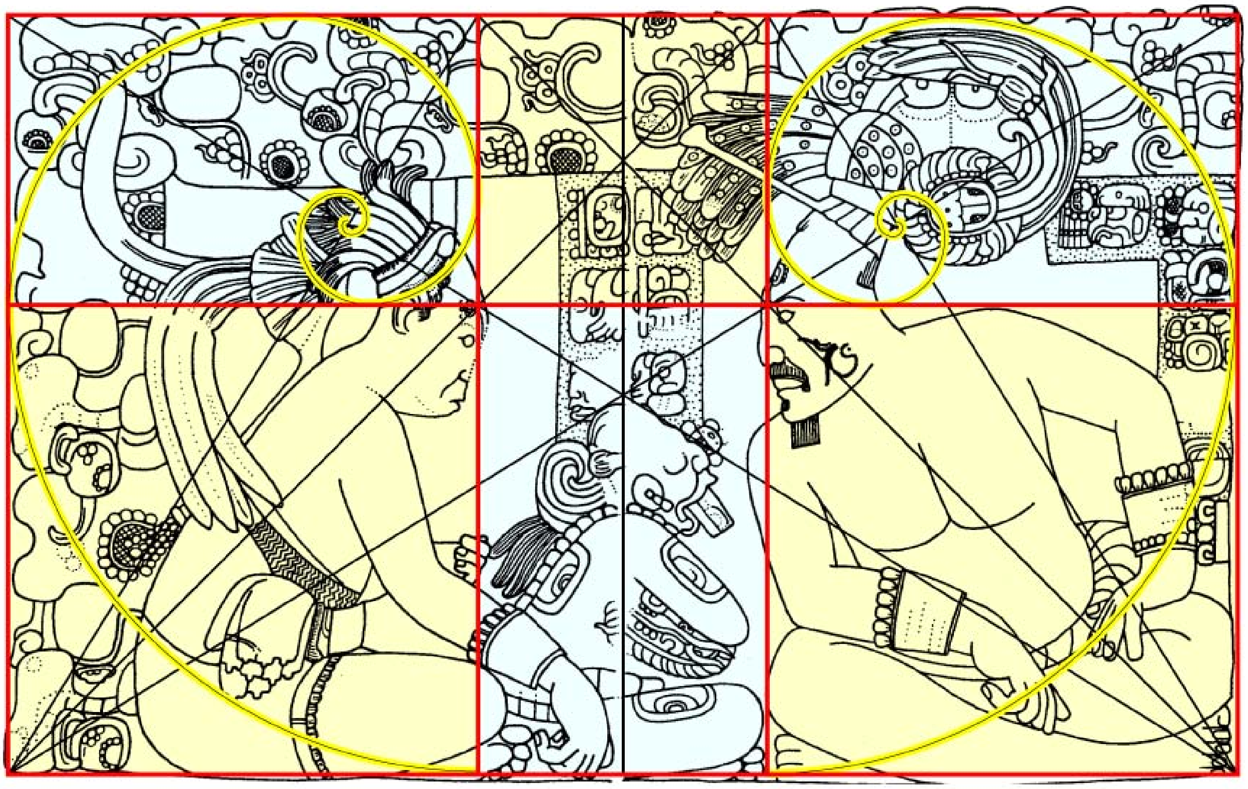
Stela 14, Nimli Punit



Hieroglyphic text block inscribed by a phi rectangle

Figure 232

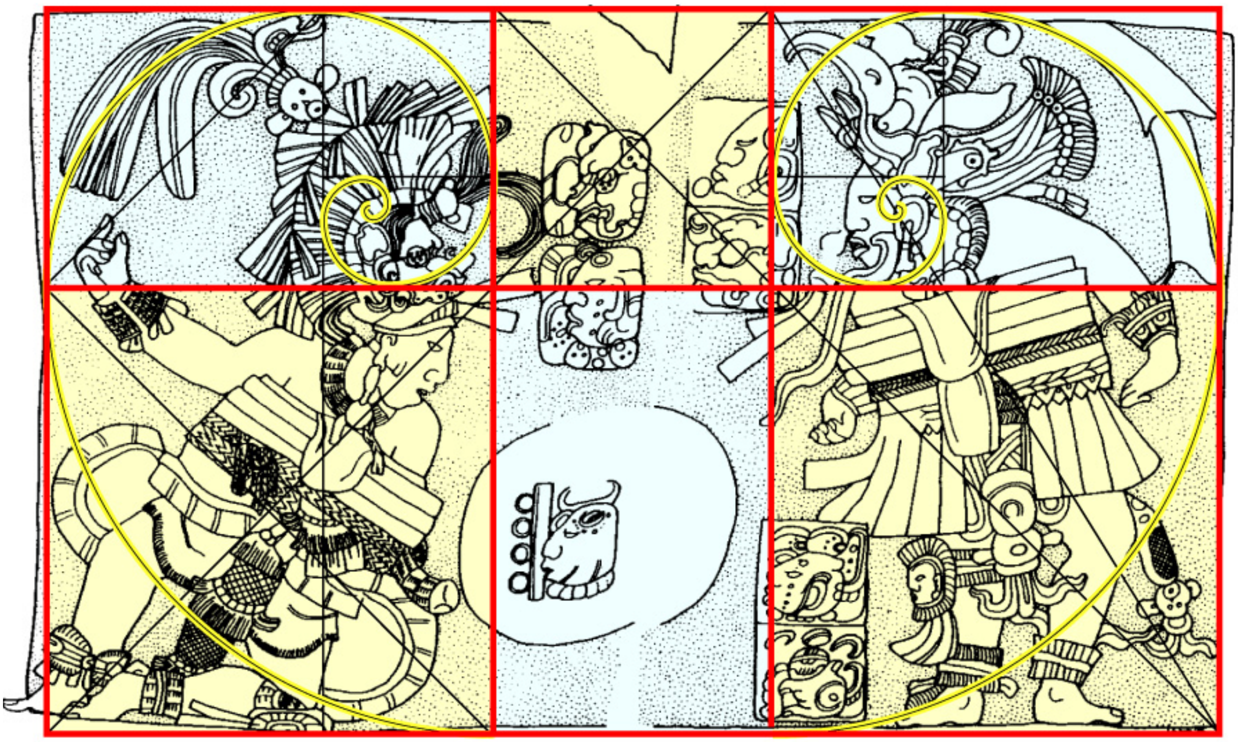
Relief Panel (Unknown Provenience)
Museo Amparo, Puebla, Mexico
Measured Drawing by Christian Prager



This relief panel is inscribed by a phi rectangle that is subdivided by squares (shaded yellow) and smaller phi rectangles (shaded blue), as per Formula Phi-2. Equiangular spirals are added to the lateral phi rectangles.

Figure 233

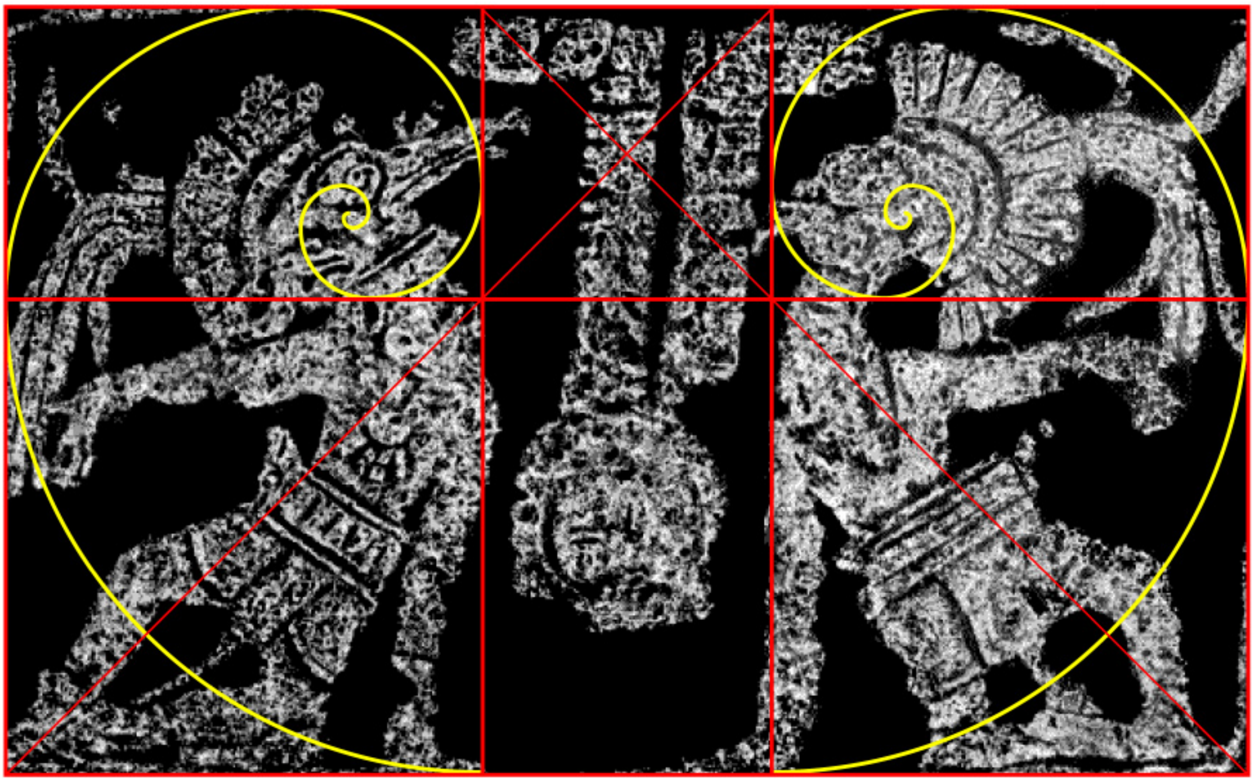
Ball Player Panel (a), (Unknown Provenience)
Measured Drawing by Merle Greene Robertson



This relief panel is inscribed by a phi rectangle that is subdivided by squares (shaded yellow) and smaller phi rectangles (shaded blue), as per Formula Phi-2. Equiangular spirals are added to the lateral phi rectangles.

Figure 234

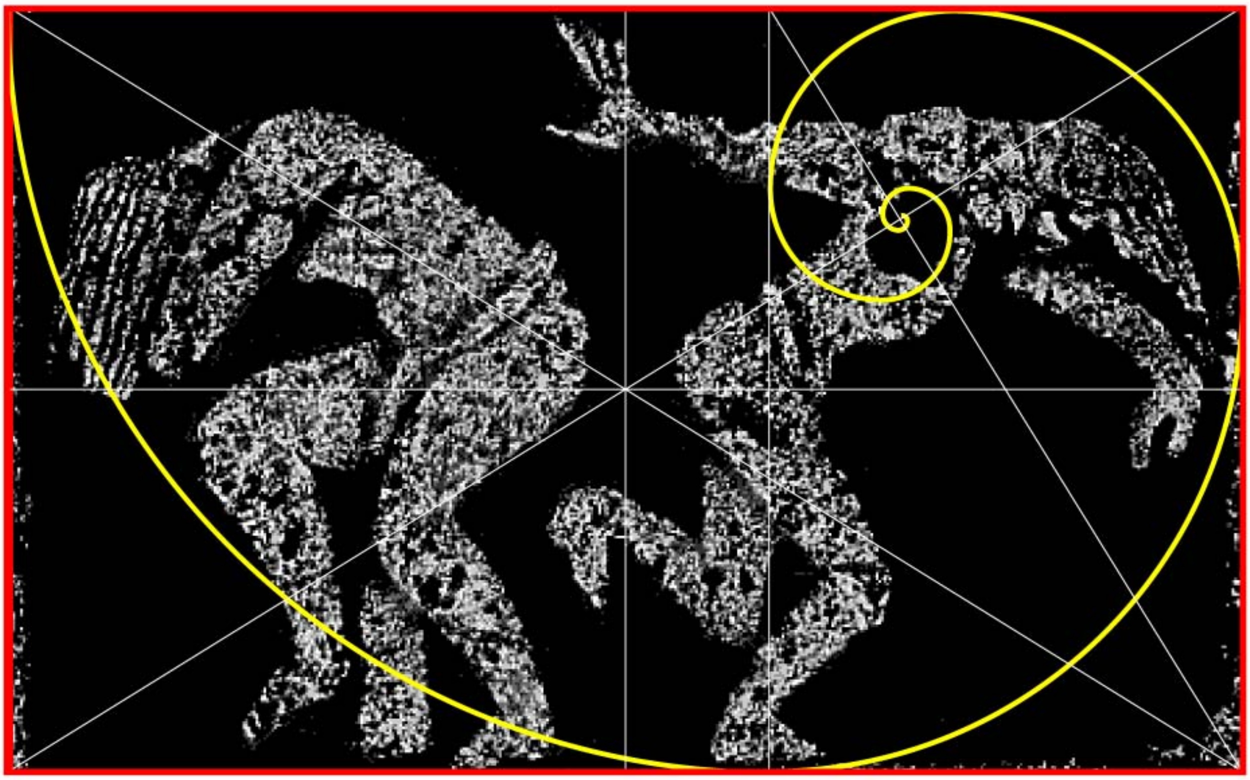
Ball Player Panel (b), (Provenience Unknown)
Rubbing by Merle Greene Robertson



This rubbing of a relief panel is inscribed by a phi rectangle that is subdivided by squares and smaller phi rectangles, as per Formula Phi-2. Equiangular spirals are added to the lateral phi rectangles.

Figure 235

Two Dancers, Yaxchilan area, Chiapas, Mexico
Rubbing by Merle Greene Robertson



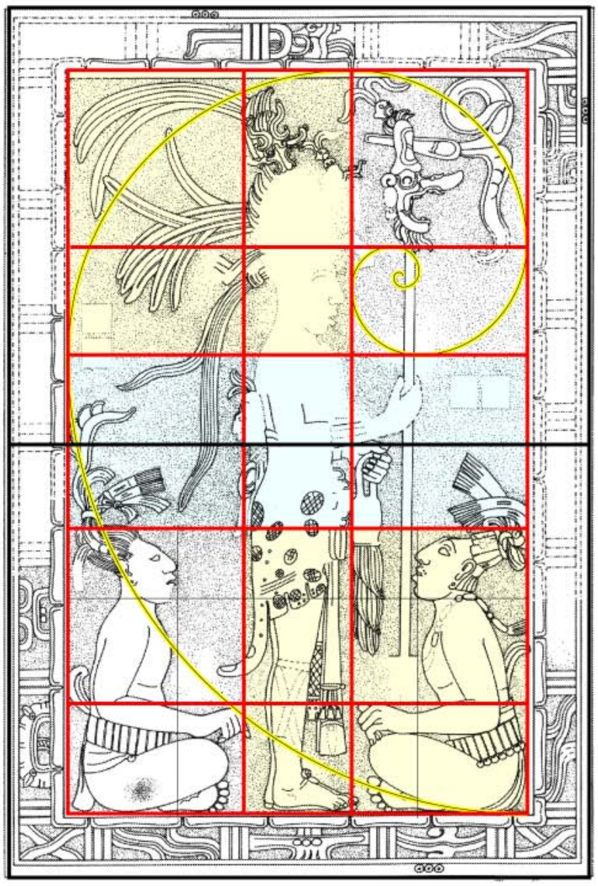
This rubbing of a relief panel is inscribed by a phi rectangle with an inscribed equiangular spiral.

Figure 236

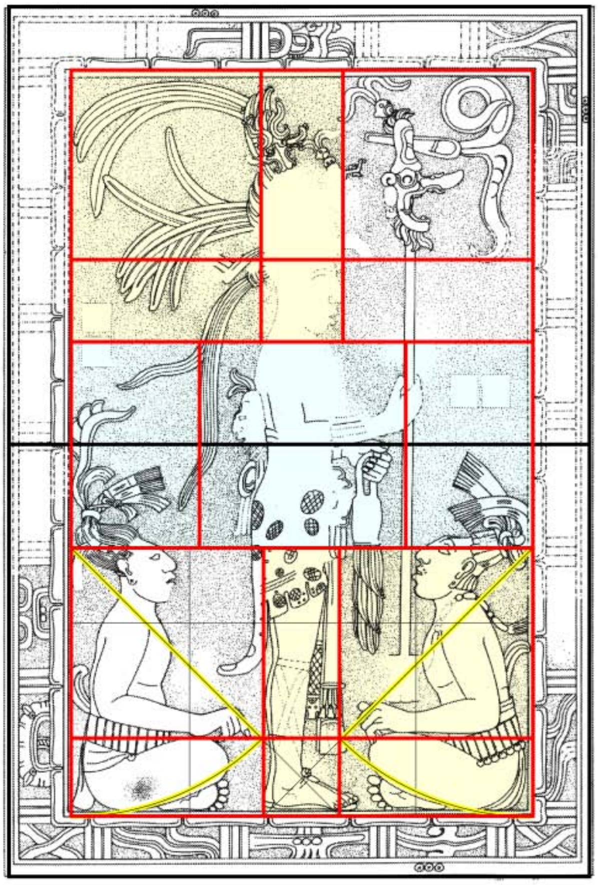
House A, Pier B, Palenque, Chiapas, Mexico

Measured Drawing by Merle Greene Robertson

a



b



The exteriors of the sky bands of both diagrams are inscribed by rectangles with a ratio of two to three that are divided in half to form two Pythagorean 3,4,5 rectangles (bold black lines).

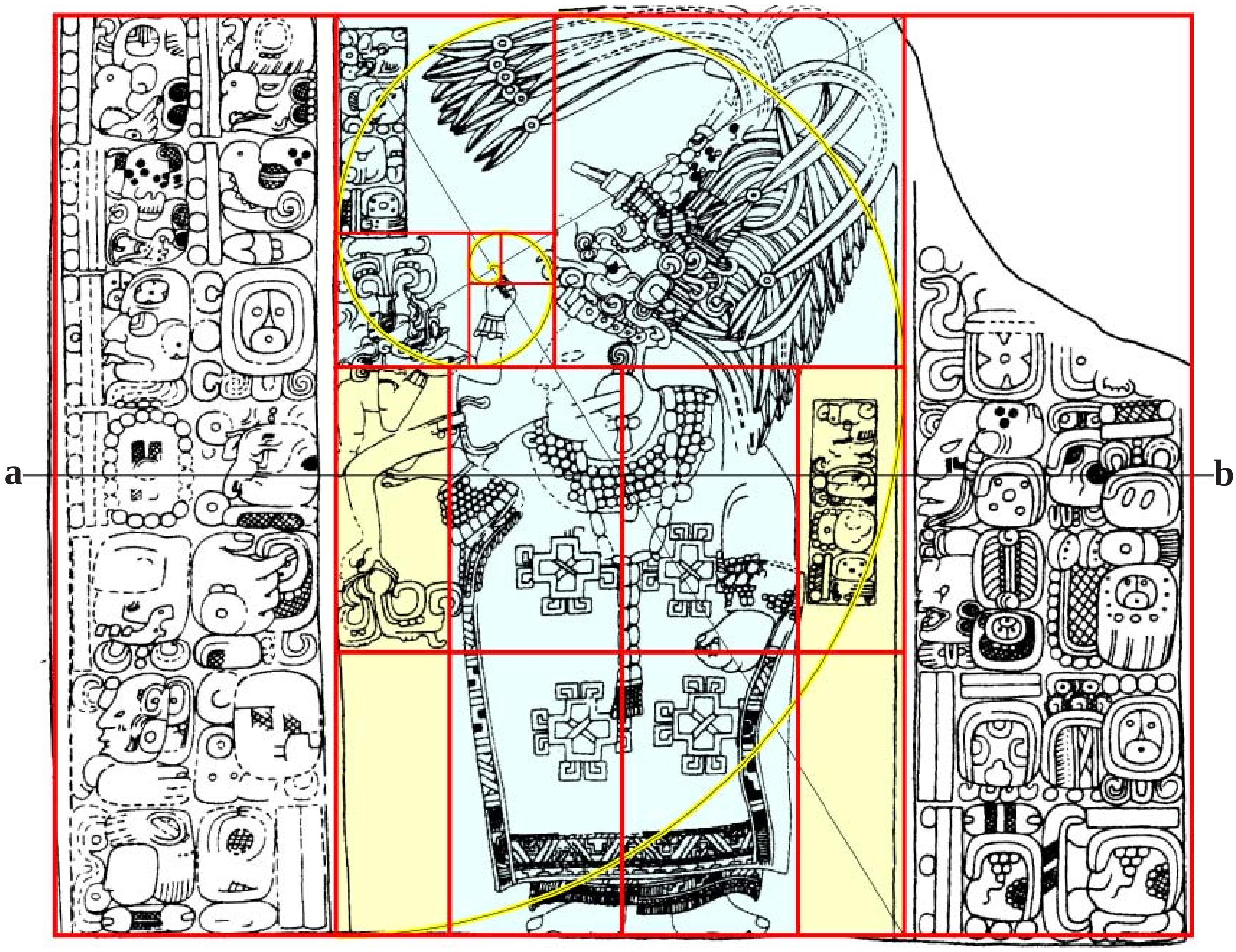
In diagram a, the standing and seated figures are inscribed by a phi rectangle that is subdivided by a variation of Formula Phi-2. Two squares are shaded yellow, two phi rectangles are in white, and a phi squared rectangle is shaded blue.

In diagram b, the standing and seated figures are inscribed by a virtually perfect phi rectangle that is subdivided by a variation of Formula Phi-3. Two squares are shaded yellow, two root two rectangles are in white, and a root five rectangle is shaded blue.

Piers C,D, and E of House A are very similar to Pier A, and can be subdivided identically.

Figure 237

Relief Panel (a), El Peru area
Museo Amparo, Puebla, Mexico



This relief panel is inscribed by a rectangle with a ratio of two to phi. The standing figure holding the Kawil god is inscribed by a phi rectangle that is the same height and one half the width of the panel. The hieroglyphic text blocks are inscribed by rectangles with a ratio of one to two times phi.

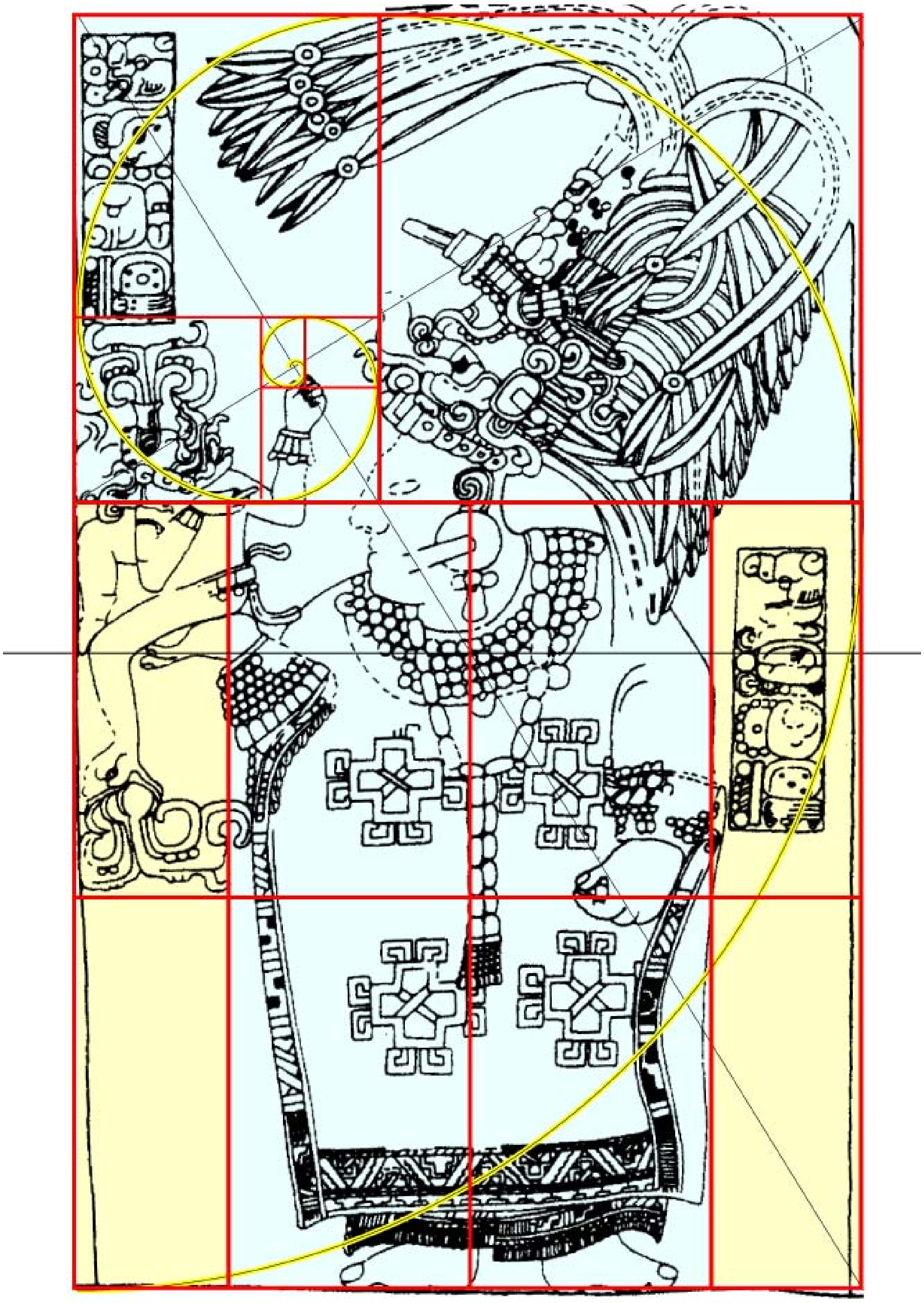
The phi rectangle that inscribes the standing figure is subdivided by horizontal and vertical phi rectangles of equal size (shaded blue) and four phi squared rectangles (shaded yellow). The vertical phi rectangle and the four phi squared rectangles form a square.

An equiangular phi spiral is inscribed within the central phi rectangle.

The thin black line (a,b) divides the panel in half horizontally.

Figure 238

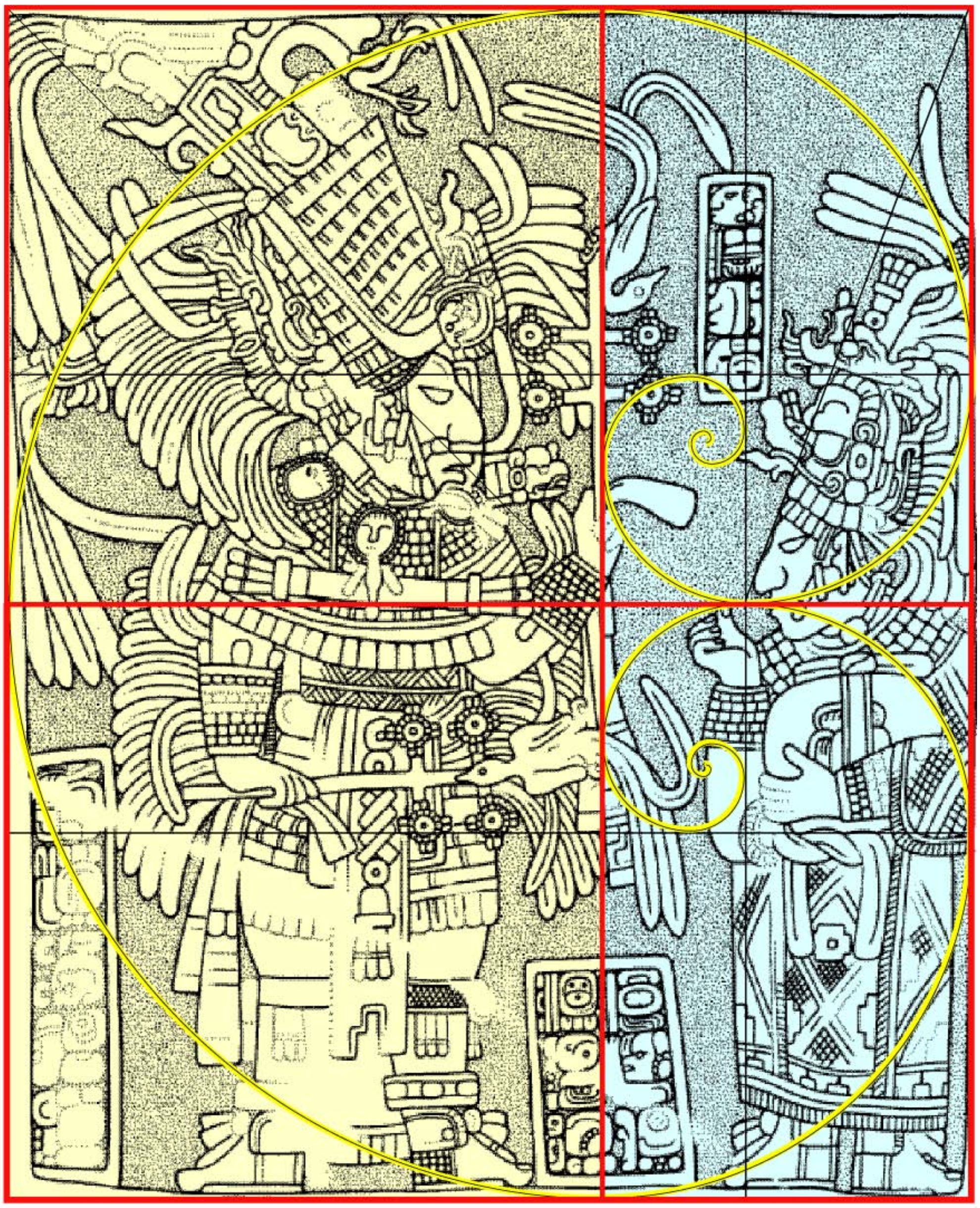
Relief Panel (b) El Peru area
Museo Amparo, Puebla, Mexico



In this close-up of the central portion of the relief panel, note the placement of the hands of the standing figure and the Kawil god. The index finger of the deity is virtually touching the vanishing point of the phi equiangular spiral.

Figure 239

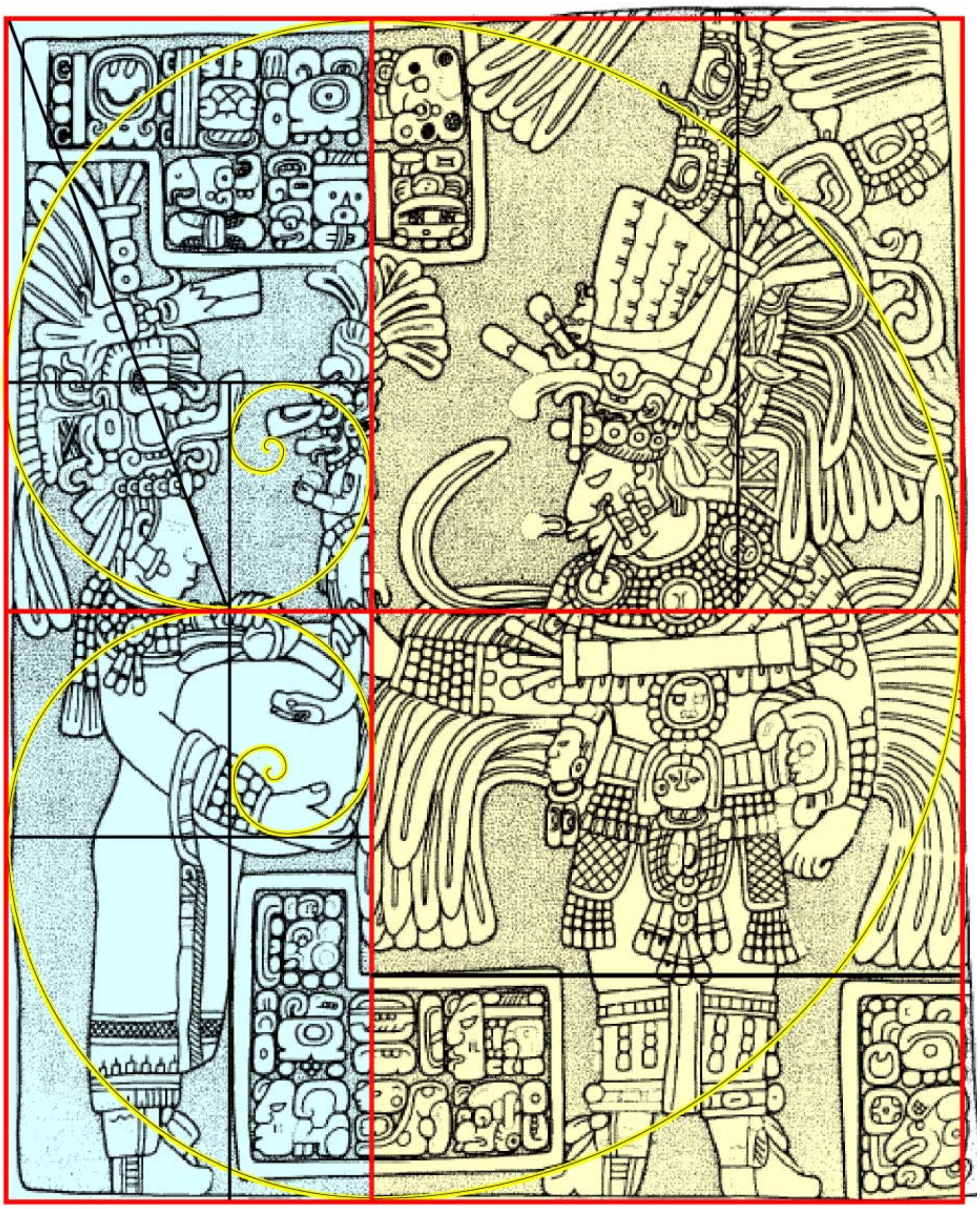
Lintel 5, Yaxchilan, Chiapas, Mexico
Measured Drawing by Ian Grahm



Lintel 5 from Yaxchilan is inscribed by a two over phi rectangle that is subdivided by two squares (shaded yellow) and two phi rectangles (shaded blue).
Note the placement of the hands relative to the principle subdivisions.

Figure 240

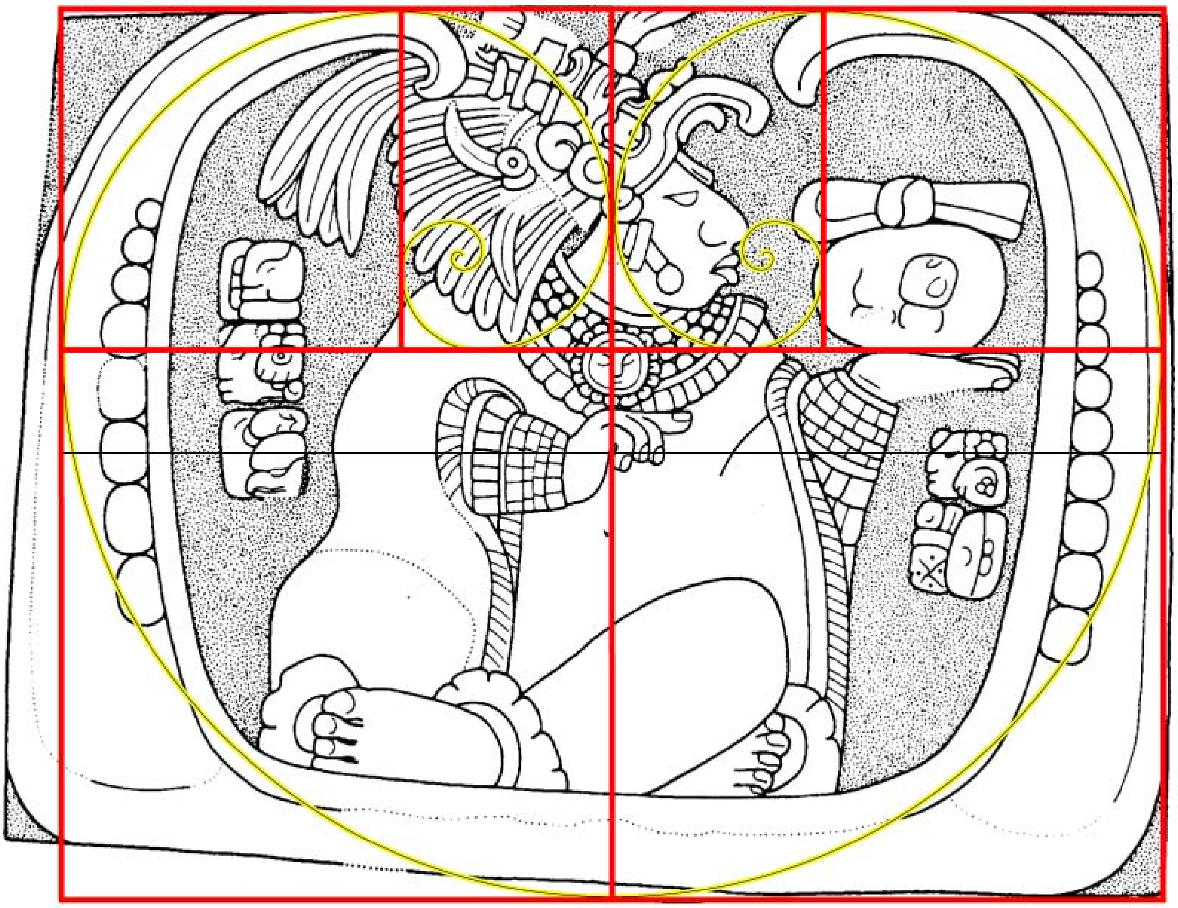
Lintel 3, Yaxchilan, Chiapas, Mexico
Measured Drawing by Ian Grahm



Lintel 5 from Yaxchilan is inscribed by a two over phi rectangle that is subdivided by two squares (shaded yellow) and two phi rectangles (shaded blue). The double square is further subdivided into two phi and phi-squared rectangles by black lines.
Note the placement of the hands relative to the principle subdivisions.

Figure 241

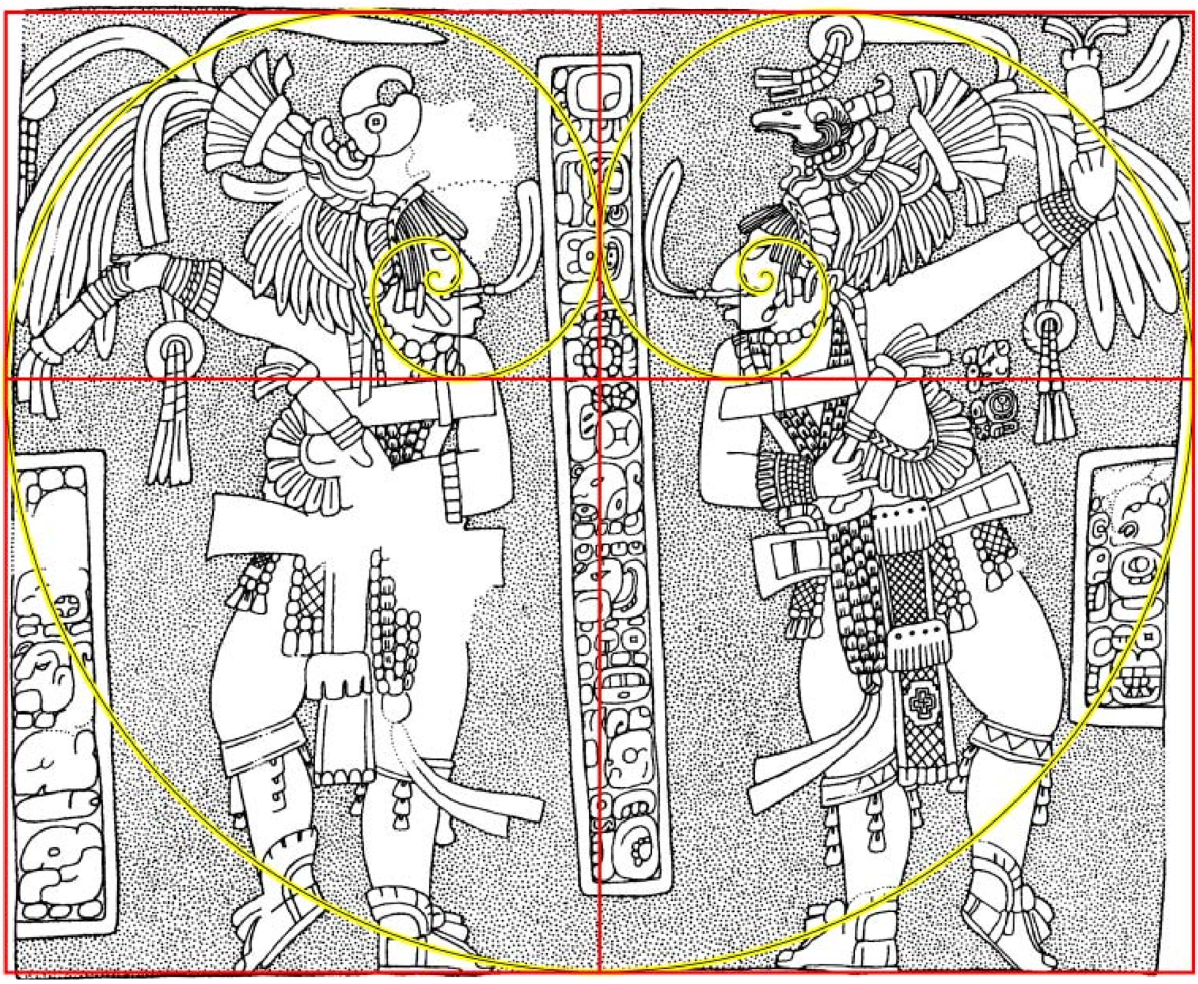
Relief Panel (Provenience Unknown)



This relief panel is inscribed by a rectangle with a length of two to a width of phi. The paired phi rectangles are subdivided by squares and phi rectangles, and equiangular spirals are inscribed within. Note the probable intentional asymmetry of the lower left-hand corner.

Figure 242

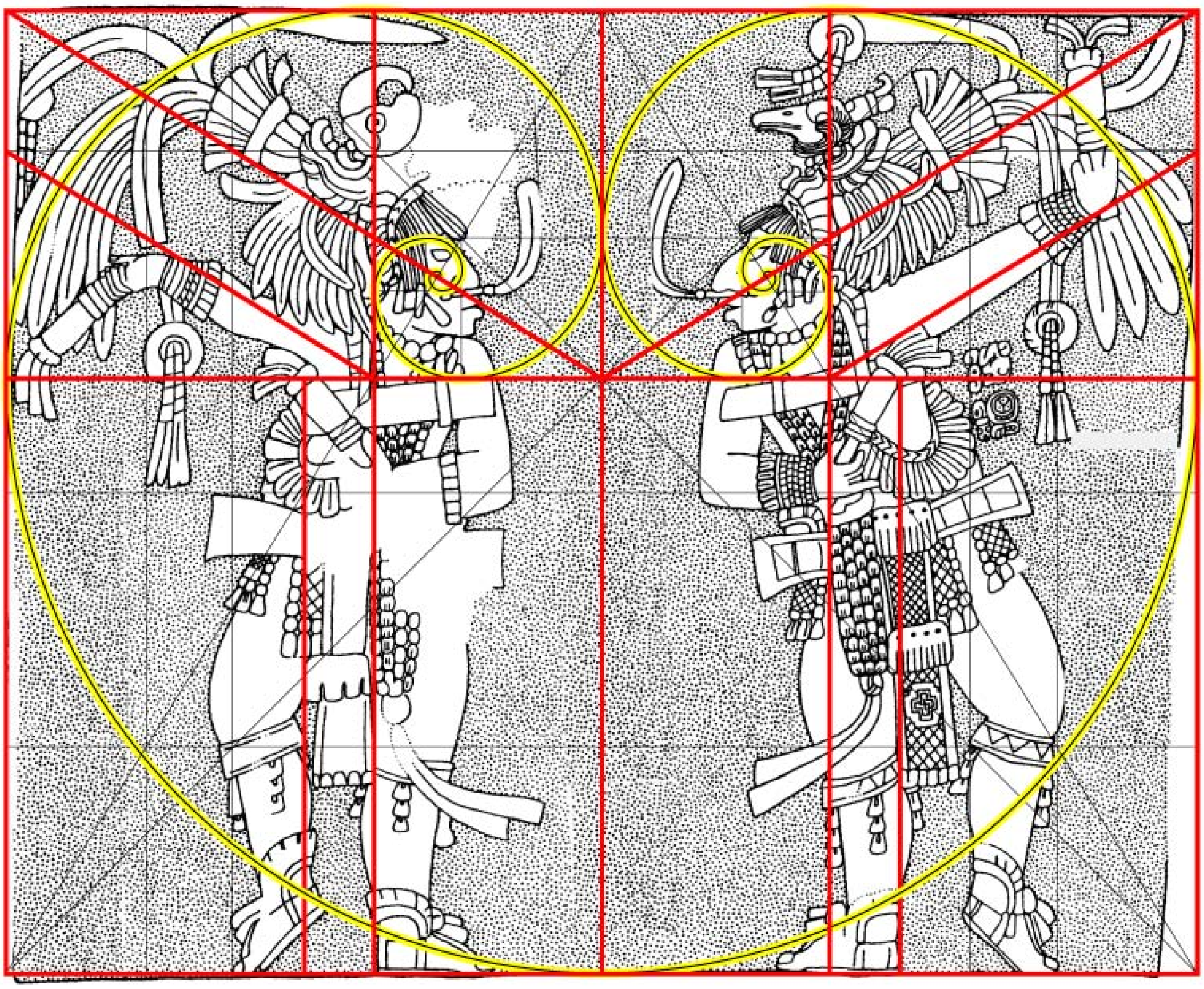
Two Dancers (a), Yaxchilan area, Chiapas, Mexico
Measured Drawing by Berthold Riese



This relief panel is inscribed by a rectangle with a length of two to a width of phi. The paired phi rectangles are subdivided by squares and phi rectangles, and equiangular spirals are inscribed within.

Note the probable intentional asymmetry of the lower right-hand corner.

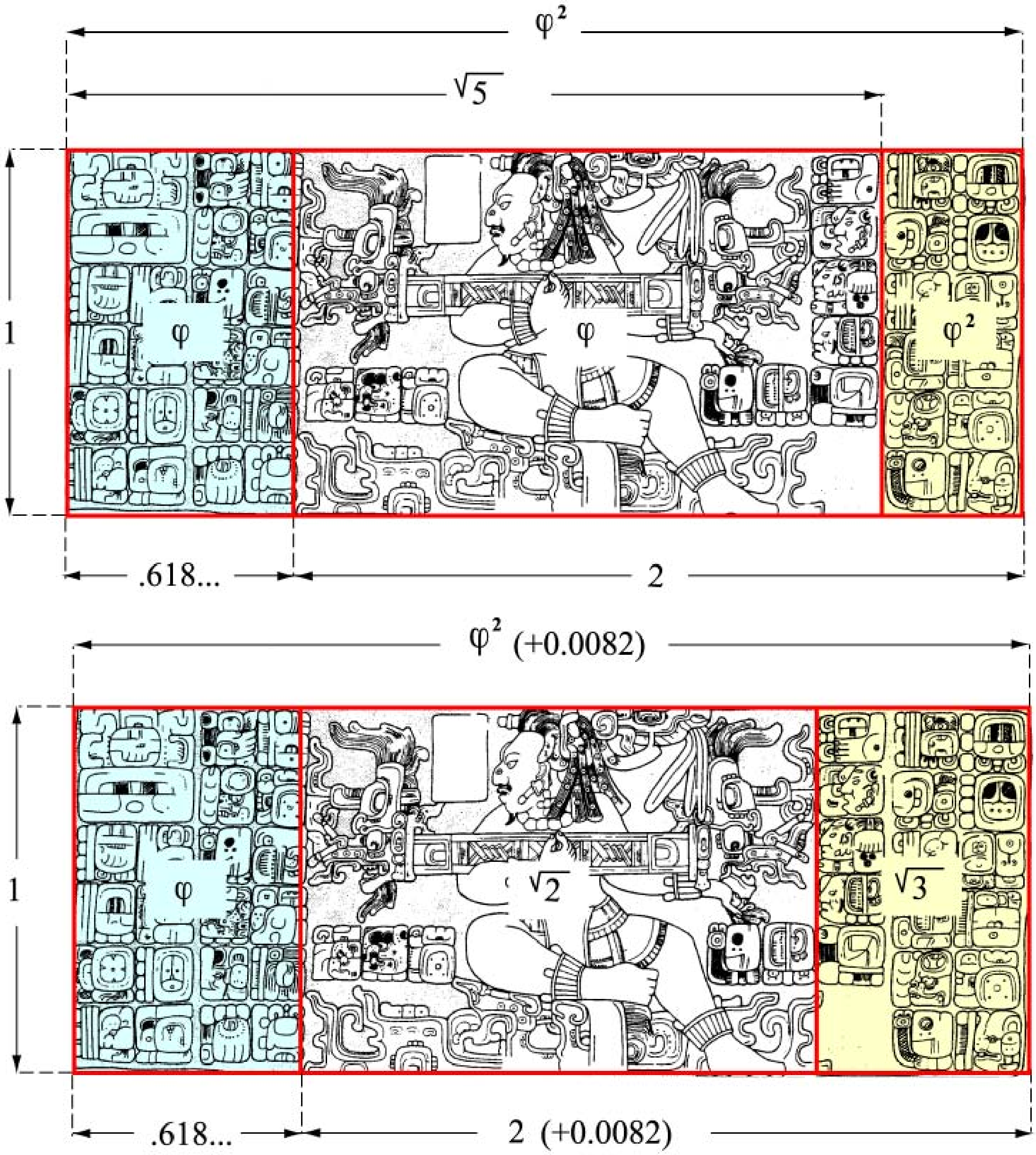
Figure 243
Two Dancers (b), Yaxchilan area, Chiapas, Mexico
Measured Drawing by Berthold Riese



Here, the hieroglyphic texts are removed from the relief panel and the phi rectangles are harmonically subdivided into squares and phi rectangles and their diagonals, in order to highlight the dynamic postures of the dancing figures.

Figure 244

Unprovenienced Lintel, Bonampak area, Chiapas, Mexico

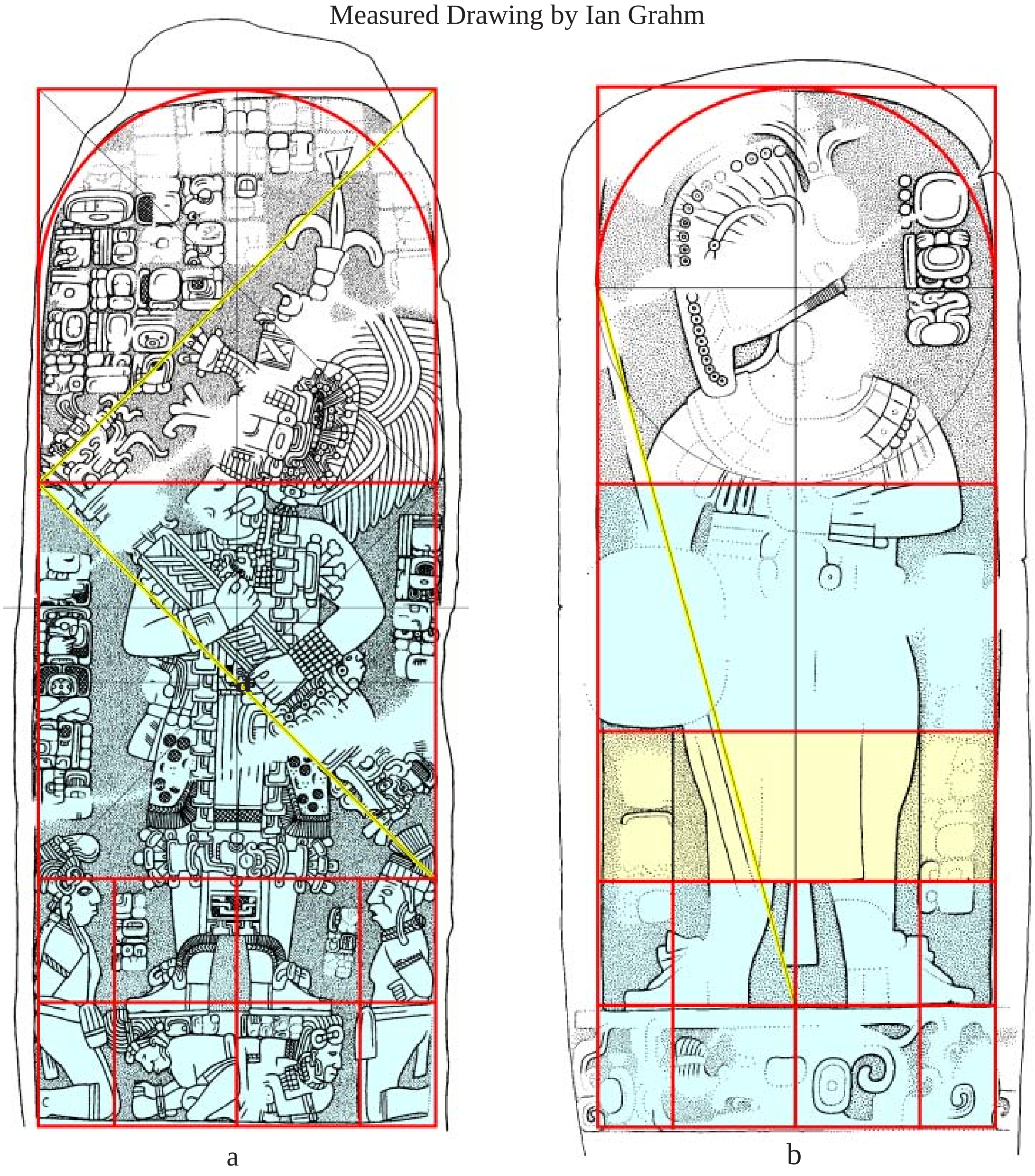


This lintel from the Bonampak area clearly indicates Maya knowledge of the perfect and virtually perfect subdivisions of a phi squared rectangle. Above, the two principal text blocks that begin with a date and a distance number are inscribed by a phi rectangle (shaded blue) and a phi-squared rectangle (shaded yellow) that flank a horizontal phi rectangle (in white) to produce a perfect subdivision of a phi-squared rectangle.

Below, the tertiary and secondary text blocks are inscribed by a root three rectangle (shaded yellow), and the iconography is inscribed by a root two rectangle (in white) to produce the virtually perfect subdivision.

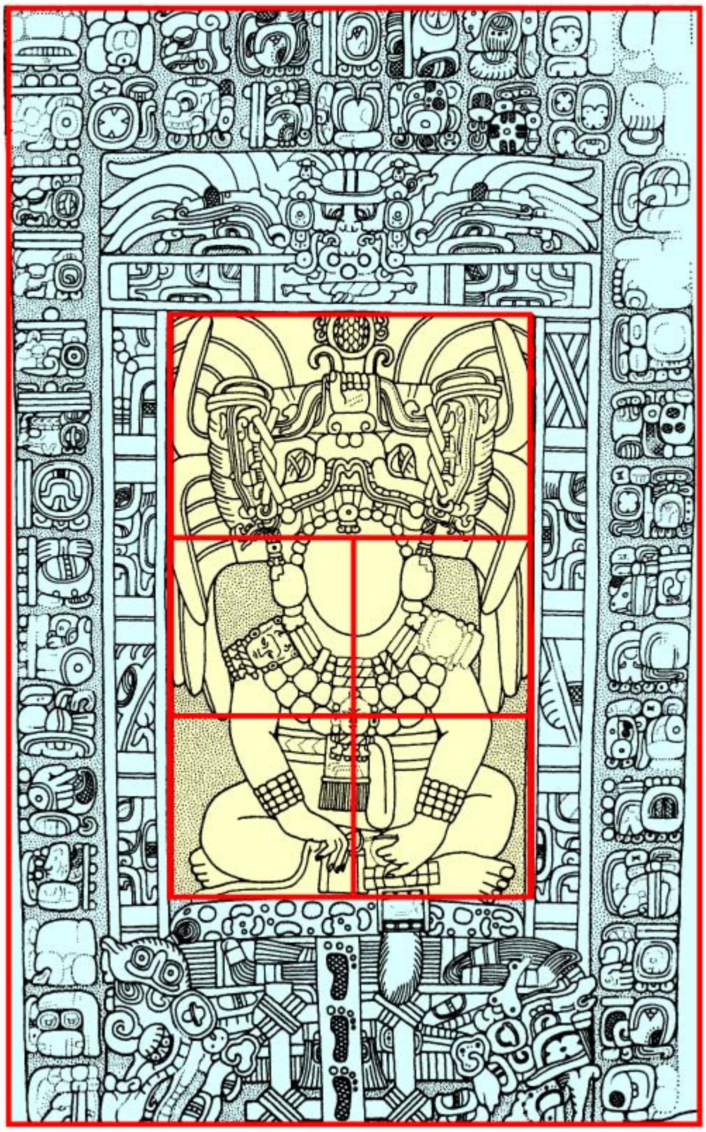
Figure 245

Stela 20, Coba, and Stela 1, Naranjo
Measured Drawing by Ian Grahm



Stela 20 from Coba (a) is inscribe by a phi-squared rectangle that is subdivided by a square with an inscribed half circle (in white) and a phi rectangle (shaded blue). The phi rectangle is further subdivided by a square (with a yellow highlighted diagonal) and a smaller phi rectangle that is subdivided by squares and yet smaller phi rectangles.
Line a,b is the horizontal center line. Note the angle of the double-headed serpent bar and the placement of the hands and the brows of the three human figures.
Stela 1 from Naranjo is identically proportioned and subdivided, except for the addition of a phi squared rectangle (shaded yellow), which, added to either phi rectangle (shaded blue), produces a square..

Figure 246
Stela 25 (a), Piedras Negras, Guatemala

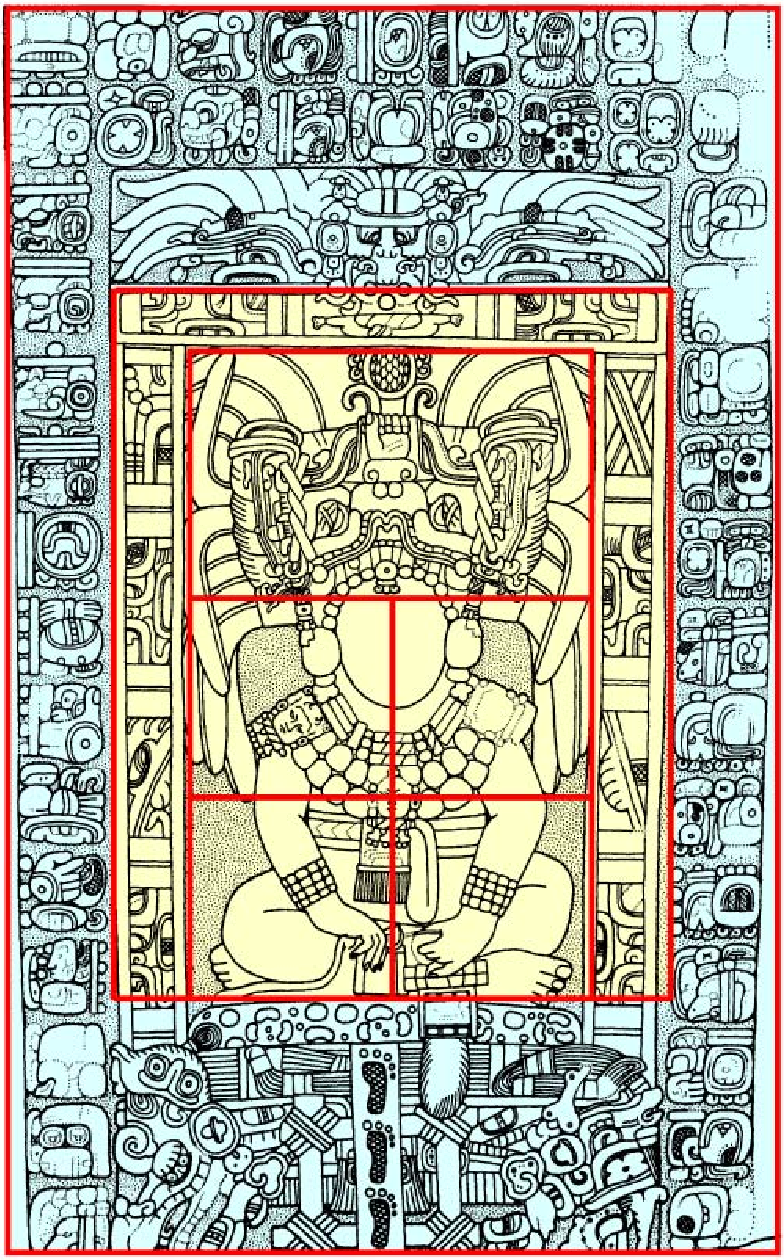


Stela 25 from Piedras Negras, the last of the carved stone monuments analyzed in this dissertation, is a marvelous example of how al of the square root, phi and Pythagorean 3,4,5 rectangles can be ingenuously incorporated into the geometrical layout of a single work of art. Each of the proportions will be highlighted in the following series of figures:

Above, the stela is inscribed by a phi rectangle (shaded blue), and the seated figure with headdress is also inscribed by a smaller phi rectangle (shaded yellow) that is subdivided at the brow by a quartered square and a smaller phi rectangle.
Note that the bottom of the stela is somewhat narrower than the top, perhaps an example of an intentional asymmetry.

Figure 247

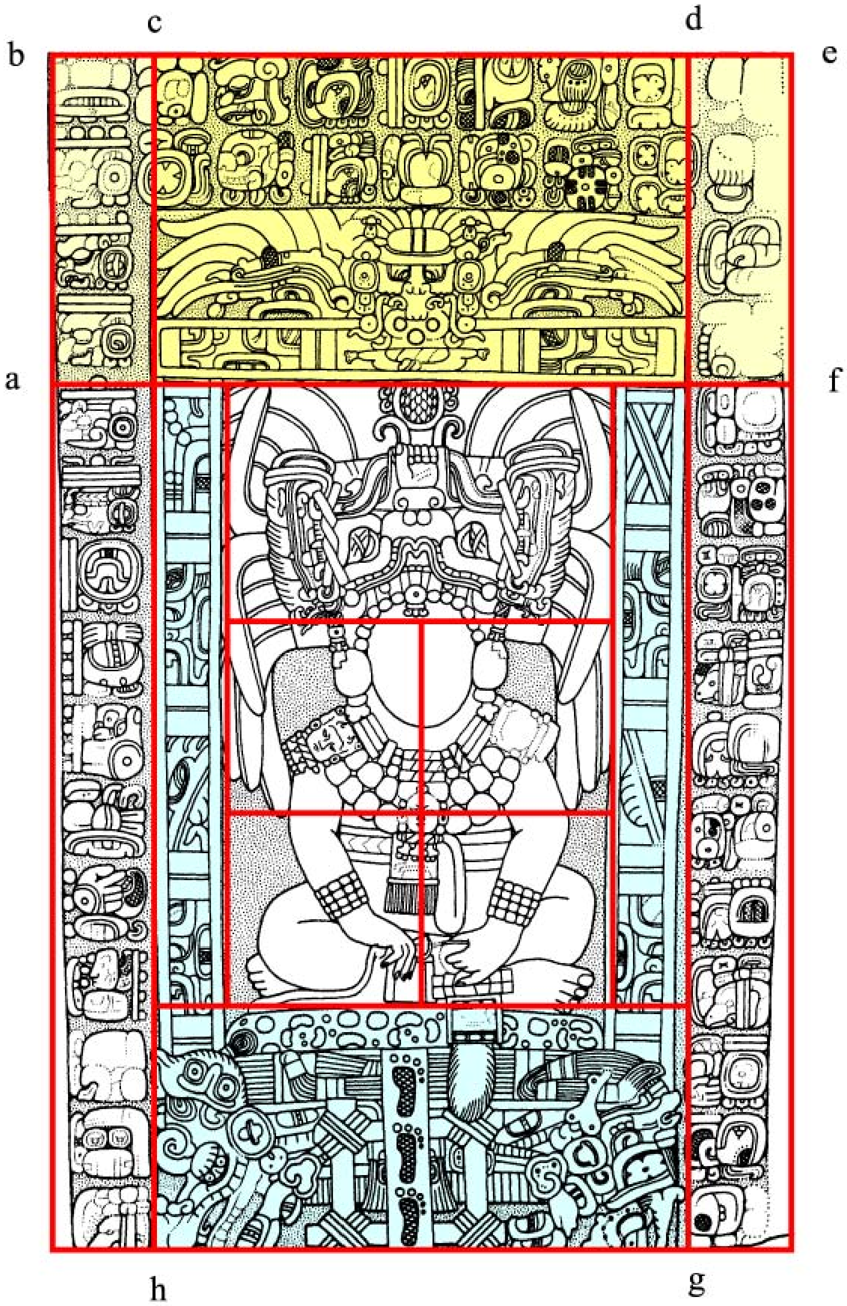
Stela 25 (b), Piedras Negras, Guatemala



In Stela 25 (b), the sky band that surrounds the seated figure is inscribed by a root phi rectangle.

Figure 248

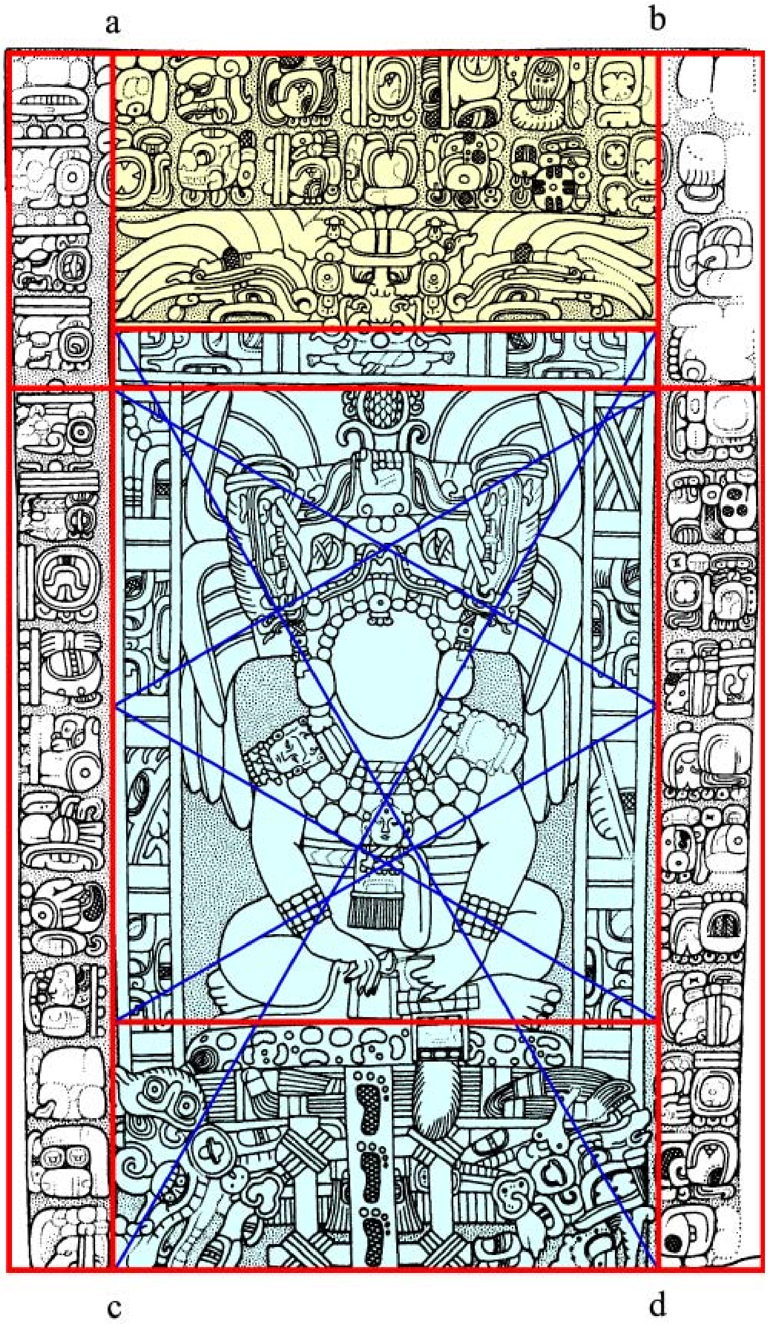
Stela 25 (c), Piedras Negras, Guatemala



In Stela 25 (c), the central phi rectangle (in white) is embedded within a larger phi rectangle (shaded blue) that is topped by a horizontal phi rectangle (shaded dark yellow). Because a root five rectangle may be subdivided by two phi rectangles, rectangle c,d,h,g is a root five rectangle. Rectangle a,b,c,f is also a root five rectangle.

Figure 249

Stela 25 (d), Piedras Negras, Guatemala

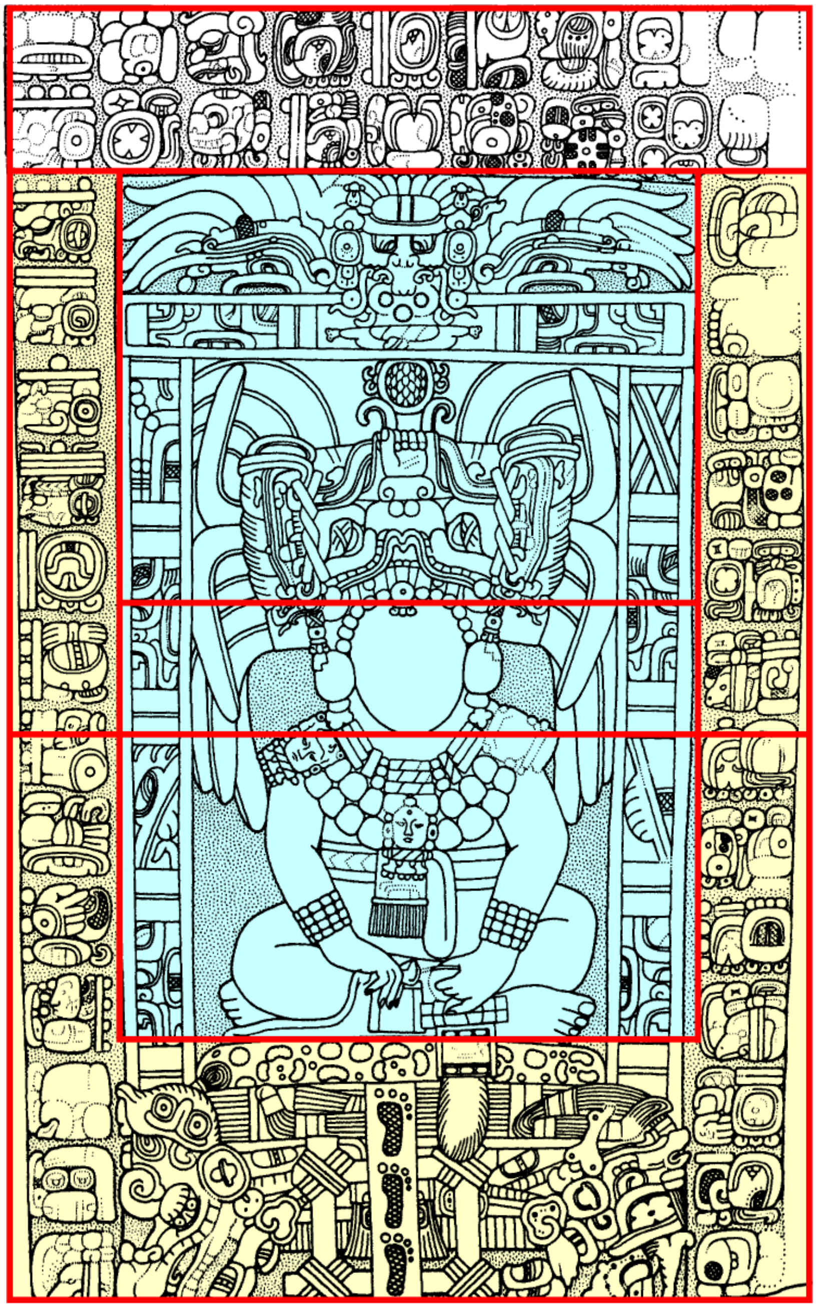


The same root five rectangle (a,b,c,d) that is subdivided by phi rectangles in the previous diagram is subdivided here by a root three rectangle (shaded blue) and a root four rectangle (shaded yellow) via the virtually perfect Formula Square Root of Five-4.

Within the root three rectangle is a two over root three rectangle that is the height of the seated figure and the width of the sky bands. All of the dark blue lines form equilateral triangles.

Figure 250

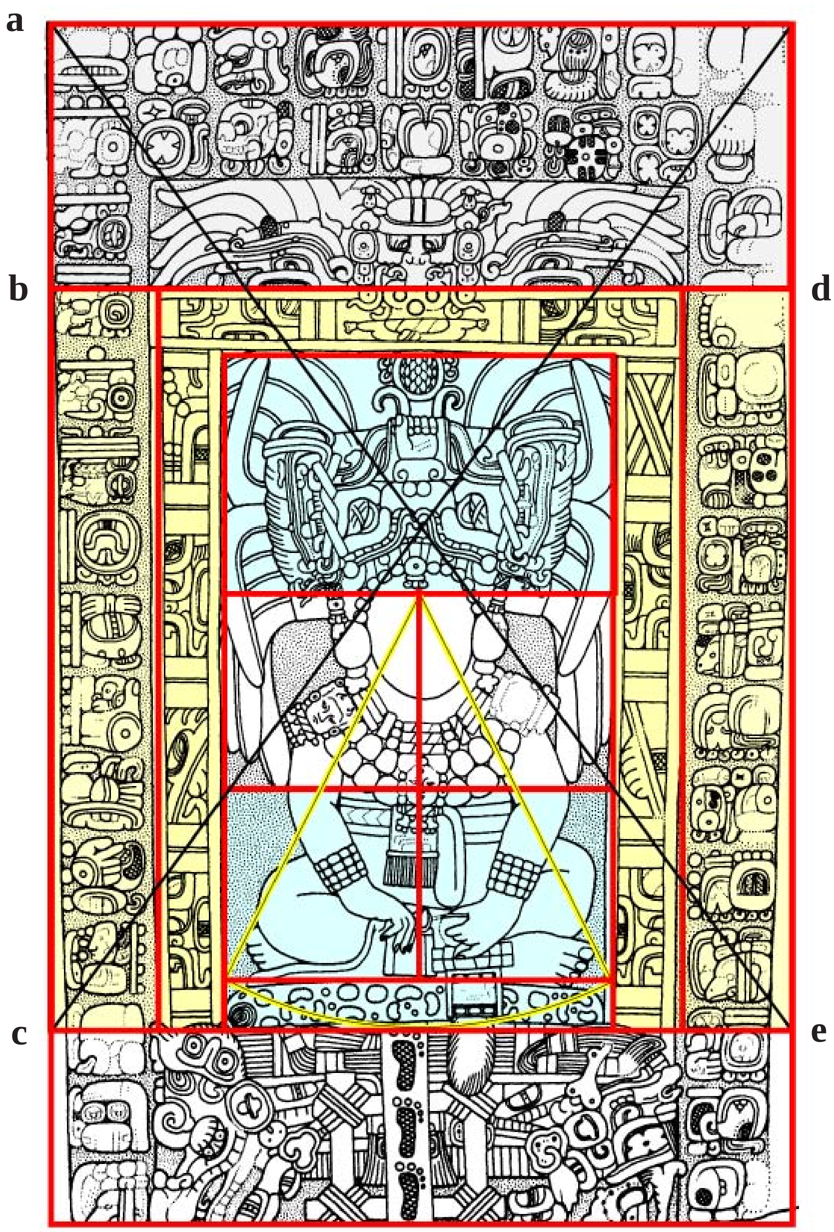
Stela 25 (e), Piedras Negras, Guatemala



In Stela 25 (e), a three over two rectangle is divided in half at the brow of the seated figure into two Pythagorean 3,4,5 rectangles (shaded blue), which are embedded in a root two rectangle divided in half into two smaller root two rectangles at the chin of the seated figure (shaded yellow).

Figure 251

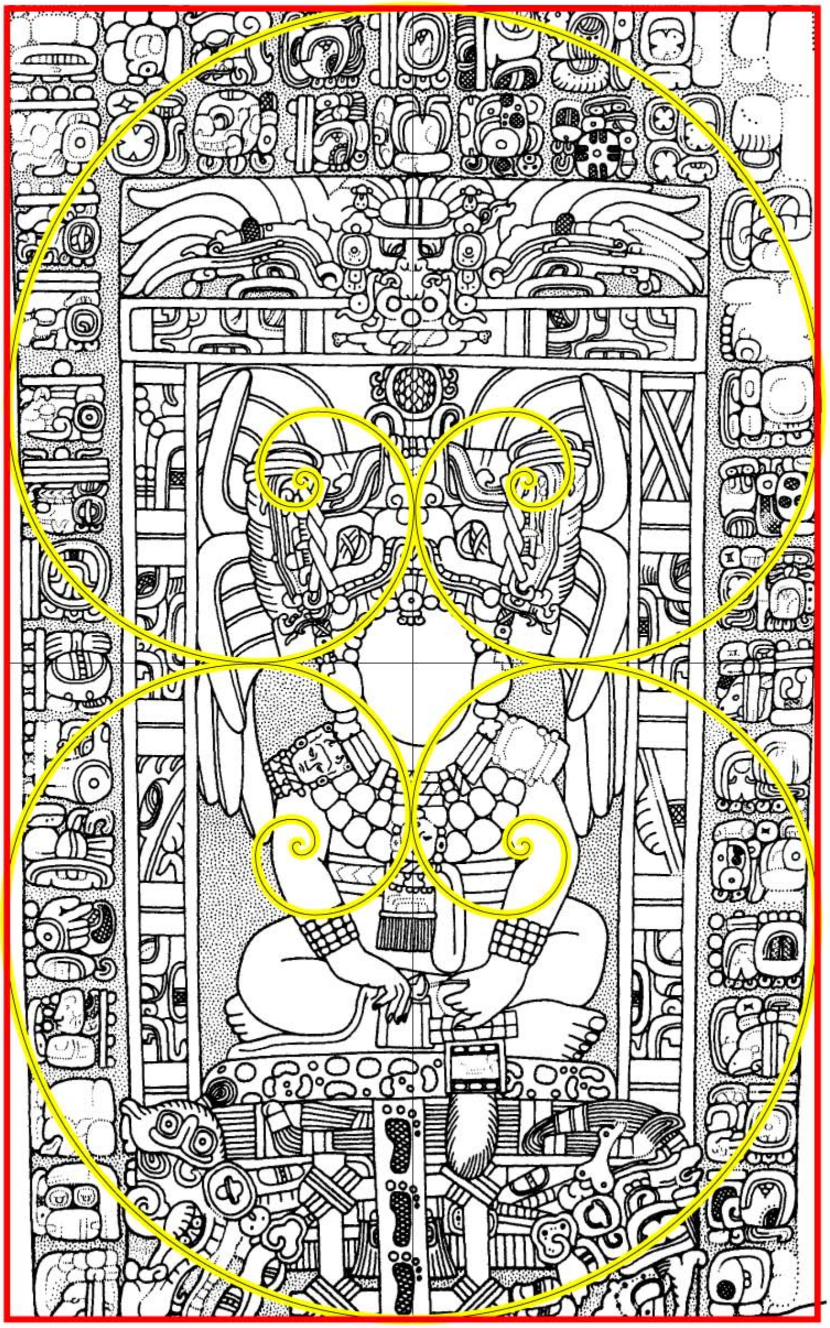
Stela 25 (f), Piedras Negras, Guatemala



In Stela 25 (f), rectangle b,c,d,e is a square (shaded light yellow). The sky band is inscribed by a root two rectangle (shaded dark yellow). The seated figure, including his jaguar pelt cushion, is inscribed by a root three rectangle that is subdivided by two phi rectangles (shaded blue) and a root four rectangle (in white) via the virtually perfect Formula Square Root of Three-3.
The space above square ab,c,d,e, is inscribed by a rectangle with a width of one to a length of three (shaded grey). Thus, c,e is three, a,c is four, and diagonal a,e is five.

Figure 252

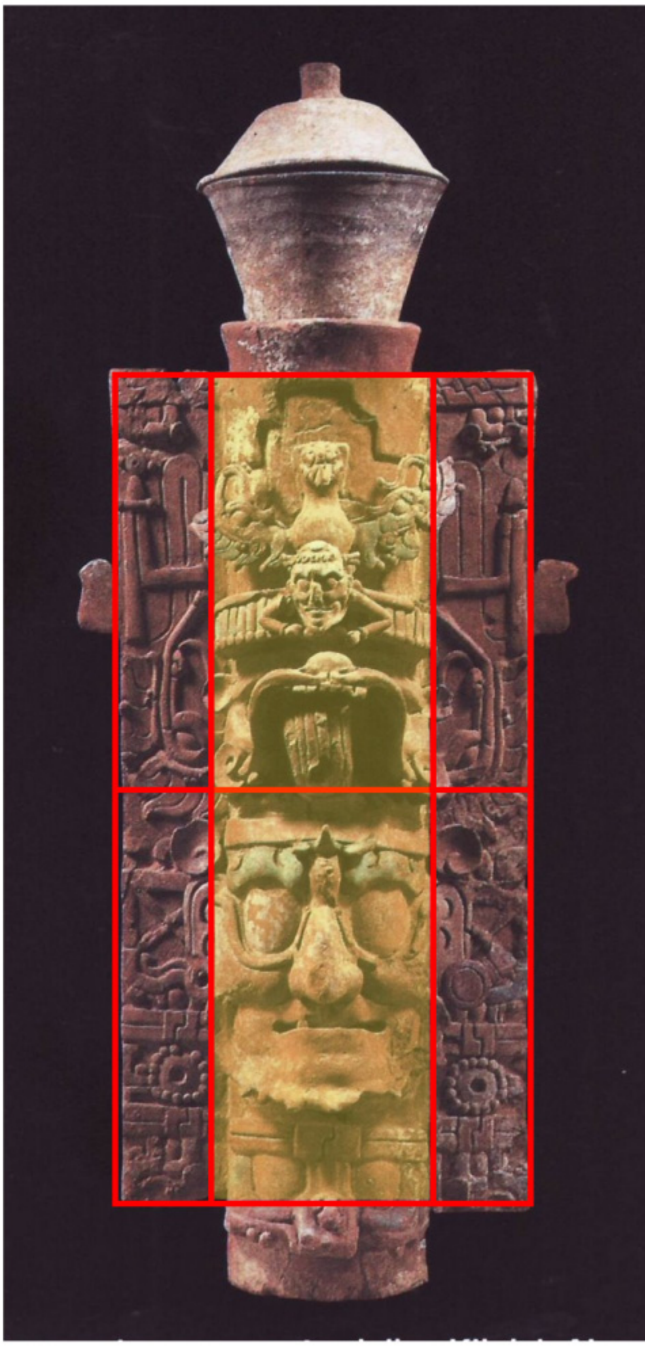
Stela 25 (g), Piedras Negras, Guatemala



The phi rectangle that inscribes Stela 25 from Piedras Negras is quartered. Four phi equiangular spirals, which frame the face of the seated figure, are added.

Figure 253

Incensario 1, Palenque, Chiapas, Mexico



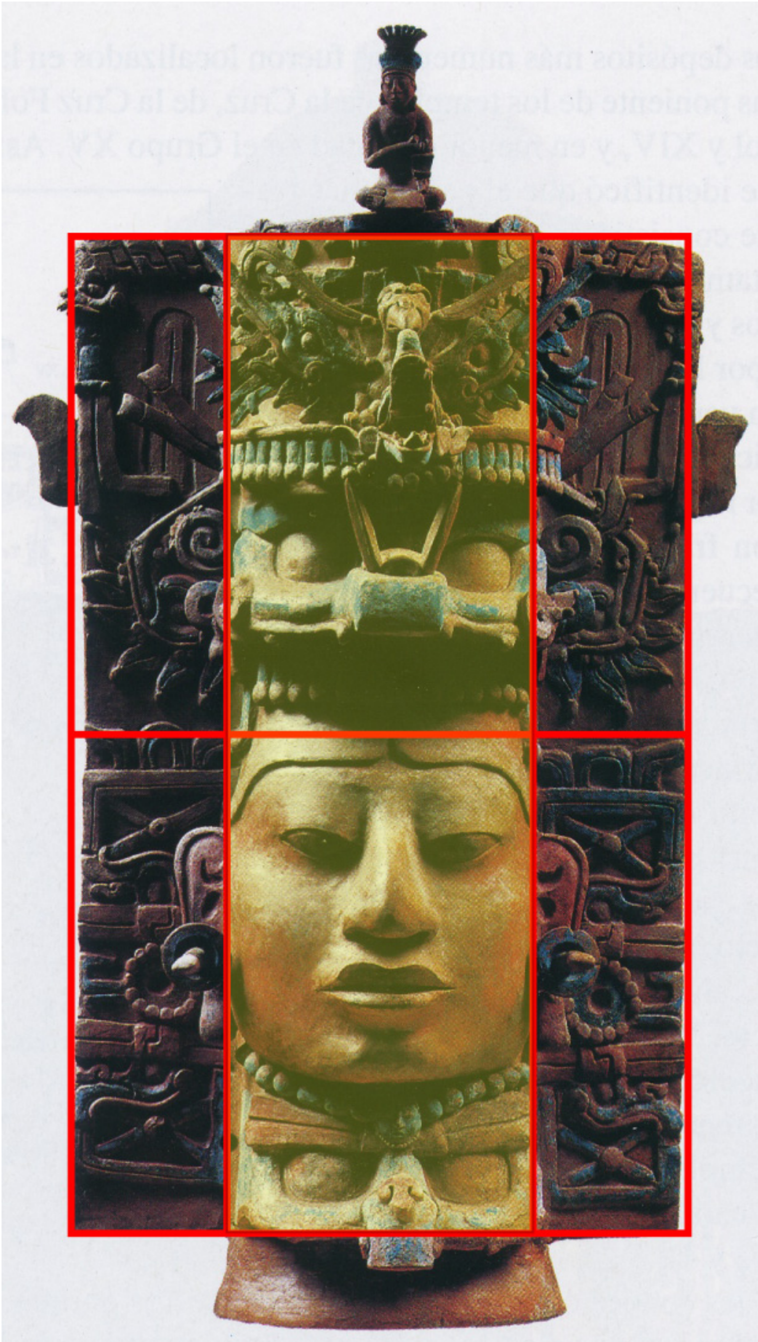
The flanges of Incensario-1 are inscribed by a root four rectangle that is divided in half at the brow of the deity. The central design elements between the flanges are inscribed by two root four rectangles that are one half the width of the root four rectangle that inscribes the flanges (highlighted yellow).

Figure 254

Incensario 2, Palenque, Chiapas, Mexico

Like Incensario 1, the flanges of Incensario 2 are inscribed by a root four rectangle that is divided in half at the brow of the Sun God. The central design elements between the flanges are inscribed by two root four rectangles that are one half the width of the root four rectangle that inscribes the flanges (highlighted yellow).

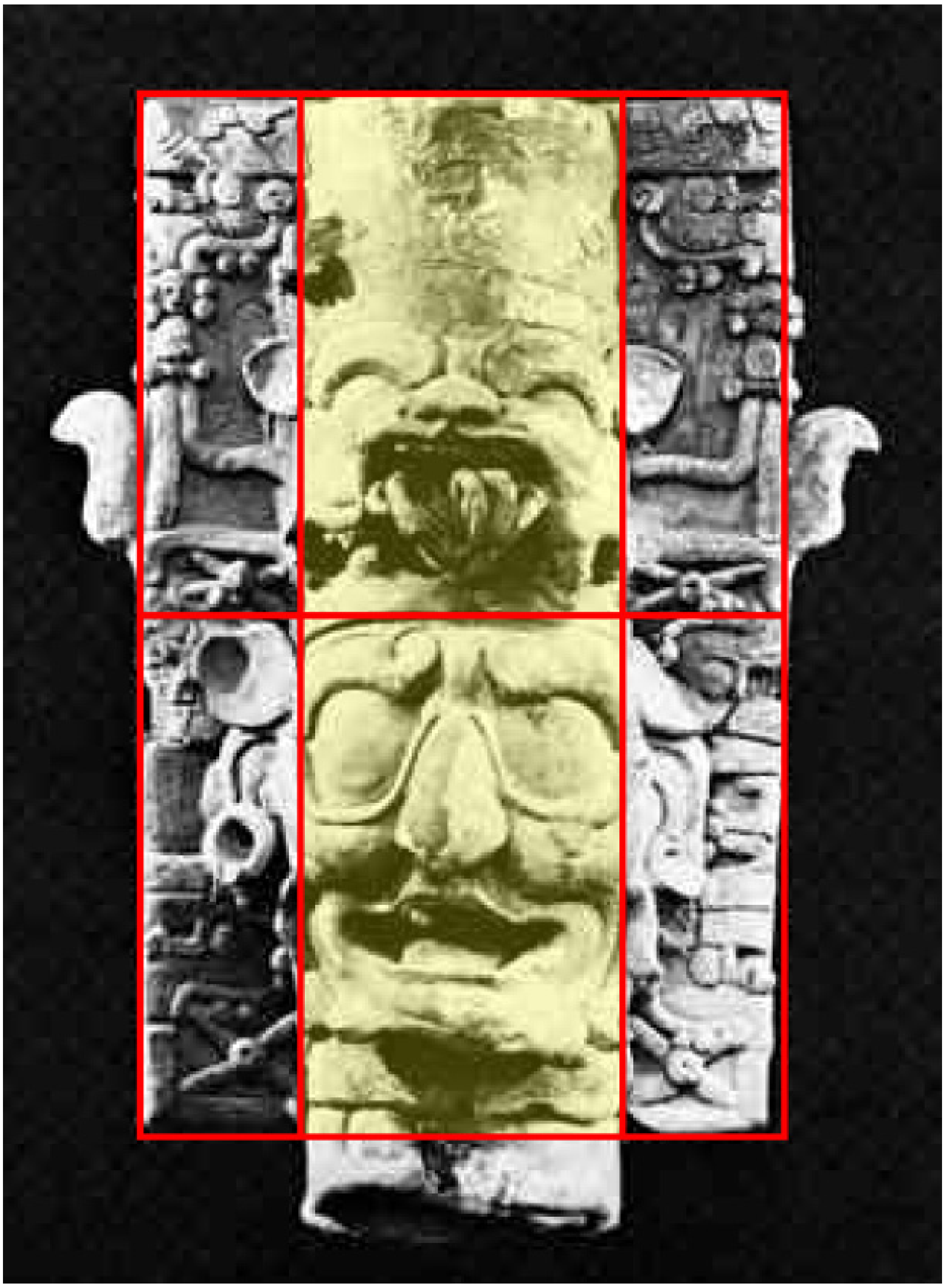
Figure 255
Incensario 3, Palenque, Chiapas, Mexico



The flanges of Incensario 3 are inscribed by a phi rectangle that is divided in half at the brow of the human face. The central design elements between the flanges are inscribed by two phi rectangles that are one half the width of the phi rectangle that inscribes the flanges (highlighted yellow).

Figure 256

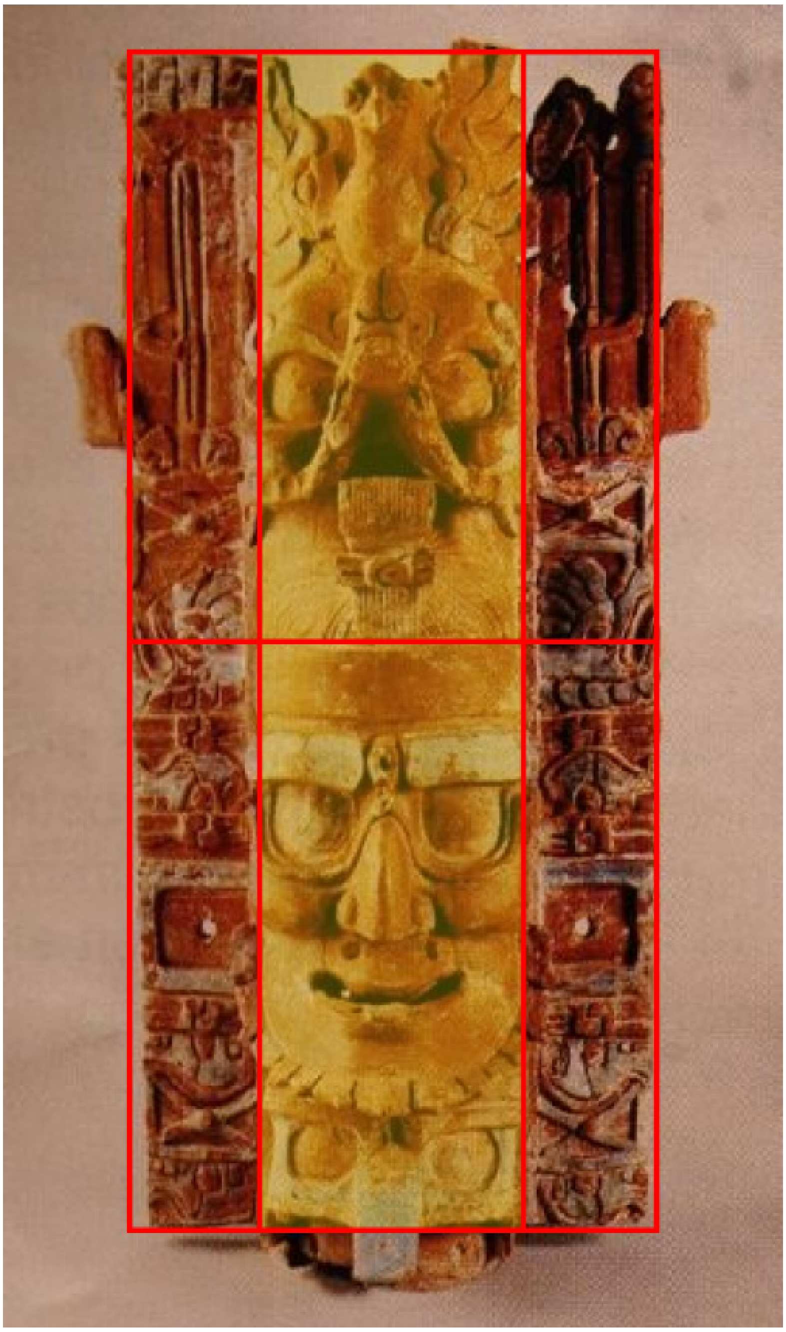
Incensario 4, Palenque, Chiapas, Mexico



As in Incensario 3, the flanges of Incensario 4 are inscribed by a phi rectangle that is divided in half at the brow of the face of the deity. The central design elements between the flanges are inscribed by two phi rectangles that are one half the width of the phi rectangle that inscribes the flanges (highlighted yellow).

Figure 257

Incensario 5, Palenque, Chiapas, Mexico



The flanges of Incensario 5 are inscribed by a root five rectangle that is divided in half at the brow of the face of the deity. The central design elements between the flanges are inscribed by two root five rectangles that are one half the width of the root five rectangle that inscribes the flanges (highlighted yellow).

Figure 258

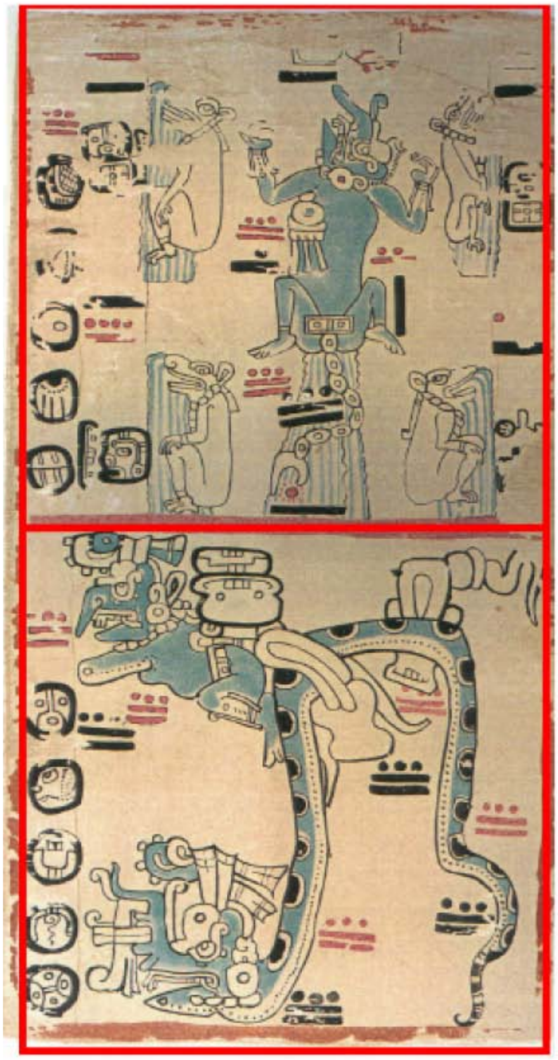
Incensario 6, Palenque, Chiapas, Mexico

The flanges of Incensario 6 are inscribed by a two over root three rectangle that is divided at the brow of the face of the Sun God. The central design elements between the flanges are inscribed by two, two over root three rectangles that are one half the width of the two over root three rectangle that inscribes the flanges and that are further subdivided by a vertical root three rectangle (highlighted yellow) and a horizontal root three rectangle (highlighted blue). Equilateral triangles are drawn in thin yellow lines.

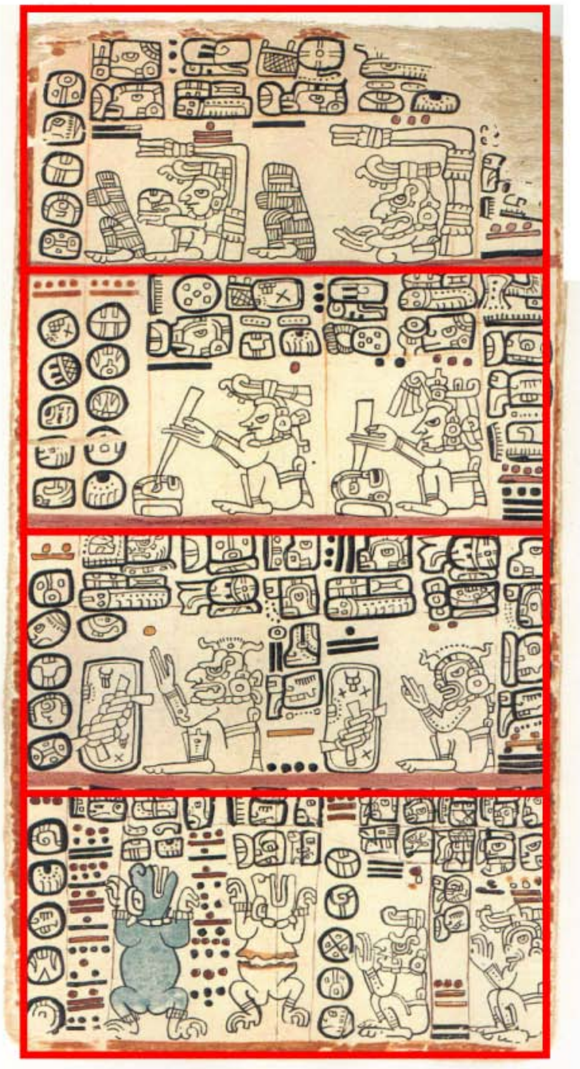
Figure 259

Madrid Codex, Pages 31 and 101

Page 31



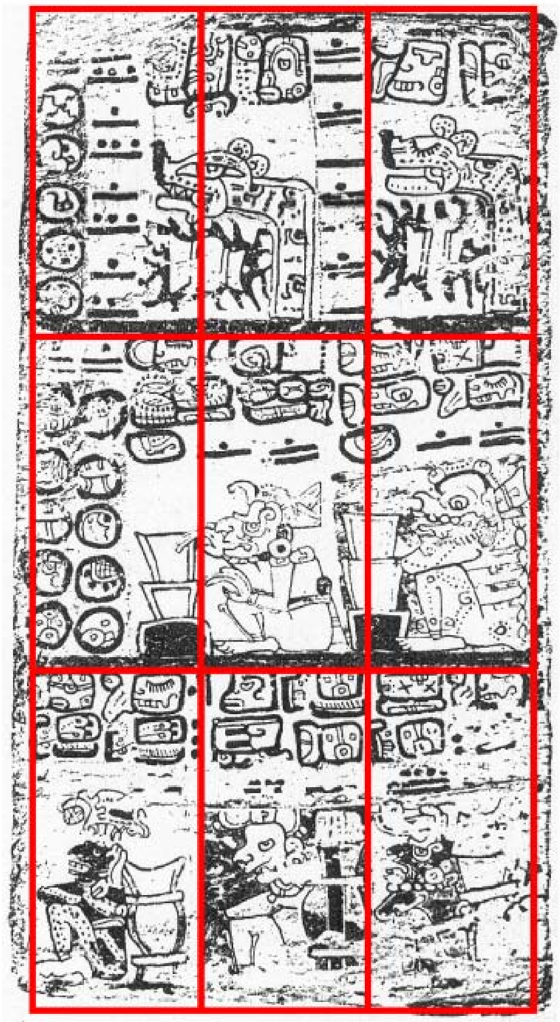
Page 101

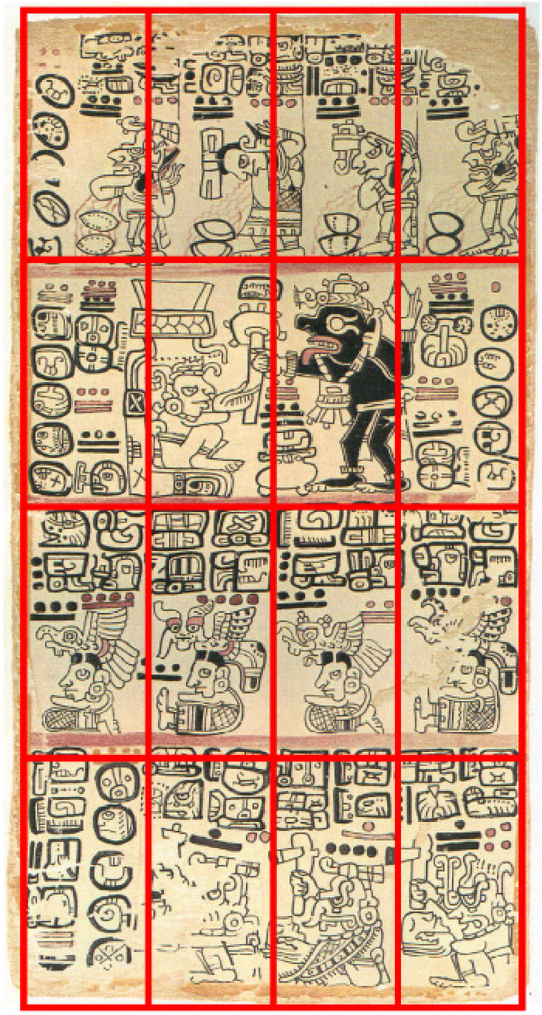


Page 31 is a square root of four rectangle divided in half into two squares. Page 101 is a square root of four rectangle subdivided into four equal-sized square root of four rectangles, as per Formula Square Root of Four-b.

Figure 260

Pages 28 and 103 of the Madrid Codex

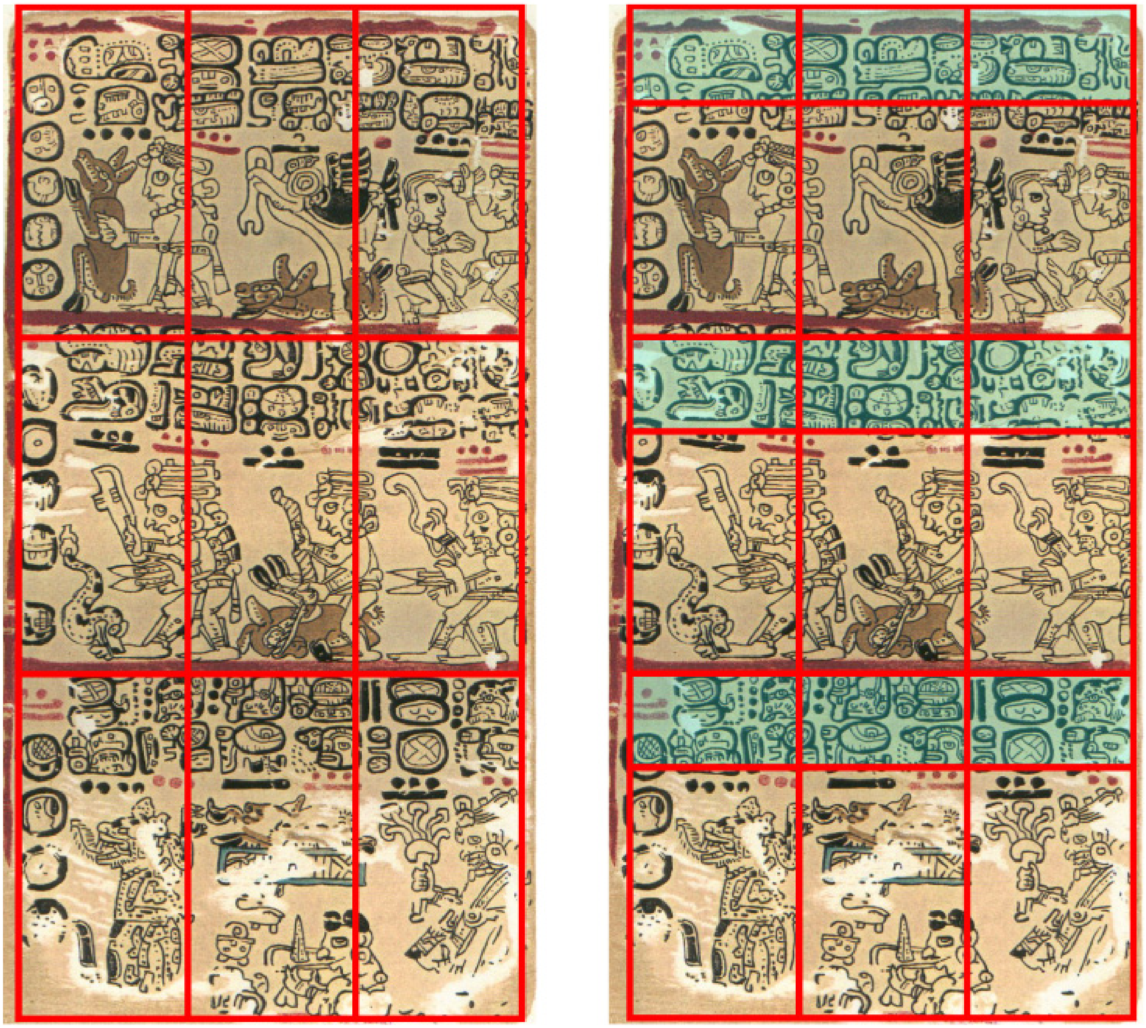




Page 28 is trisected into nine root four rectangles, and page 103 is quadrasected into sixteen root four rectangles

Figure 261

Madrid Codex, Plate XVII



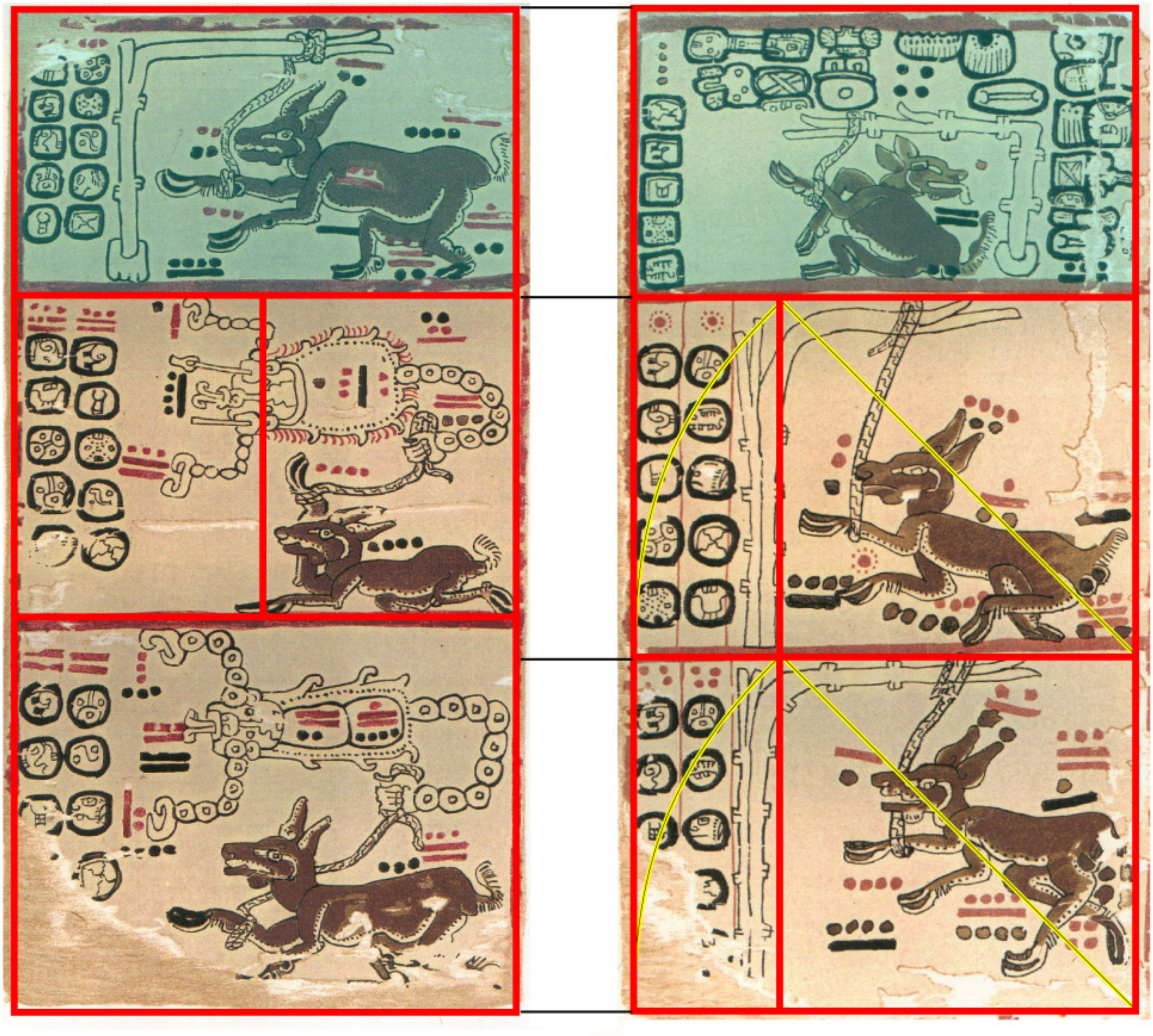
On the left, the square root of four rectangle that inscribes the page is subdivided by three, both vertically and horizontally, to produce nine equal-sized square root of four rectangles. On the right, each of these nine smaller square root of four rectangles is further subdivided into a square root of three rectangle (shaded blue) and a square root of two rectangle, as per Formula Square Root of Four-4

Figure 262

Madrid Codex, Plates XII and XI

Plate XII

Plate XI

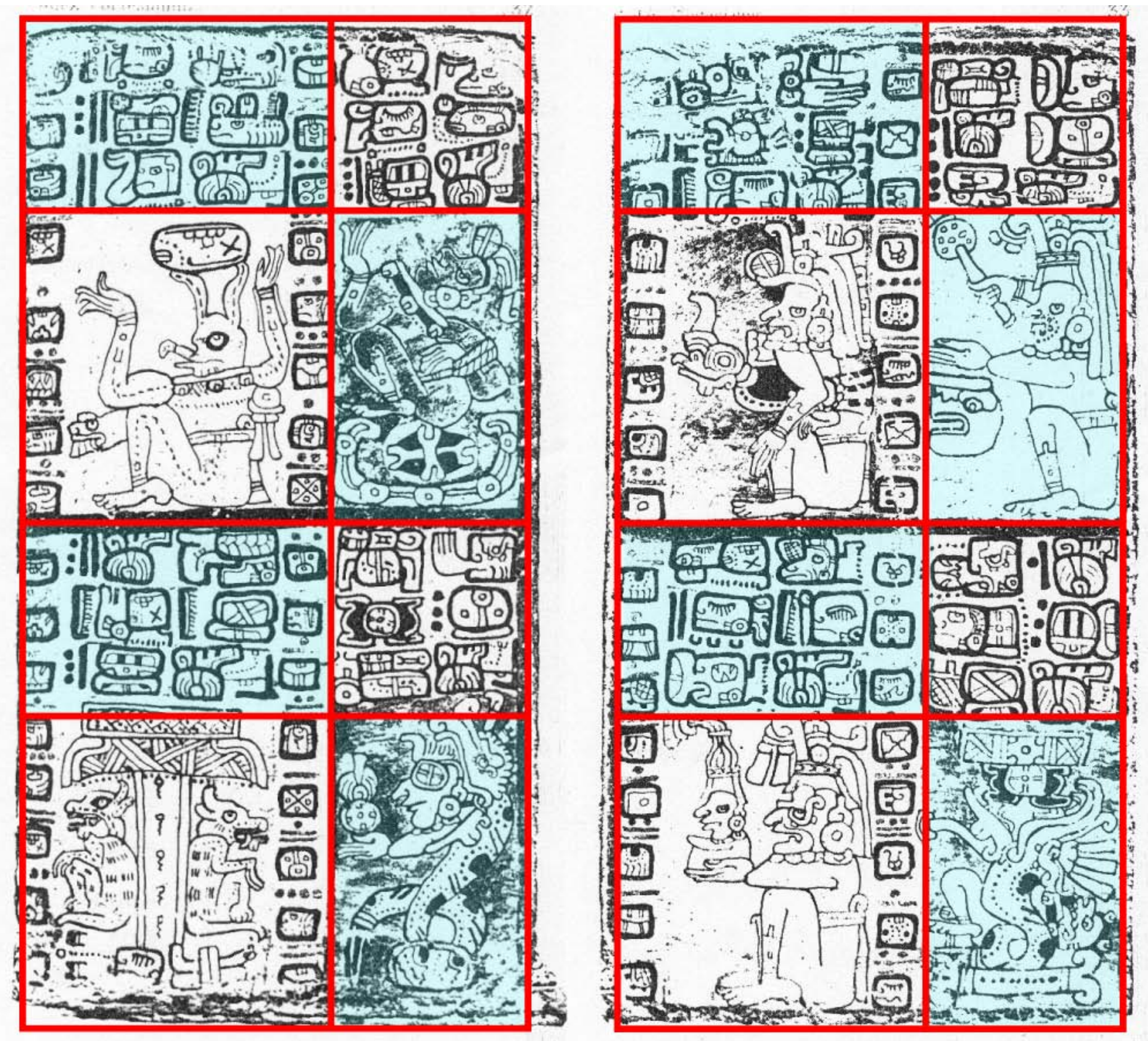


Both Plates XII and XI are inscribed by square root of four rectangles that are subdivided into square root of three rectangles (shaded blue) and root two rectangles, as per Formula Square Root of Four-4. The remaining root two rectangle of Plate XII (unshaded) is further subdivided into three Pythagorean 3,4,5 rectangles, as per Formula Square Root of Two-3. The corresponding square root of two rectangle on Plate XI is divided in half into twwo smaller root two rectangles. The diagonals of the squares within these root two rectangles are the radii of the arcs that determine their lengths (yellow highlighted lines).

Figure 263

Madrid Codex, Pages 32 and 33

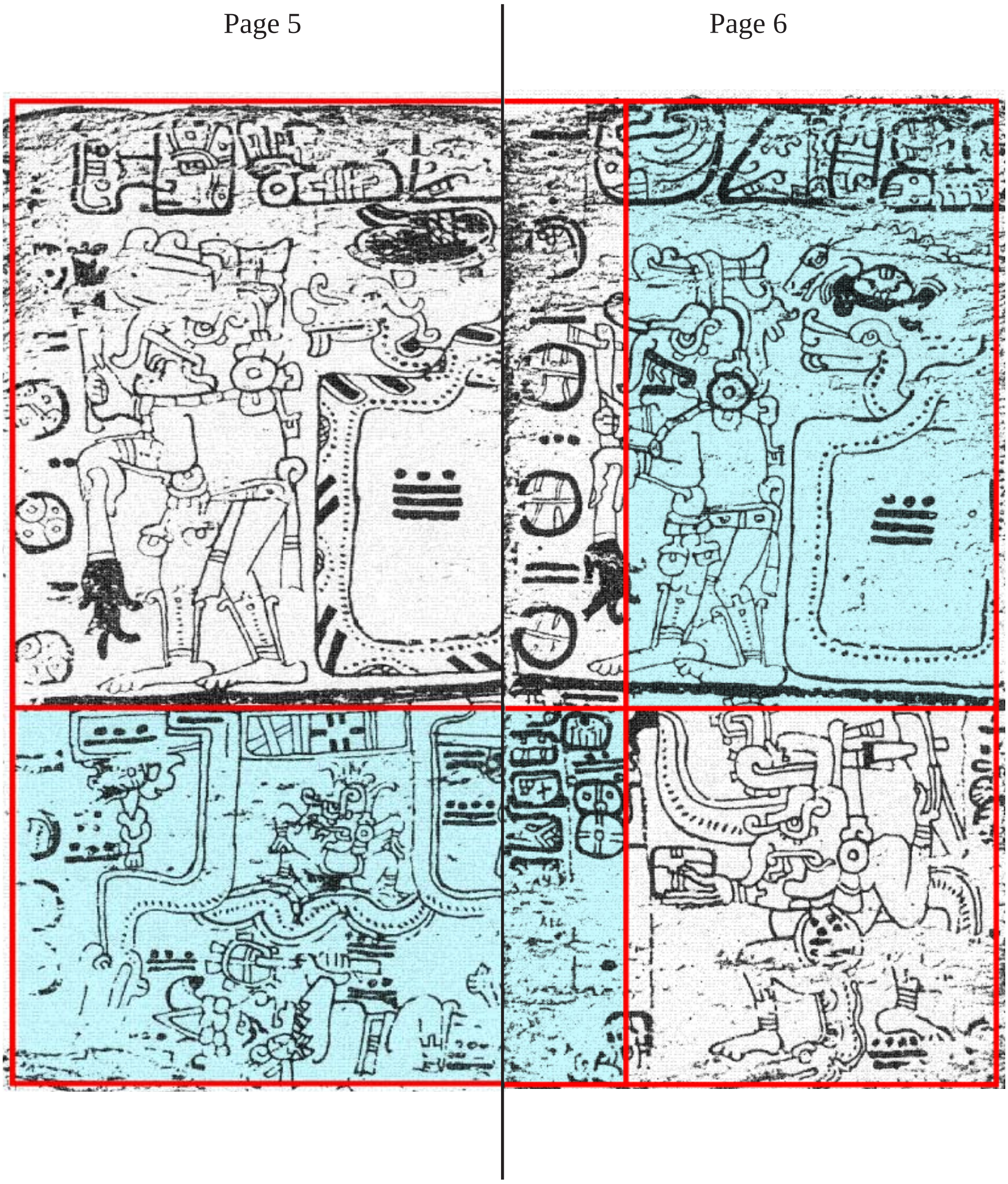
Page 32 Page 33



Pages 32 and 33 are divided into two squares that are then subdivided into smaller squares (in white) and phi rectangles (shaded blue), as per Formula Square-2.

Figure 264

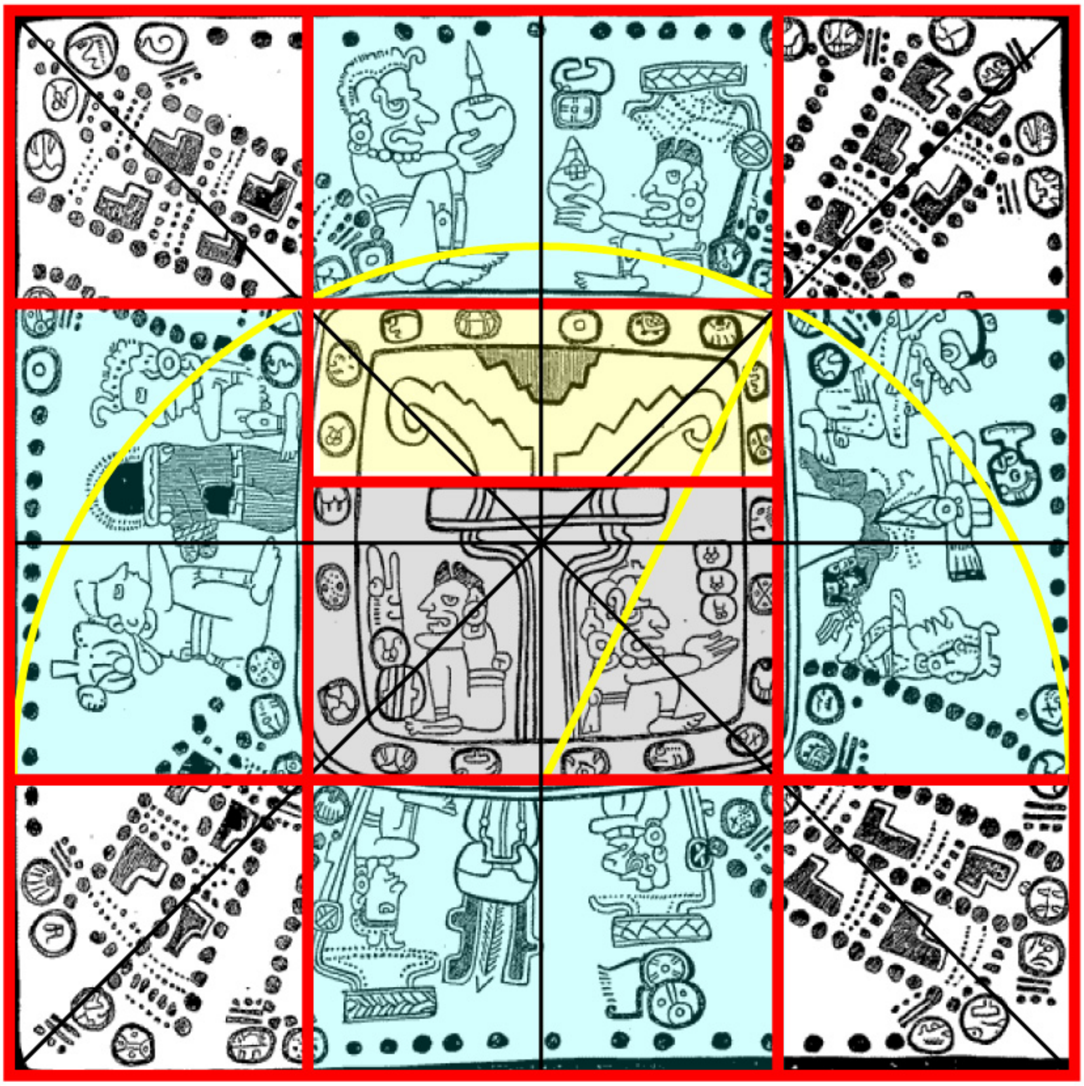
Madrid Codex, Pages 5 and 6



The square root of four rectangles that inscribe pages 5 and 6 join at the center (vertical black line) and form a square that is subdivided into squares (in white) and phi rectangles (shaded blue), as per Formula Square-2.

Figure 265

Madrid Codex, Pages 41 and 42

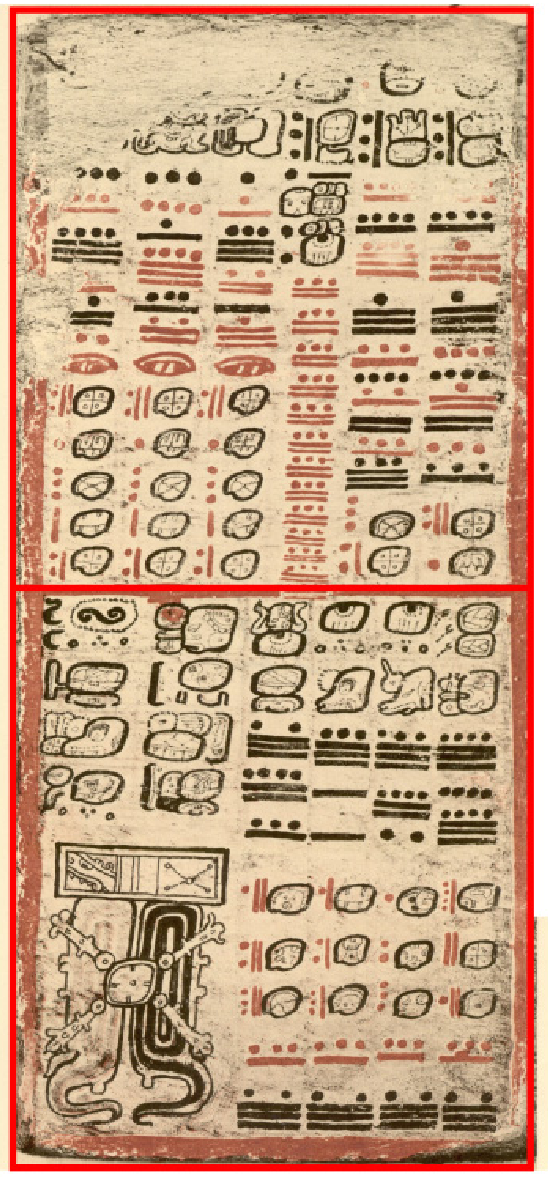


The square root of four rectangles that inscribe pages 41 and 42 are merged at the center and form a square that is subdivided into squares (unshaded) and phi rectangles (shaded blue), as per Formula Square-2. The square in the center of the diagram is divided into a phi rectangle (shaded gray) and a phi squared rectangle (shaded yellow). The yellow highlighted diagonal within this center square is the radius for the arc that determines the width of the adjacent phi rectangles.

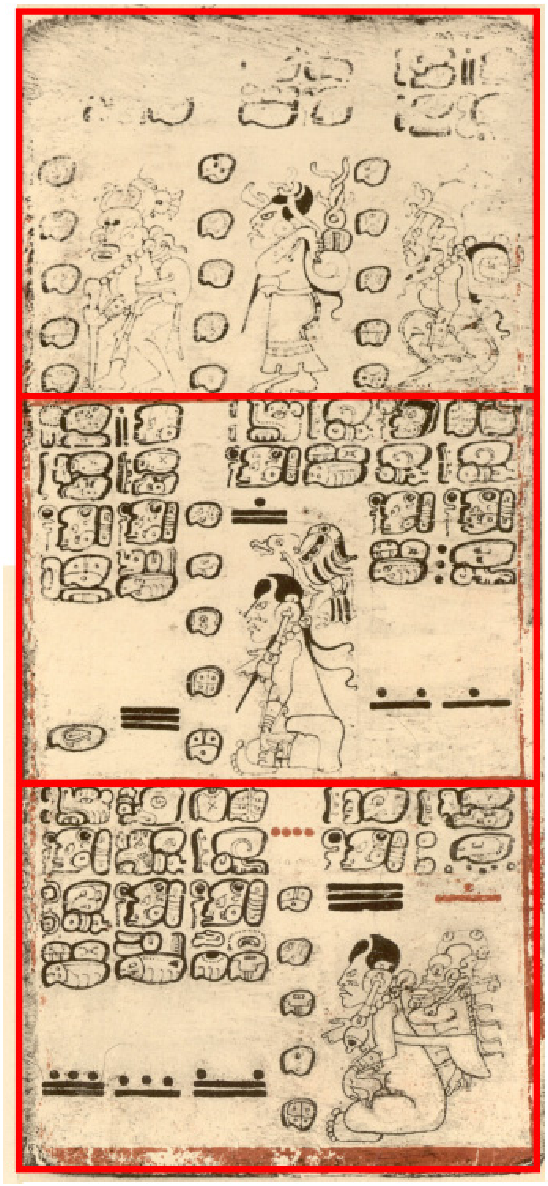
Figure 266

Dresden Codex, Pages 18 and 52

Page 52



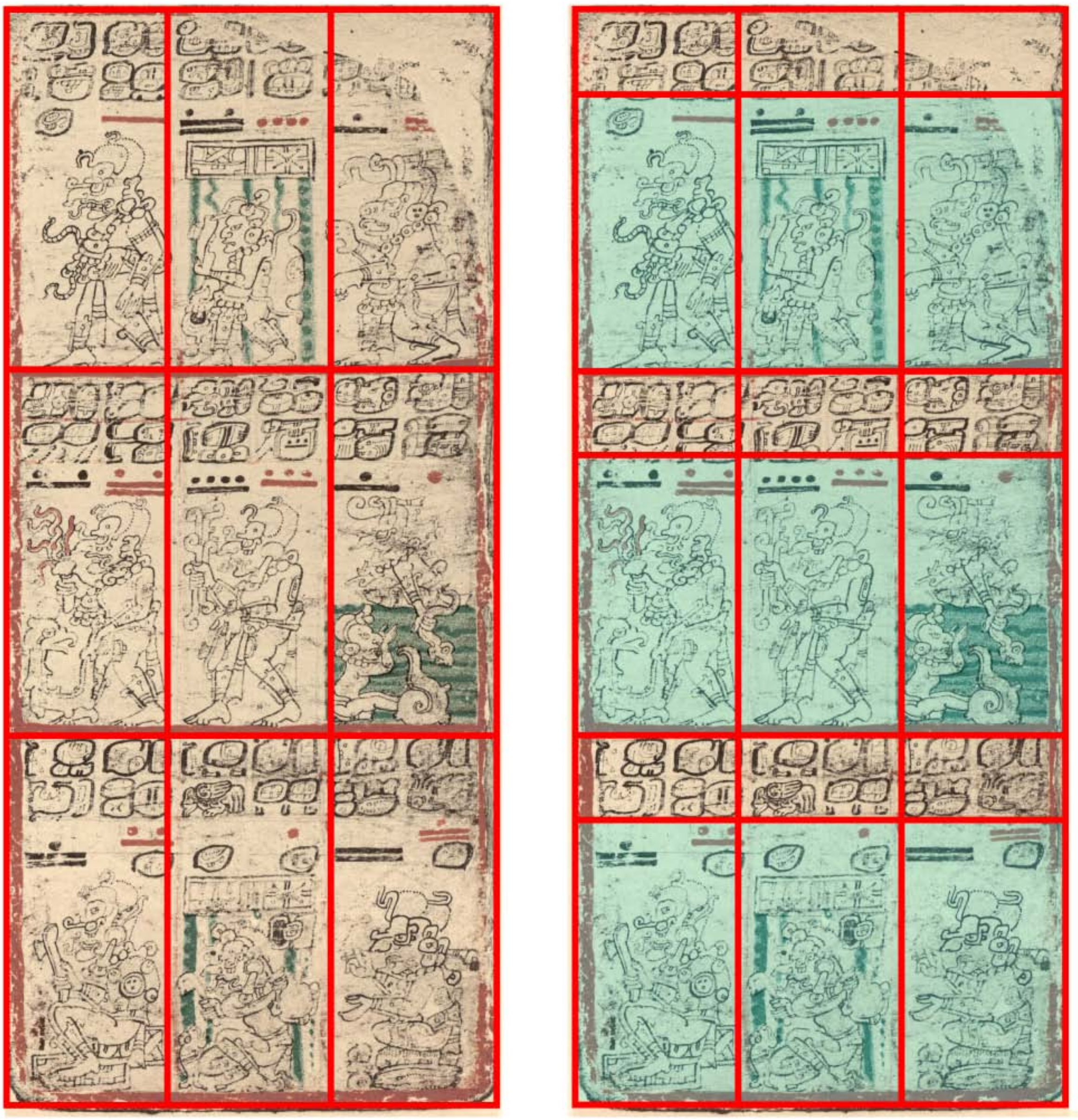
Page 18



The pages of the Dresden Codex are all inscribed by square root of five rectangles. Page 52 is divided in half, and page 18 is trisected into three virtually perfect Pythagorean 3,4,5 rectangles, as per Formula Square Root of Five-5.

Figure 267

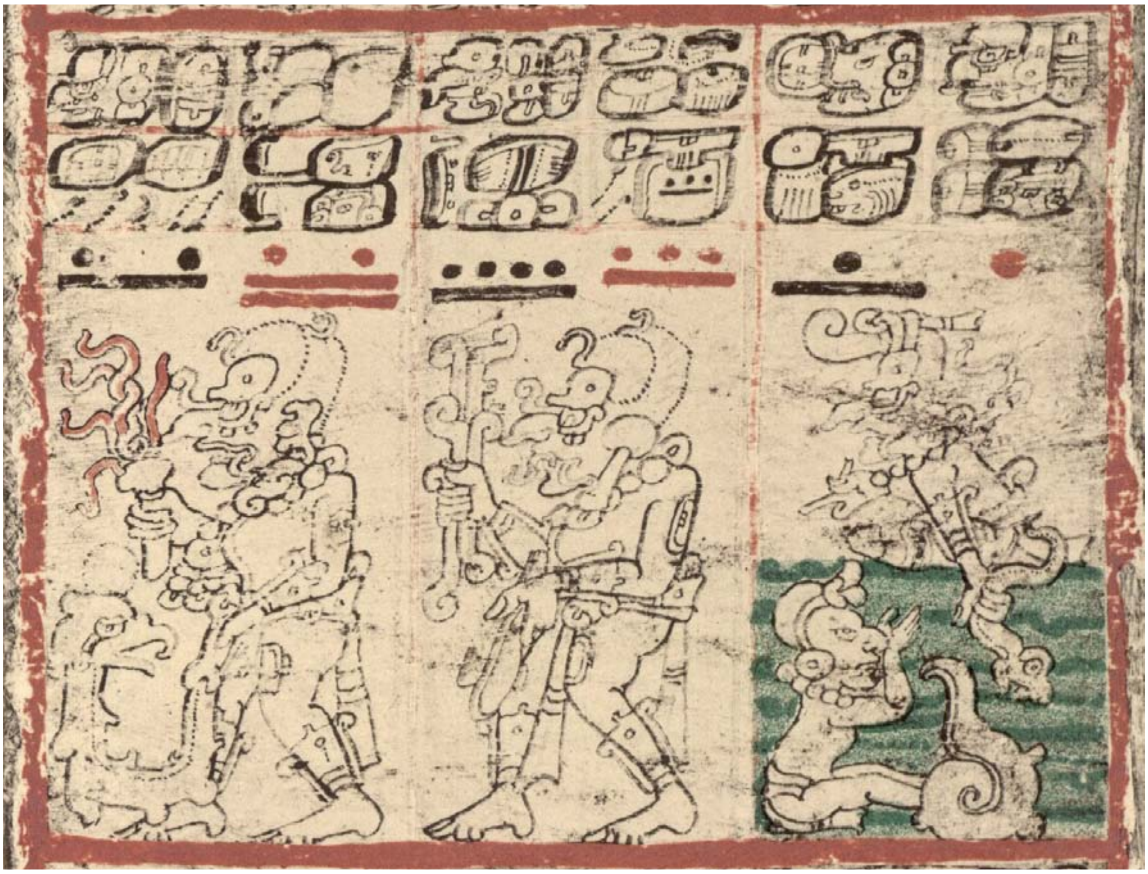
Dresden Codex, Page 39



The height and width of the square root of five rectangle that inscribes page 39 is trisected to create nine smaller square root of five rectangles (left). On the right, each of these smaller rectangles is further subdivided into square root of three rectangles (shaded blue) and square root of four rectangles (double squares) via the virtually perfect Formula Square Root of Five-4. Note that each row of three vertical square root of three rectangles produces a larger horizontal square root of three rectangle, as per Formula Square Root of Three-2.

Figure 268

Dresden Codex, Page 39 (Close-Up)

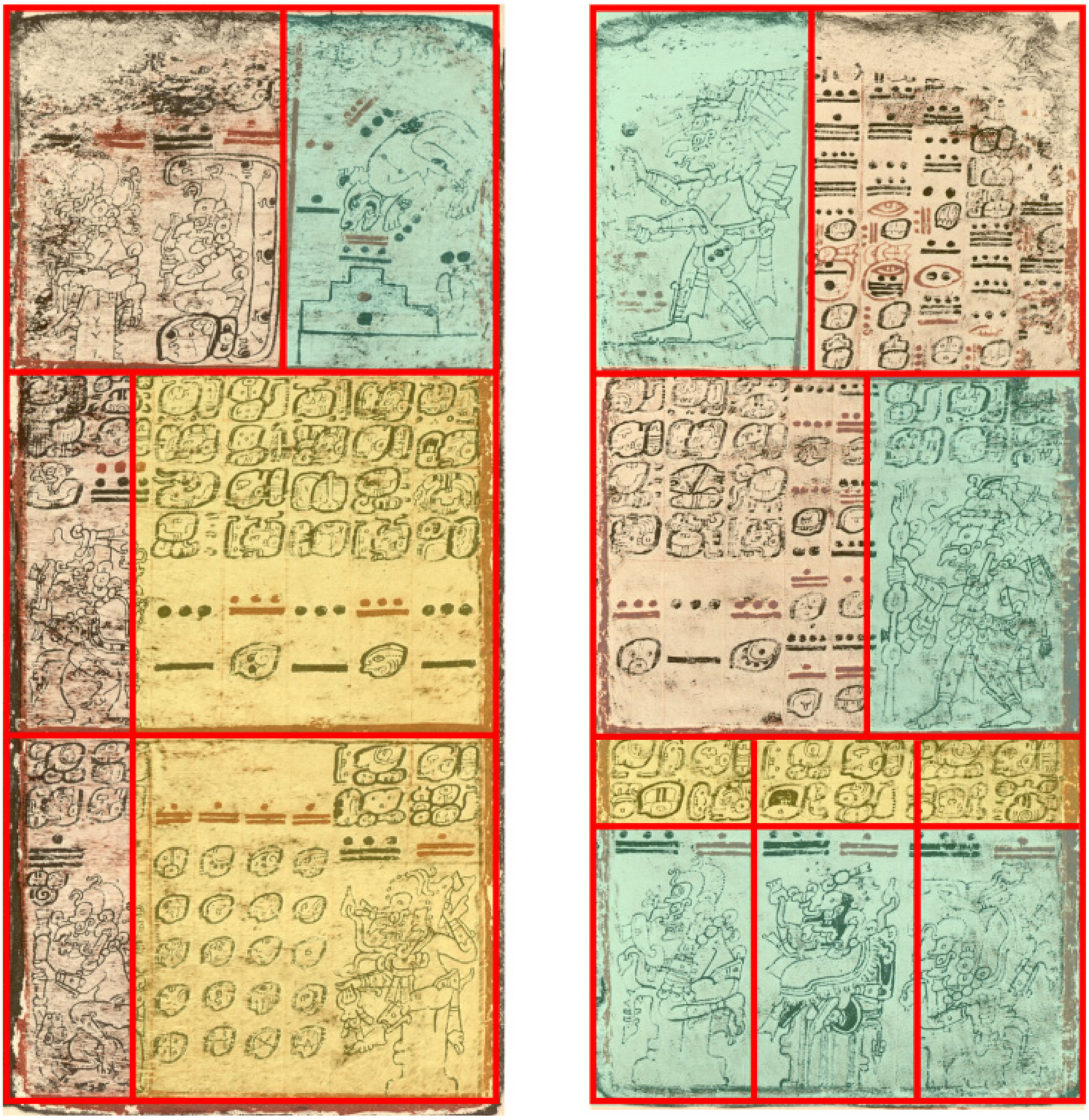


In this close up of page 39 of the Dresden Codex, you can see each of the subdivisions of the over-laid diagram in Figure 266 (right) painted in now faded red lines by the original artist.

Figure 269

Dresden Codex, Pages 31 and 32

Page 31 Page 32



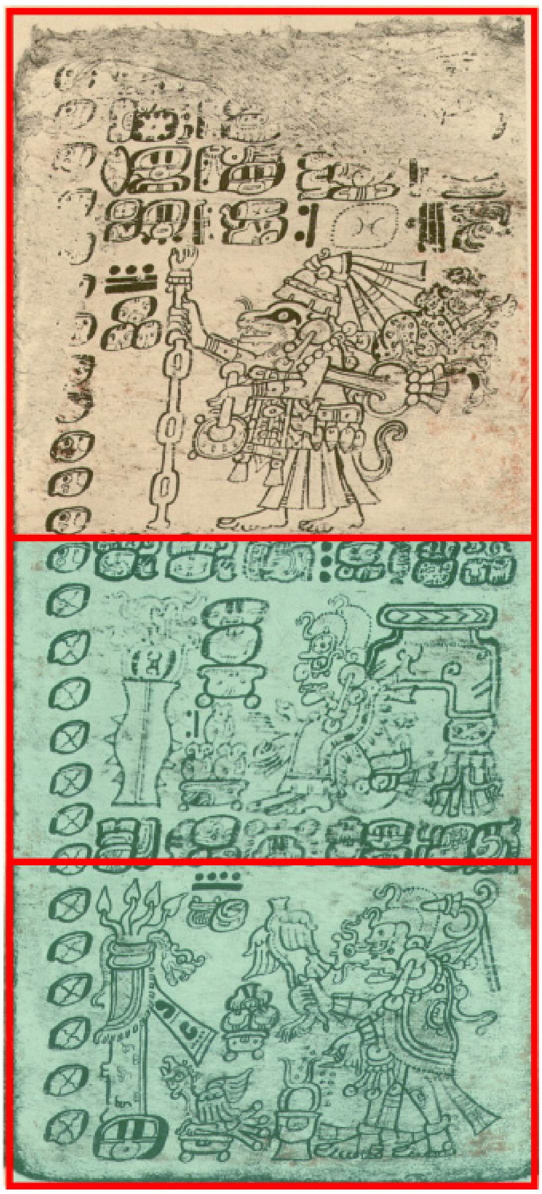
Pages 31 and 32 are each trisected into three virtually perfect Pythagorean 3,4 5 rectangles that are then subdivided by various formulae. The uppermost Pythagorean rectangle on page 31 and the upper two on page 32 are subdivided into square root of three rectangles (shaded blue) and smaller Pythagorean 3,4,5 rectangles (unshaded), as per Formula Pythagorean 3,4,5-3. The remaining two Pythagorean 3,4, 5 rectangles on page 31 are divided into squares (shaded yellow) and rectangles (unshaded) with proportions of one to three. The lowermost Pythagorean rectangle on page 32 is composed of three square root of five rectangles that are further subdivided into square root of four rectangles (shaded yellow) and square root of three rectangles (shaded blue).

Figure 270

Dresden Codex Pages 26 and 28

Page 28

Page 26

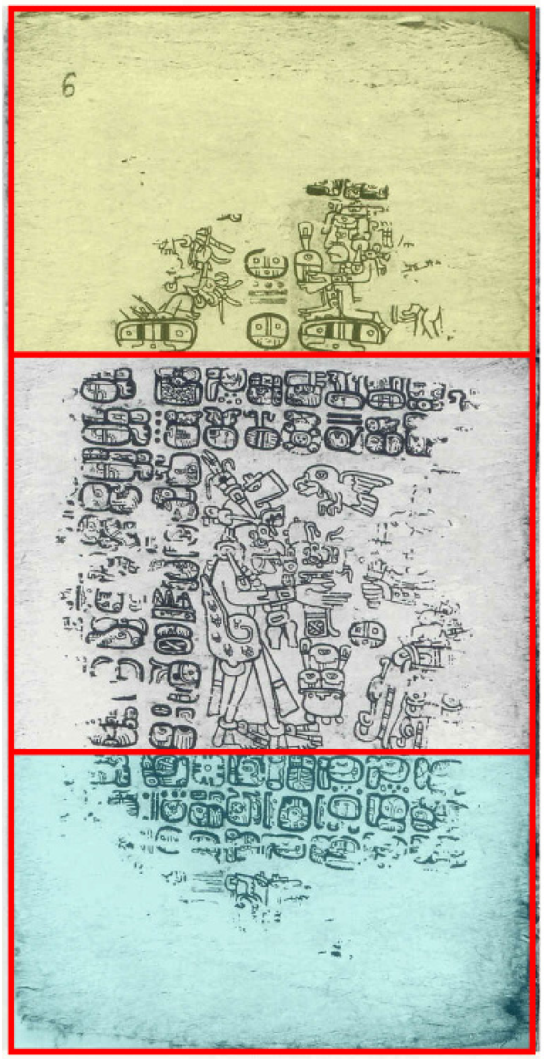


The square root of five rectangles that inscribe Pages 26 and 28 are subdivided by squares (un-shaded) and phi rectangles (shaded blue).

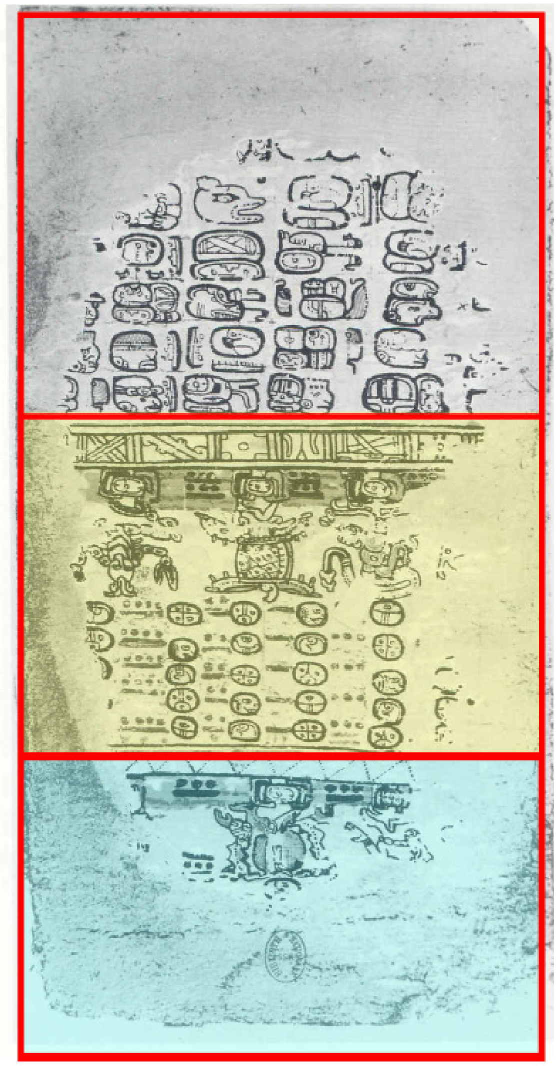
Figure 271

Paris Codex, Pages 6 and 24

Page 6



Page 24

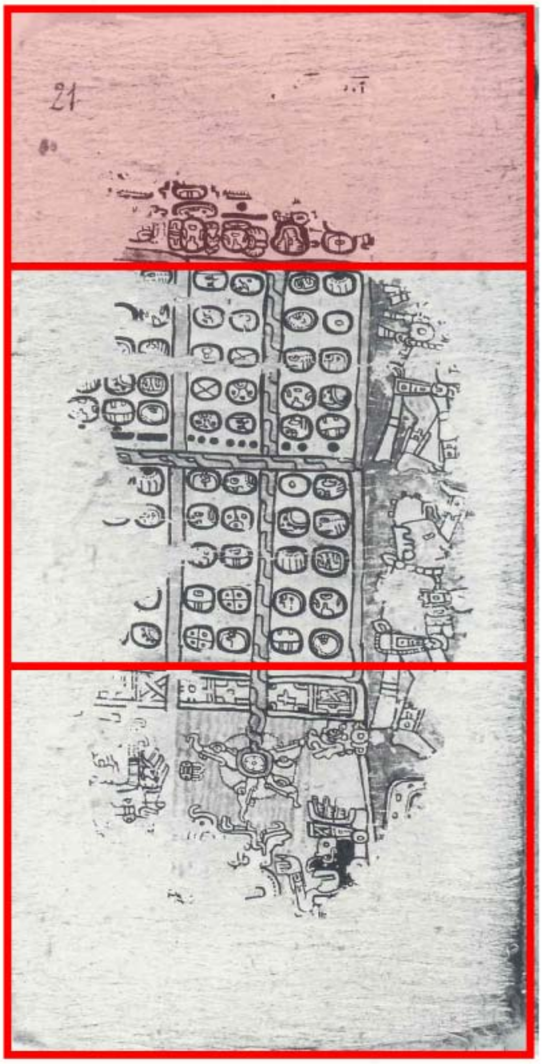


Both of the square root of four rectangles that inscribe pages 6 and 23 of the Paris Codex are subdivided virtually perfectly by square root of three rectangles (shaded blue), Pythagorean 3,4,5 rectangles (unshaded), and rectangles with a proportion of two to three (shaded yellow).

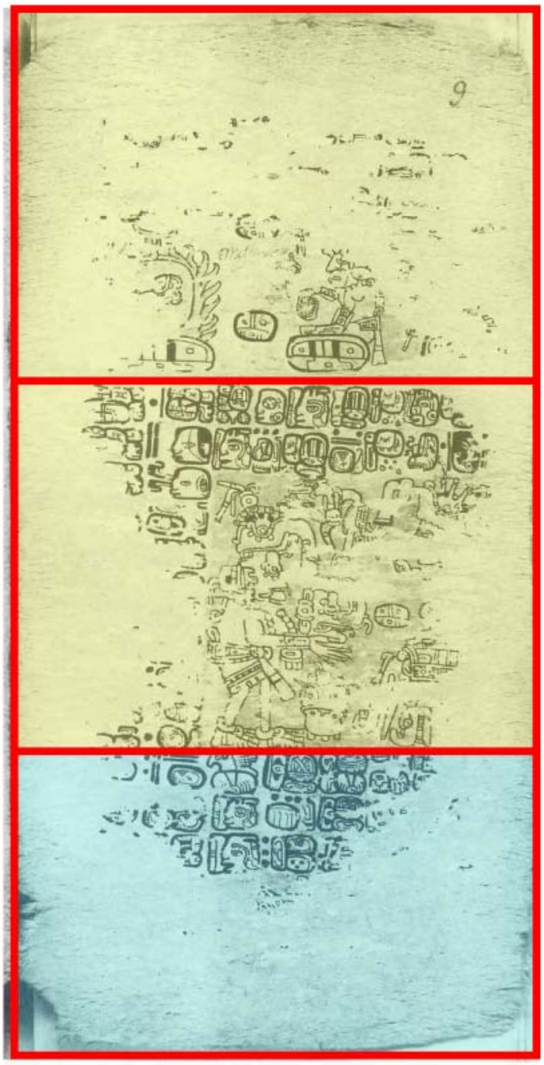
Figure 272

Paris Codex, Pages 9 and 21

Page 21



Page 9

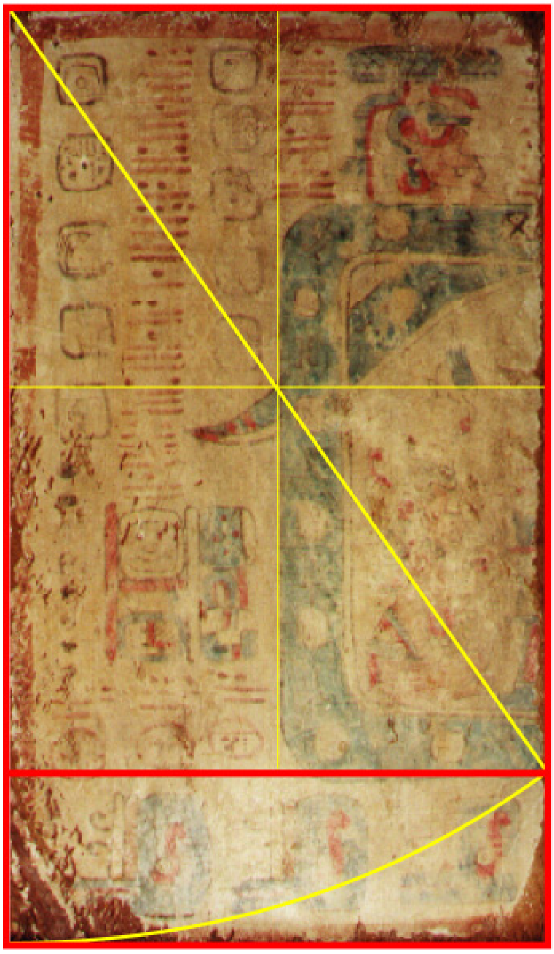


Page 21 of the Paris Codex is inscribed by a square root of four rectangle that is subdivided into a square root of four rectangle (shaded red) and two Pythagorean 3,4,5 rectangles (unshaded). Page 9 is inscribed by a square root of four rectangle that is subdivided by a square root of three rectangle (shaded blue) and a square root of two rectangle (shaded yellow) that is divided in half to form two smaller square root of two rectangles, as per Formula Square Root of Four-3.

Figure 273

The Prague Codex, Pages 1 and 4

Page 1



Page 4

Page 1 of the Prague Codex is inscribed by a square root of three rectangle that is subdivided by a quartered square root of two rectangle whose diagonal is equal to the height of the square root of three rectangle, as per Formula Square Root of Two-1. Page 4 is also inscribed by a square root of three rectangle that is subdivided by a square root of three rectangle (shaded blue), a square root of five rectangle (shaded yellow), and a square root of two rectangle (unshaded), as per Formula Square Root of Three-5.

Figure 274

Codex Vindobonensis Mexicanus, Pages 2 and 62

Page 2

Page 62

All of the pages of the Codex Vindobonensis Mexicanus can be inscribed by two over square root of three rectangles. Page 2 is simply divided longitudinally to form two square root of three rectangles. Page 62 trisects these two square root of three rectangles to create six smaller square root of three rectangles.

Figure 275

Codex Vindobonensis Mexicanus Pages 25 and 35

Page 35

Page 25

Pages 25 and 35 of the Codex Vindobonensis Mexicanus are inscribed by two over square root of two rectangles that are subdivided by square root of two rectangles (shaded yellow) and square root of five rectangles (shaded blue), as per Formula Two Over Square Root of Three-3. The thin black lines are tentatively suggested subdivisions of the square root of two rectangles in half and the square root of five rectangles into squares and phi rectangles.

Figure 276

Codex Zouche-Nuttall, Pages 28 and 48

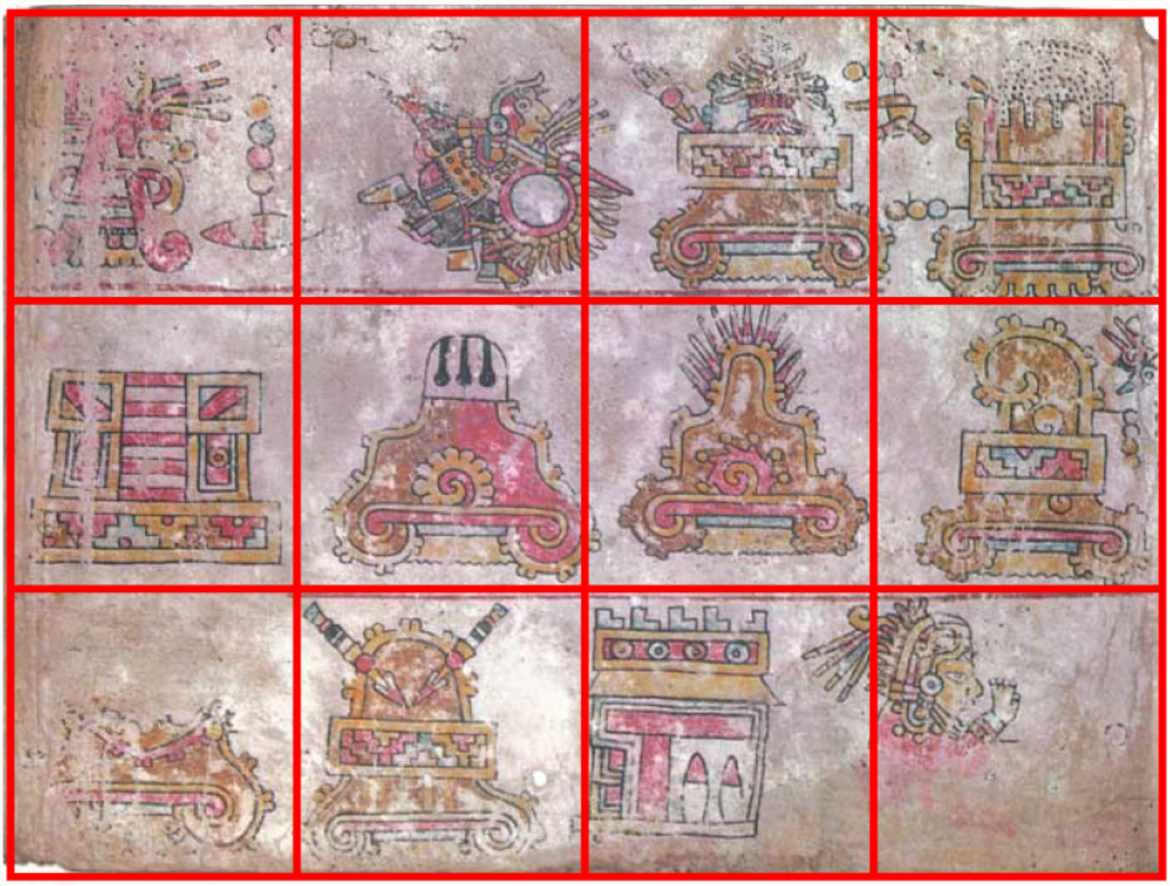
Page 28

Page 48

All of the pages of the Codex Zouche-Nuttall can be inscribed by Pythagorean 3,4,5 rectangles. The length of page 28 is quartered and its width is trisected to create twelve equal squares. The length of page 48 is trisected, creating three virtually perfect square root of five rectangles, as per Formula Pythagorean 3,4,5-5.

Figure 277

Codex Becker, Page 13

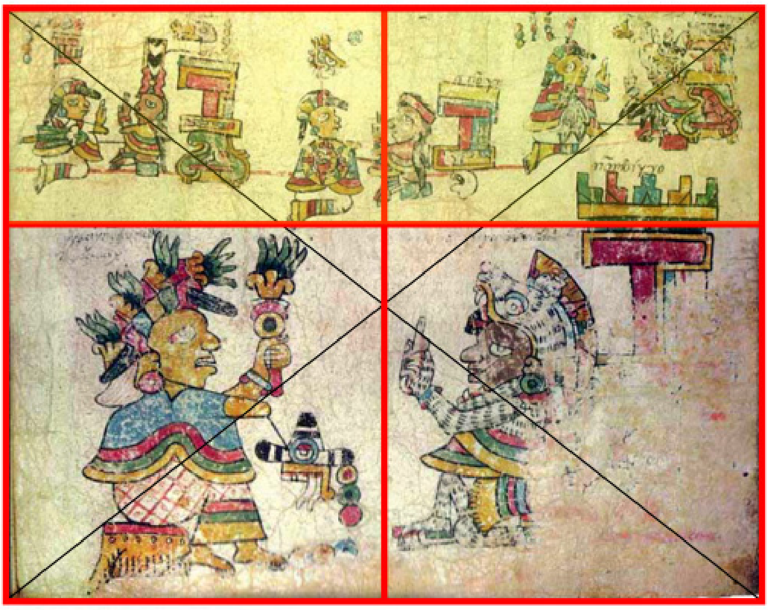


All of the pages of the Codex Becker can be inscribed by Pythagorean 3,4,5 rectangles, and on most of them the iconographic elements are arranged in groups of twelve that fit fairly neatly into twelve equal-sized squares that are created by trisecting the width of the Pythagorean rectangles and quartering the lengths, as shown on page 13 above.

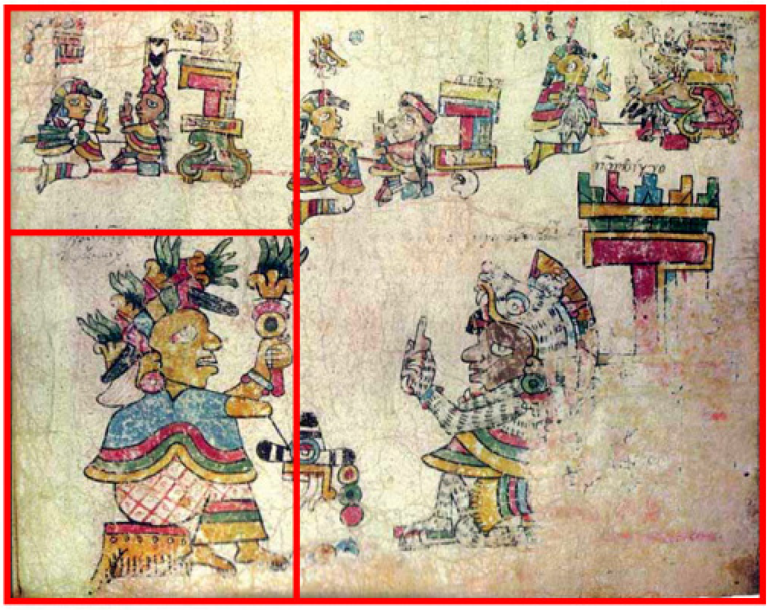
Figure 278

Codex Egerton, Pages 28

a



b

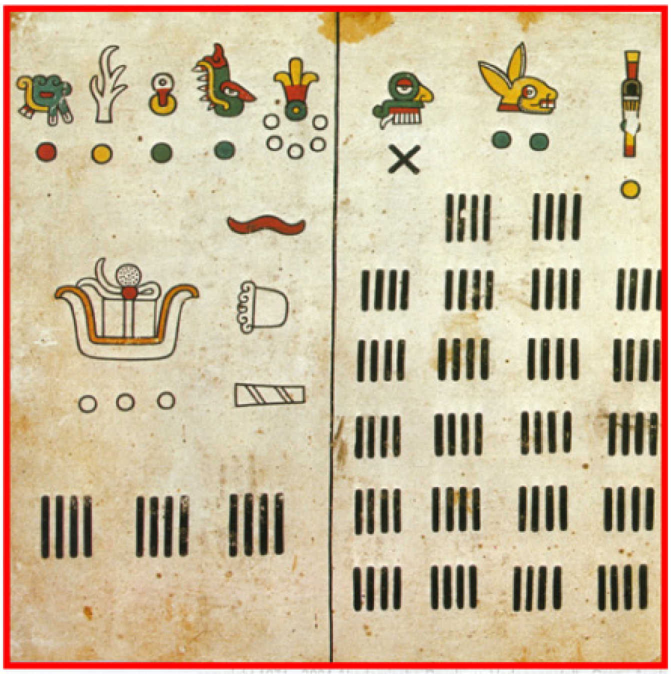


All of the pages of the Codex Egerton can be inscribed by a square root of phi rectangle. At the bottom of many of these pages, two principal seated figures occupy most of the space while smaller figures occupy the space near the top of the page. Diagram a is a suggested subdivision of the space by a square root of four rectangle, or two squares (unshaded) and two square root of three rectangles (shaded yellow), as per Formula Square Root of Phi-2. In diagram b, a second possible subdivision is suggested. Here, the initial rectangle is further subdivided into three smaller square root of phi rectangles, as per Formula Square Root of Phi-1.

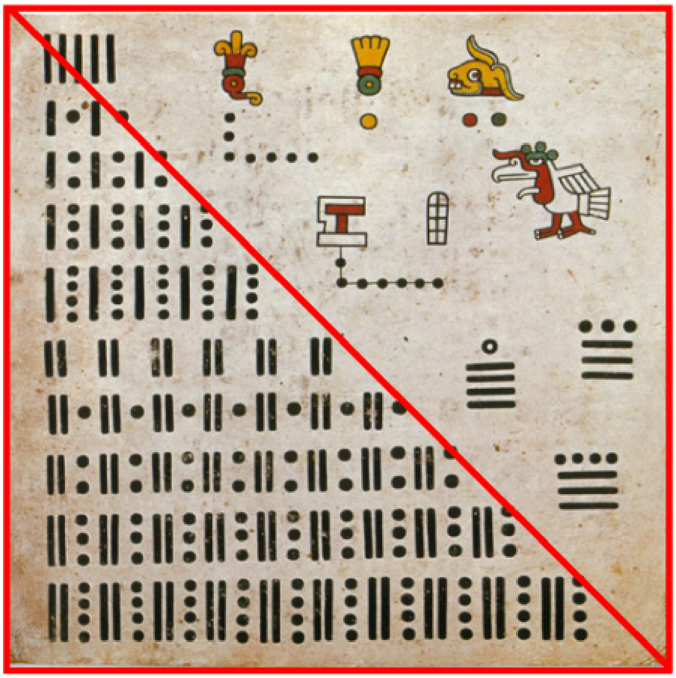
Figure 279

Codex Fejérváry-Mayer, Pages 16 and 20

Page 20



Page 16

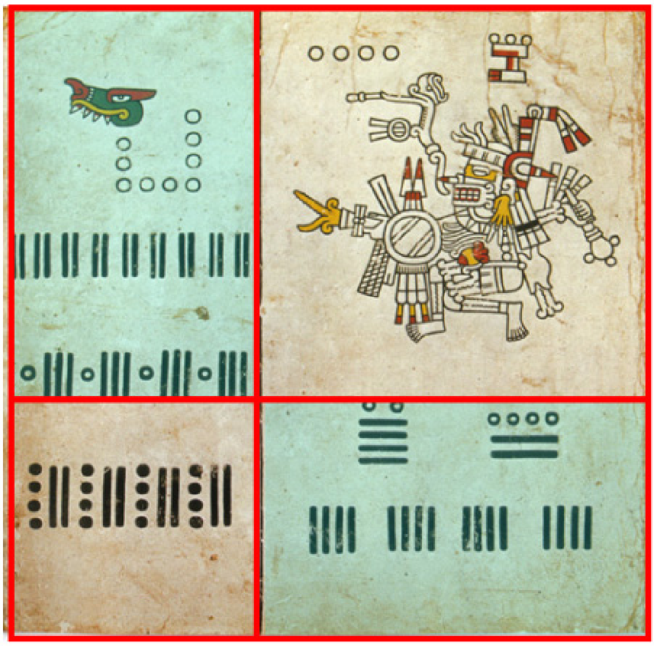


Page 20 of the Codex Fejérváry-Mayer is inscribed by a square that is bisected vertically, and Page 16 is bisected diagonally.

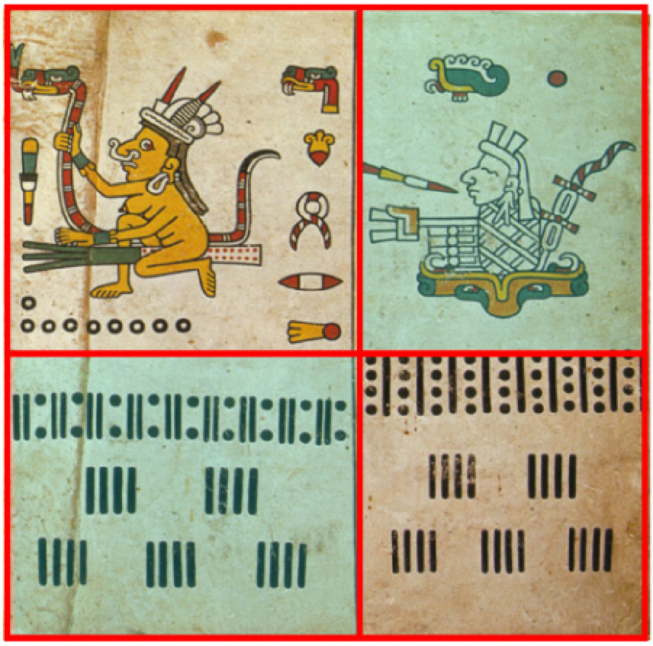
Figure 280

Codex Fejérváry-Mayer, Pages 17 and 18

Page 18



Page 17

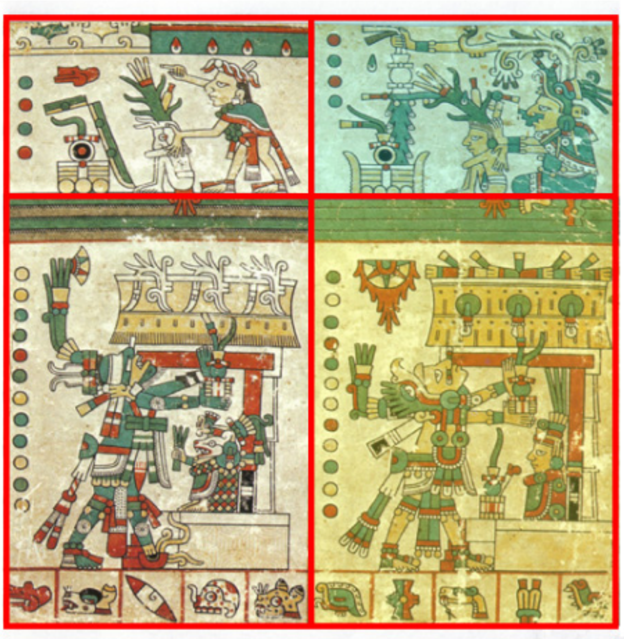


Page 18 is inscribed by a square the is subdivided into smaller squares and phi rectangles (shaded blue), as per Formula Square-2. Page 17 is subdivided by two overlapping square root of five rectangles. The smaller square in the lower right corner, when added to either of the two over phi rectangles (also shaded blue), produces square root of five rectangles.

Figure 281

Codex Fejérváry-Mayer, Pages 33 and 34

Page 34

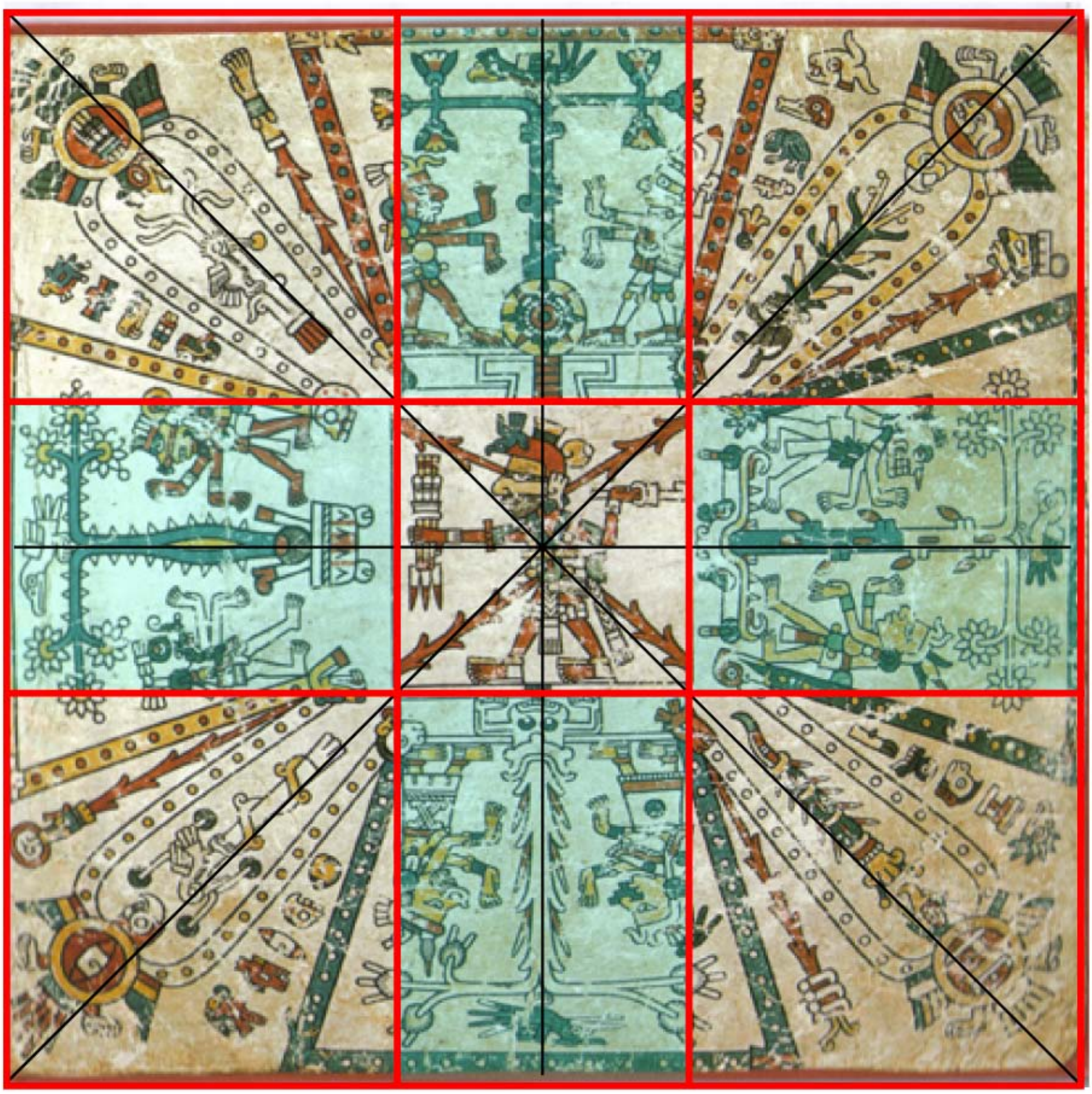


Page 33

Pages 33 and 34 are inscribed by squares. On page 34, the right half of the square is subdivided into a vertical square root of two rectangle (shaded yellow) and a horizontal square root of three rectangle (shaded blue), as per Formula Square Root of Four-4. Page 33 is identically subdivided, but the two vertical square root of two rectangles are highlighted to show that they form a single horizontal square root of two rectangle.

Figure 282

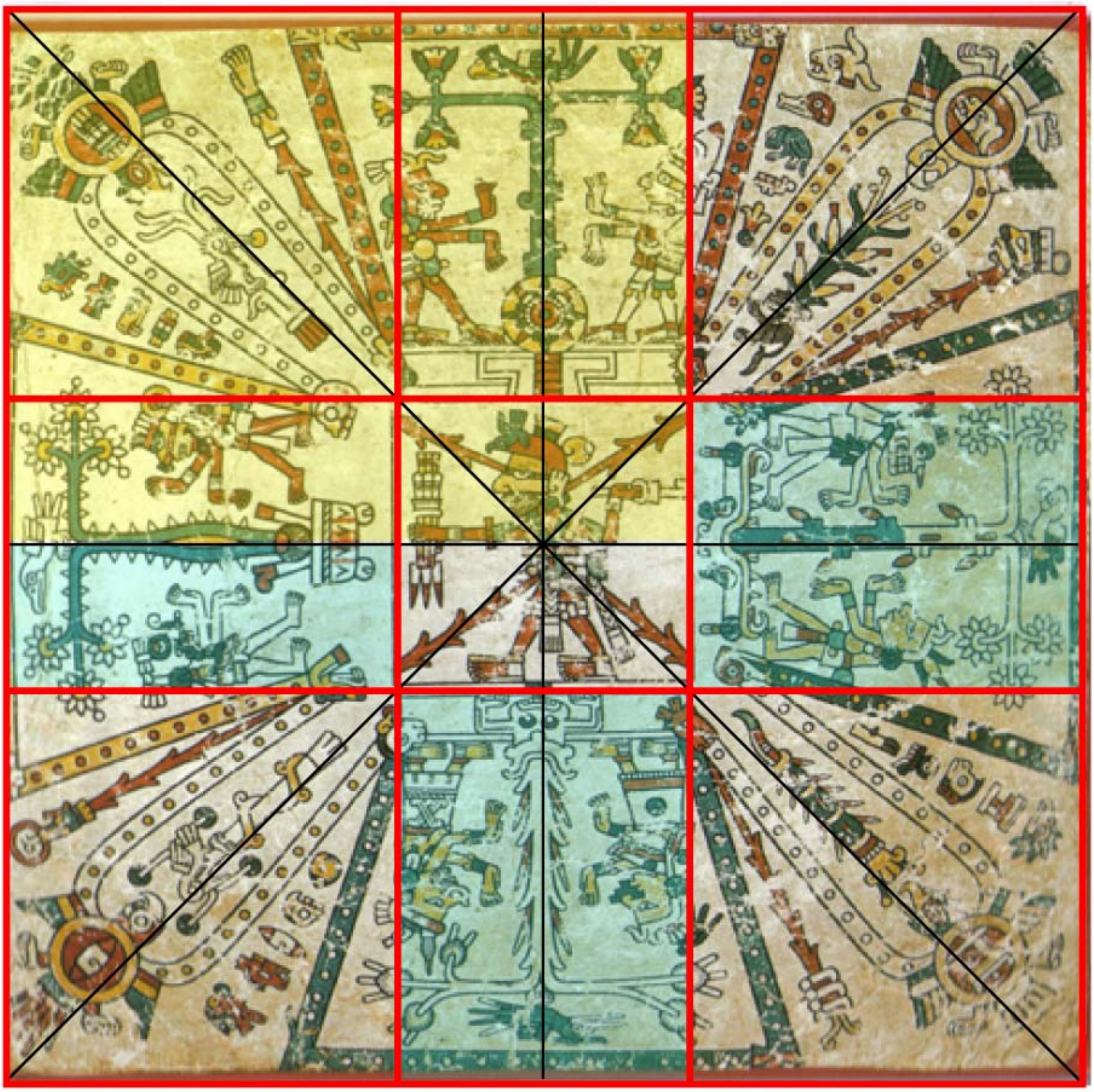
Codex Fejérváry-Mayer, Page 1 (a)



Page 1 of the Codex Fejérváry-Mayer is inscribed by a square, and bold red lines are drawn from the sides of this outer square to frame the central square that was drawn by the original artist to inscribe the Central Mexican god of fire and time, Xiuhtecuhtli, at the center of the composition. The resulting rectangles adjacent to each of the four sides of the central square (shaded blue) are Pythagorean 3,4,5 rectangles that frame the standing figures and trees adjacent to each side of the square.

Figure 283

Codex Fejérváry-Mayer, Page 1 (B)



Here, Page 1 of the Codex Fejérváry-Mayer is identically subdivided, but a virtually perfect square root of phi rectangle is highlighted in yellow. If the central square is arbitrarily assigned six units per side, then the yellow highlighted rectangle would be fourteen units long by eleven units wide, producing a virtually perfect square root of phi rectangle accurate to -0.0007 (seven ten thousandths of one percent), as per Formula Square Root of Phi-2.

Figure 284

Borgia Codex, Pages 34 and 33

Page 34

Page 33

All of the pages of the Borgia Codex may be inscribed by squares. Page 34 is simply a square quartered, and Page 33 is a square trisected into nine equal squares.

Figure 285

Borgia Codex, Pages 24 and 26

Page 24

Page 26

Pages 24 and 26 and both subdivided via variations of Formula Square-2. The square inscribing page 24 is subdivided into a phi rectangle (shaded blue) and the remaining rectangle is a phi squared rectangle. Page is a variation of this same formula with phi rectangles shaded blue and squares left un-shaded.

Figure 286

Borgia Codex, Page 23 (a)

Page 23 is also inscribed by a square, and bold red lines are drawn from the sides of this outer square to frame the central square that was drawn by the original artist to inscribe the skull and bones at the center of the composition. The resulting rectangles adjacent to each of the four sides of the central square (shaded yellow) can be viewed as paired square root of five rectangles (two over square root of five) that frame the day name symbols and seated figures adjacent to each side of the square. The blue shaded rectangle is a phi rectangle. Thus, the geometrical composition may also be viewed as paired phi rectangles that join at the center. If the width of the central square is one, then the width of the square that inscribes the page is phi times two.

Figure 287

Borgia Codex, Page 23 (b)

A second geometrical formula is also indicated for Page 23 of the Borgia Codex. Bold red lines are drawn from the sides of the square that inscribes this page to frame a central square that was drawn by the original artist to inscribe the 20 day-name symbols surrounding the skull and bones at the center of the composition. The resulting rectangles adjacent to each of the four sides of the central square (shaded blue) are phi rectangles, and if the width of the central square is one, then the width of the square that inscribes the page is square root of five. The yellow highlighted diagonal in one half of the central square is the radius for the yellow highlighted arc that determines the width of the adjacent phi rectangles via Formula Square Root of Five-2. Note that the hands of each of the blood-vomiting, star-farting figures at the extreme corners of the composition reach into the adjacent phi rectangles (shaded blue).

Figure 288

Cospi Codex, Pages 9 and 10

Page 9

Page 10

All of the pages of the Cospi Codex may be inscribed by squares. The fantastical, atlatl-wielding warriors on both pages, the wounded human figures on page 9, and the dart-pierced toponyms on page 10 are inscribed by two over root of three rectangles, which, in turn, inscribe equilateral triangles (thin black lines). The remaining space on the left of each page is occupied by a column of dates. The sides of the equilateral triangles are the radii for the arcs that define the widths and lengths of these columns. Note the tips of the atlatl darts at the interstices of the equilateral triangles.

CHAPTER 9

THE SQUARE ROOT AND PHI PROPORTIONS IN THE MAYA UNIVERSE

The Maya are, and were, keen observers of nature. In addition to monitoring the movements of the sun, moon, planets and constellations, they note changes in the weather, the seasonal cycles of flowering plants, and the migration patterns of birds and animals. Intimate knowledge of changes in their environment is the basis of their daily lives. The timing of the burning, planting, weeding and harvesting of corn and other crops, when to cut wood and harvest thatch, when and where to build a home, when and what to hunt and where, when and why to perform ceremonies: these activities are precisely orchestrated relative to their profound knowledge of their physical universe.

With this in mind, I pose a question: Where in the Maya universe might allegories for Maya geometry be observed? Answers to this question are not immediately obvious. You do not find square root and phi proportions in the shapes of the irregular coastlines and mountain ranges, in the shapes of meandering streams and rivers, or in the apparently chaotic brachiations of plants and trees in the dense forests of the Maya world.

But a profound clue was provided earlier, in the chapter on Maya vernacular architecture, by Don Guadelupe Lucio's response to my question, "*Que puede decir me sobre las flores*?" ("What can you tell me about flowers?") Remember that he began his response by explaining that flowers were very important to the Maya and that most Maya did not cultivate flowers but rather harvested wild flowers for use in specific ceremonies. He told me that the word "flowery" in Yucatec Maya meant "sacred." He said that the human soul was a white flower and that one's ancestors and offspring were called flowers. He concluded by saying that his grandfather, who was also a shaman and who had taught him how to use a measuring cord, had told him, "*Las formas de las flores son dentro nuestros casas*." ("The shapes of the flowers are in our houses").

In the first two diagrams of this dissertation (Figures 1 and 2) I illustrated how the square root and phi proportions are rectangular expressions of the proportions inherent in the equilateral triangle, the square, and the pentagon. Most flowers have three, four, or five petals or multiples of three, four, and five petals. In fact, botanists categorizes flowers into four basic categories: Three Somes, Four Somes, and Five Somes (flowers whose petals are arranged in multiples of threes, fours, and fives), and the much less common Irregulars, which are usually bi-petaled (mostly found in the flowers of legumes) or cone-shaped (mostly found in the petals of a few species of lilies). Thus, the set of proportions seen in most flowers is the same set of proportions utilized to design Maya art and architecture. Nowhere in nature are these proportions more clearly and abundantly expressed than in the shapes of flowers (Figure 289).

In Maya and Teotihuacan art from the Classic Period, flowers are often paired with sea shells in sacred contexts. These sea shells are usually cut in half to show a cross section of the equiangular phi spiral within. A flower and a cut shell are almost always at either side of the stingray spine in the headdress of the Quadripartite Monster that often rests at the base of the world cross. The world cross, and crosses in general (for example, those held in the hands of kings in the art of Yaxchilan), are frequently depicted with flowers at their extremities. Modern “Christian” crosses in the Maya highlands invariably display carved or painted flowers at their extremities. I have often seen these modern crosses adorned with fresh flowers. Churches, too, prominently display geometrically stylized flower iconography (Figures 290-292).

Flowers and shells often hang or float in the background of sacred scenes, particularly in the art at Palenque (Figure 293). Also at Palenque, the exterior facade of House E, where coronation ceremonies were performed, was adorned with a painted mural of geometrically stylized flowers (Figure 294). The murals at Teotihuacan are often framed by alternating flower and shell motifs, and speech scrolls or spirals are also often adorned with flowers and shells (Figure 295).

Ear flares, worn ubiquitously by Maya nobles and royalty, are flowers made of of jade. The flower ear flares of the gods are almost always surmounted by cut shells; cut shells are also worn as diadems and necklaces by gods and kings. The Maya Gods K and N, as well as human figures, are sometimes depicted as emerging form sea shells. Ancestors are portrayed emerging from flowers and flowering trees (Figures 296-304)

Figure 289

Flowers

Three some, four some, and five some flowers (flowers with multiples of three, four, and five petals).

Figure 290

Flowered and Foliated Crosses

Flowered and foliated crosses in the Highlands of Chiapas

Figure 291

Flower Iconography and Churches 1

Flowers and geometrically stylized flowers on the facade of the church at San Juan Chimula, Chiapas, Mexico.

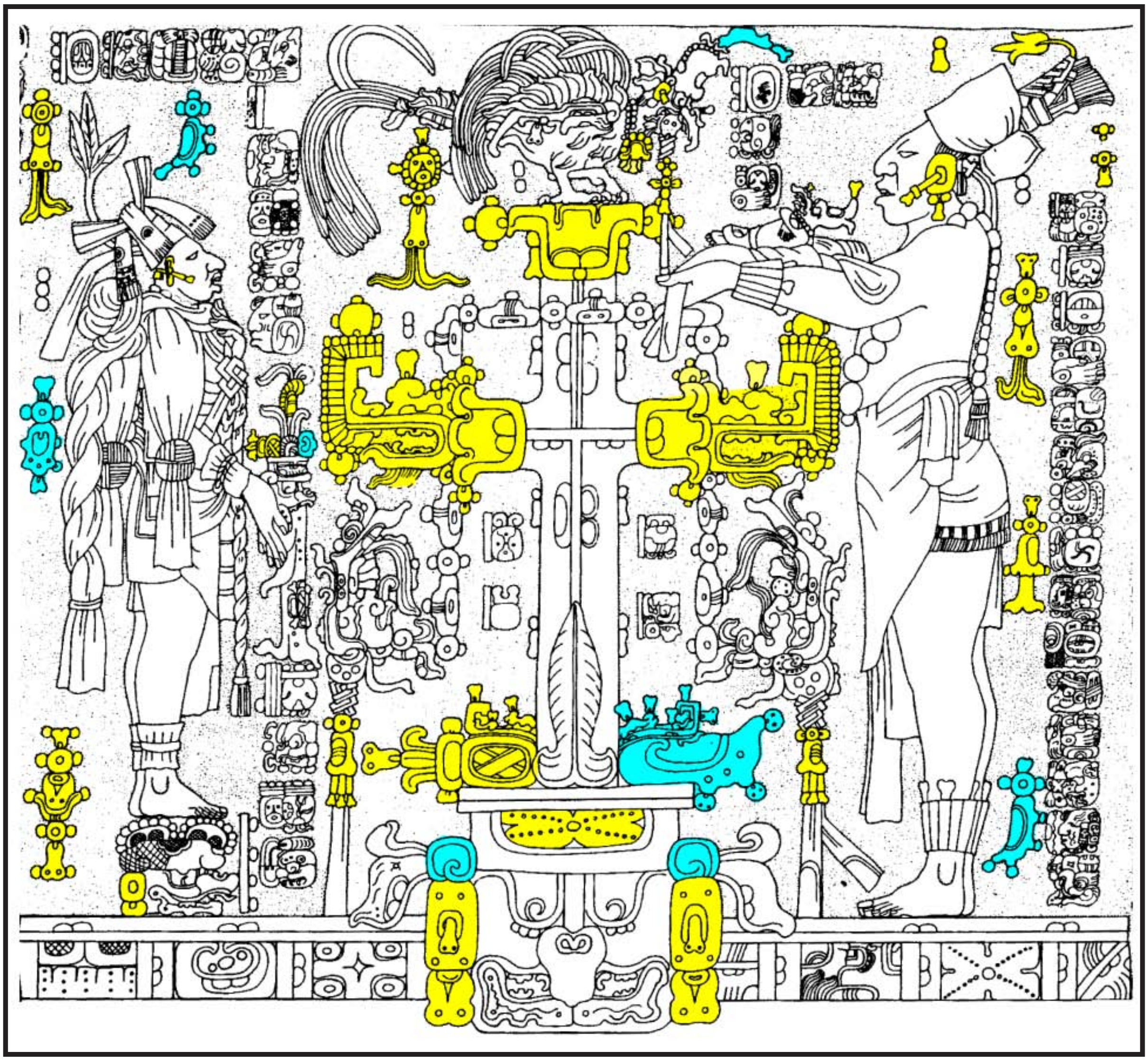
Figure 292

Flower Iconography and Churches 2

Geometrically stylized flowers on the facade of a church in Yucatan, Mexico.

Figure 293

The Cross Tablet, Temple of the Cross, Palenque, Chiapas, Mexico



Flowers, “bone seeds”, and roots are shaded yellow and shells are shaded blue.

Figure 294

Facade of House E, Palenque, Chiapas, Mexico

Geometrically stylized flowers cover the front wall of House E in the Palace at Palenque.

Figure 295

Teotihuacan Mural, (Tepantitla, Room 2, Mural 3)

Shells are on the interior and flowers are hanging from the exterior of the spirals emerging from the hands of the standing figure.

Figure 296
Jade Ear Flares

Round, four-sided, five-sided, and six-sided jade flower ear flares.

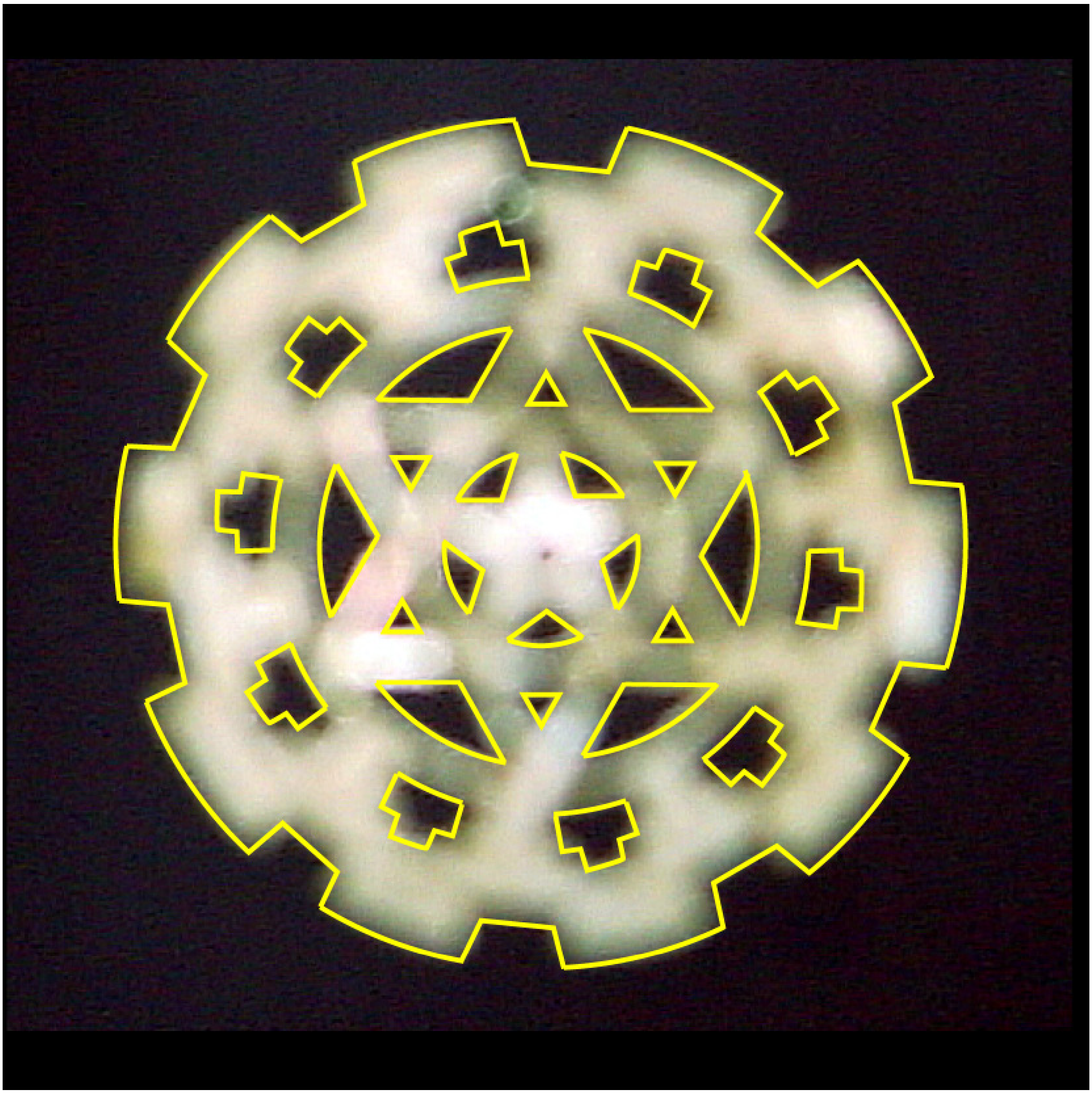
Figure 297

Painted Maya Vase (Provenience Unknown)

This painted vase shows a seated figure with a jeweled pentagram ear flare instead of the usual jade flower ear flare. A fish is nibbling at a waterlily flower in the figure’s headdress.

Figure 298

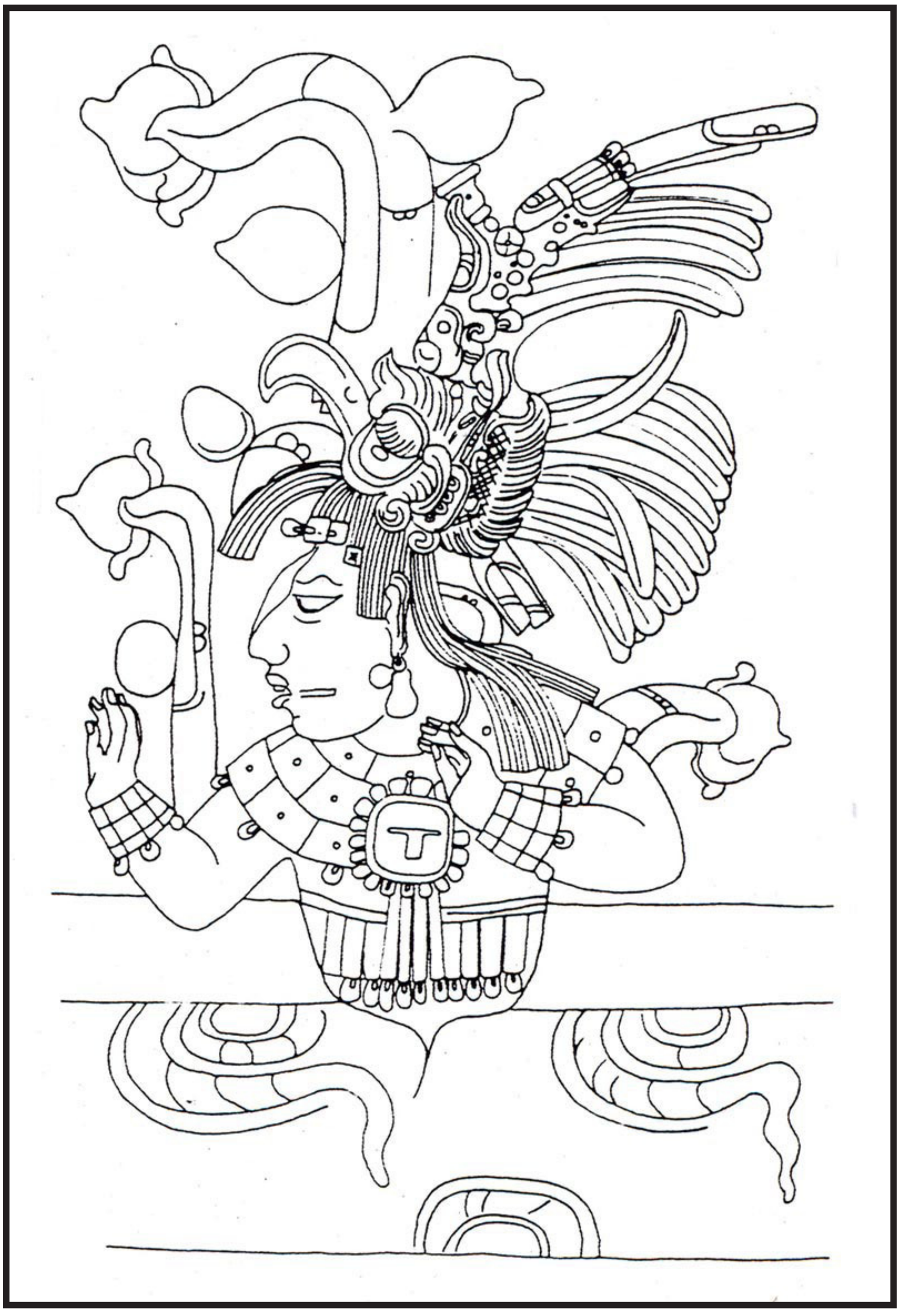
Shell Jewel (Provenience Unkown)



This is a blurry photo of an exquisitely carved shell jewel, about ten centimeters in diameter, that I took through a glass case in the Anthropology Museum in Guatemala City. Because the photograph is blurred, I outlined it. The decagonal exterior ring encloses opposing equilateral triangles that inscribe a pentagram.

Figure 299

Detail of Pacal's Sarcophagus, Temple of the Inscriptions,
Palenque, Chiapas



Ancestors are depicted as flowering trees on Pacal's sarcophagus.

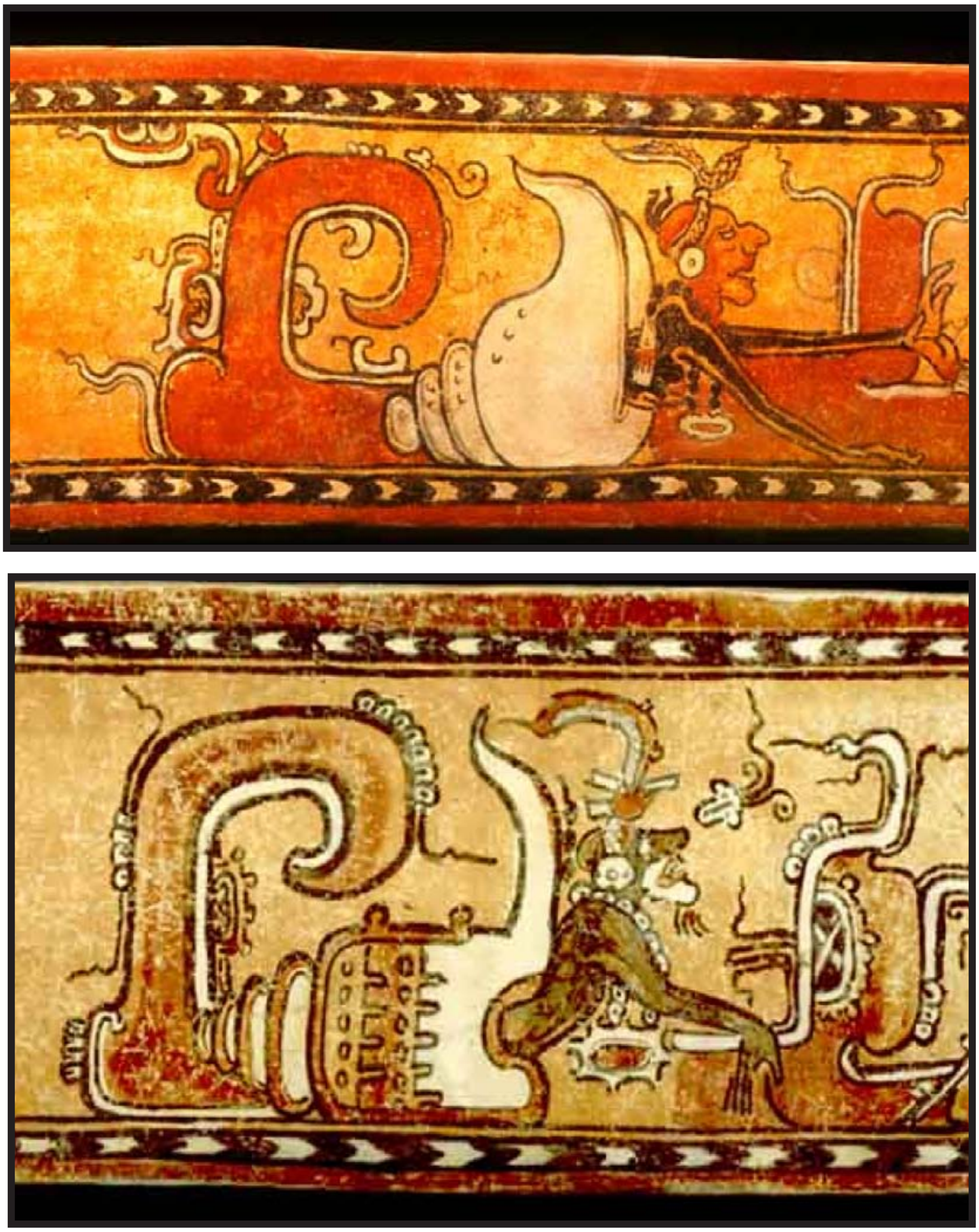
Figure 300

Humans Emerging from Flowers (Provenience Unknown)

These are a few of many examples of ceramic depictions of humans (possibly ancestors) emerging from flowers.

Figure 301

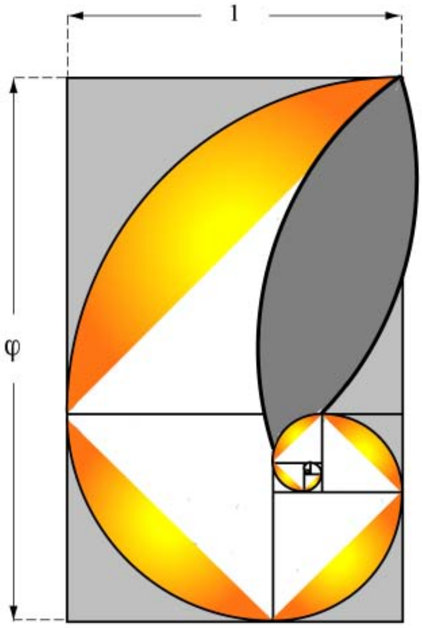
Two Painted Vases (Provenience Unknown)



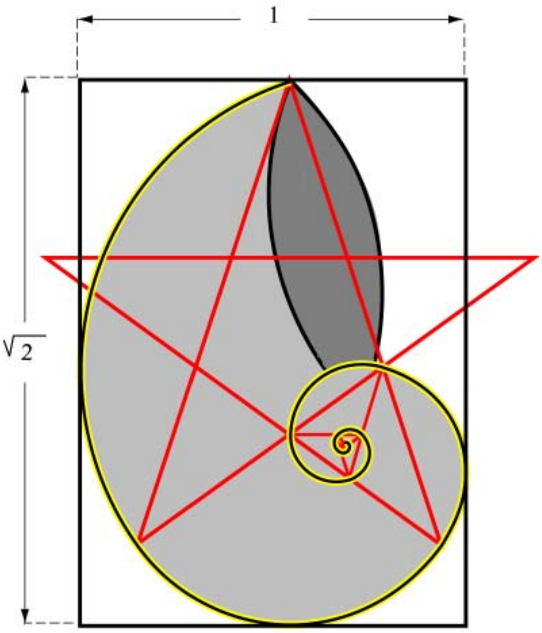
These painted vases depict God N emerging from sea shells.

Figure 302

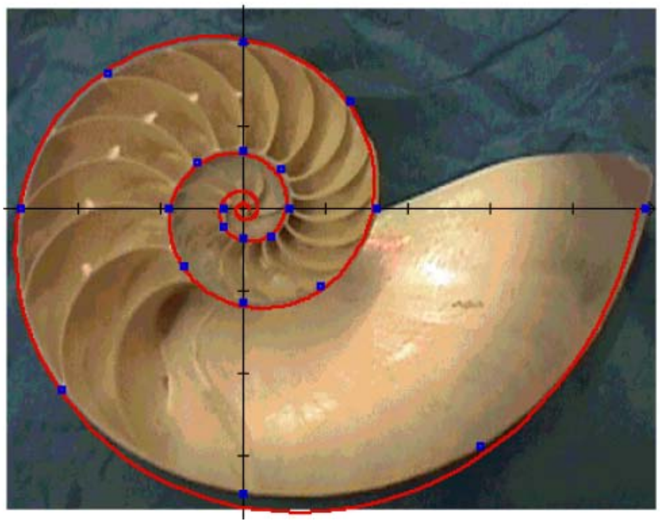
The Geometry of Sea Shells



a



b



c

Diagram a represents a sea shell with an equiangular spiral that inscribes a phi rectangle. Diagram b is a drawing of a sea shell with an equiangular spiral that is derived from a pentagram and is inscribed by a root two rectangle. Diagram c is a photograph of a sea shell with an equiangular spiral where each 180-degree turn has a ratio of one to phi relative to the previous 180-degree turn.

Figure 303

Cut Shell Jewel (Provenience Unknown)

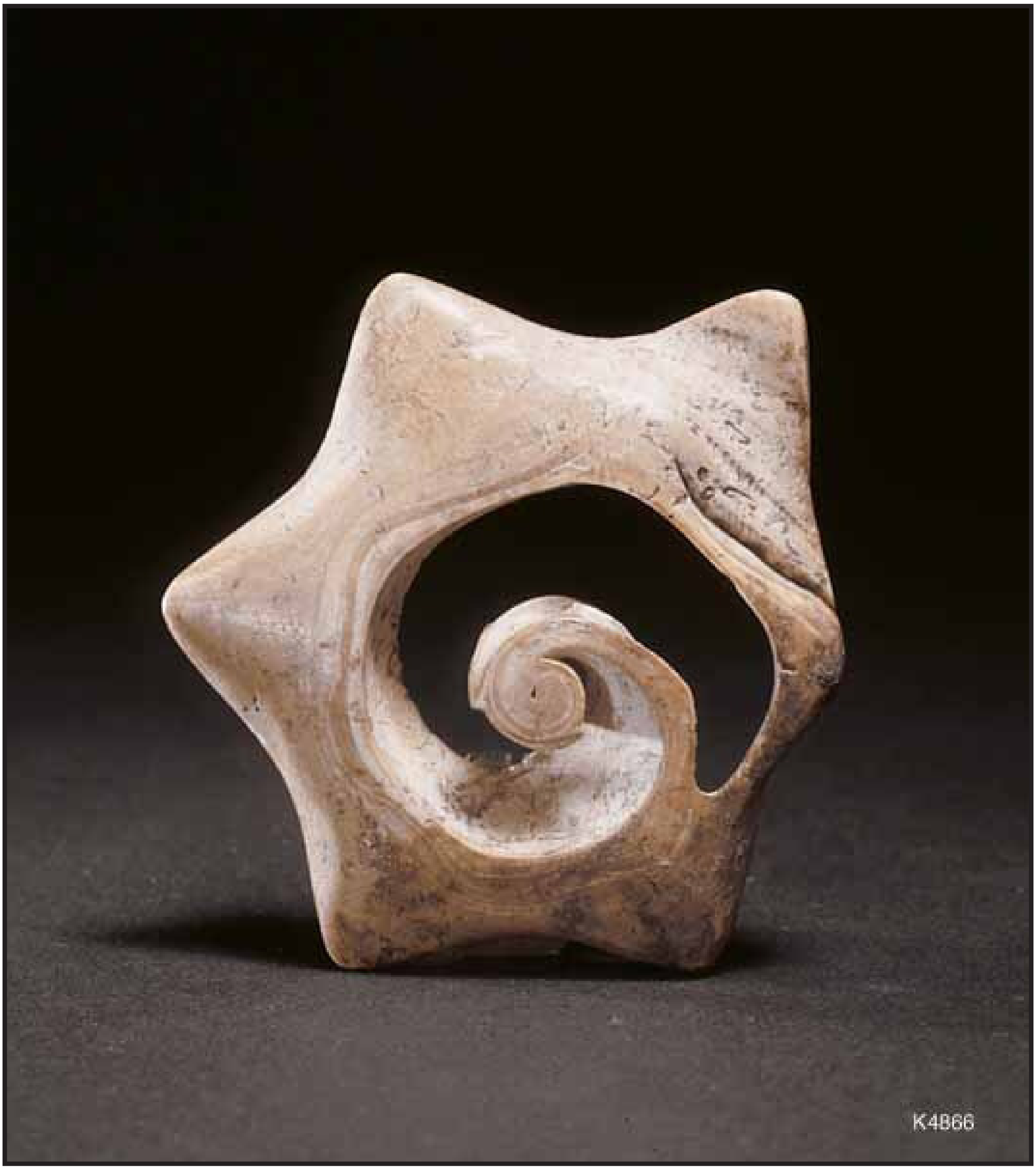
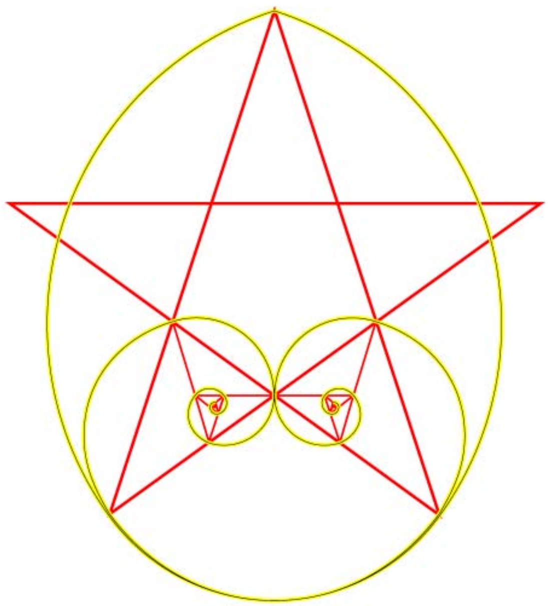
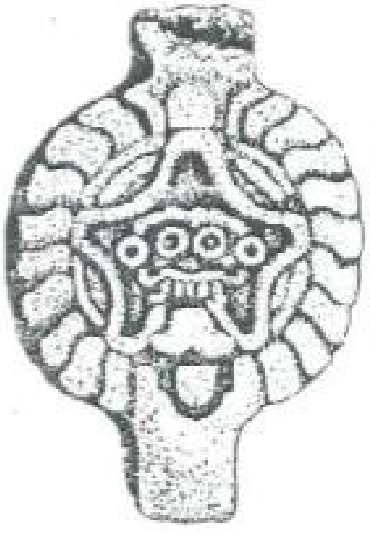


Figure 304

The Bifurcated Spiral of a Pentagram



a



b

c

Diagram a depicts opposing equiangular spirals generated from a pentagram. The spirals bifurcate at the base of the pentagram. Diagram b is a drawing of a stone carving of the god Tlaloc in a pentagram from Teotihuacan. Note the bifurcated tongue at the base of the pentagram. Diagram c is a Maya stamp of a pentagonal flower with a bifurcated spiral emerging from its base.

Figure 305

Tlaloc and the Pentagram

a

b

Diagram a is a detail from a lintel at Yaxchilan. Note the pentagram in the eye of the Tlaloc headdress.
Diagram b is a mural painting of Tlaloc in a pentagram with a bifurcated tongue emerging from its base.

Flowers and shells are not the only expressions of the square root and phi proportions to be found in the Maya universe. A Maya interested in observing these forms in nature might begin by noting the shapes and proportions inherent in his or her own body. The width of most Maya vernacular houses in the Yucatan Peninsula consists of two units called uinics ("humans"), which are measured by stretching a cord from fingertip to fingertip, with arms outstretched and perpendicular to the body. The Maya shamans and house builders whom I worked with knew that one uinic was also virtually equal to the height of the person who was measuring it. Thus, a human being with arms outstretched and perpendicular to the body may be inscribed by a square.

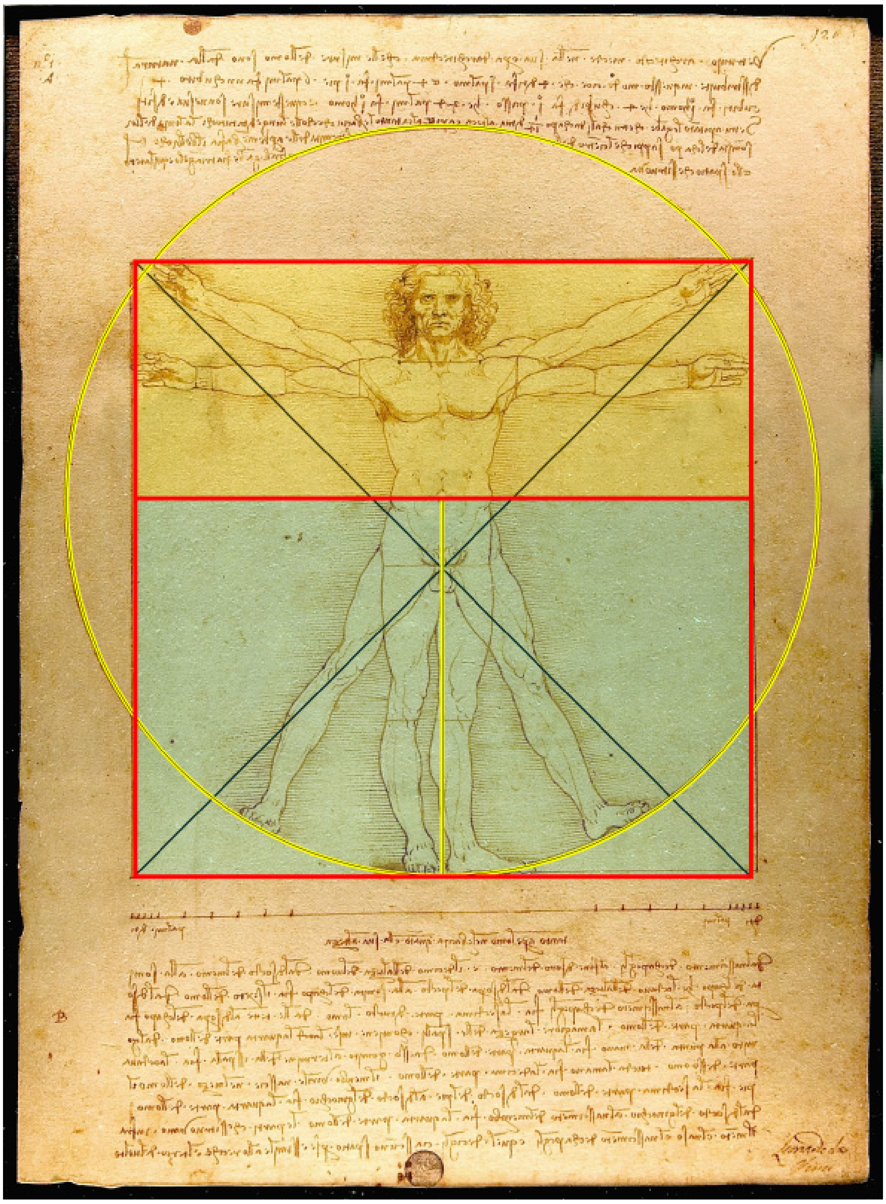
The center of this square occurs at the pubic bone or genital area, and the phi division of this square (as per Formula Square-2) happens to occur at the height of the navel. (I have measured dozens of people to check the accuracy of these observations and have found them to be accurate to within a few centimeters).

In the Yucatec dialect the word for navel is "tzuk," which is also translated as "division place." (The God C or "Tzuk" head also demarcates the intersections of the crosses depicted in the carved stone panels of the Cross Group and the sarcophagus lid of Pacal at Palenque). Your navel is also where you were once attached to your mother by a cord, so to speak, and when some of the Maya I have known in Chiapas ask, “Where are you from?” they often phrase it as, “Where is your umbilicus buried?”

If the Maya did observe these proportions in the human form, they were not the only people to do so. Though it is beyond the scope of this dissertation to thoroughly examine the history and use of these proportions in non-Maya cultures, an examination of Leonardo da Vinci’s drawing, “The Vitruvian Man,” is presented to help illustrate how these proportions relate to the human body (see figure 306).

Figure 306

"Vitruvian Man"
Leonardo da Vinci, 1492



The square (in red lines) and the yellow highlighted circle are superimposed over the original square and circle drawn by da Vinci. I have added to the drawing the diagonals of the square (black lines) that center on the genital area of the human form; the horizontal division at the navel that divides the square (via Formula Square-2) into a phi rectangle (shaded blue) and a phi-squared rectangle (shaded yellow); and the radius of the circle (yellow highlighted line from the navel to the feet) that bisects the phi rectangle.

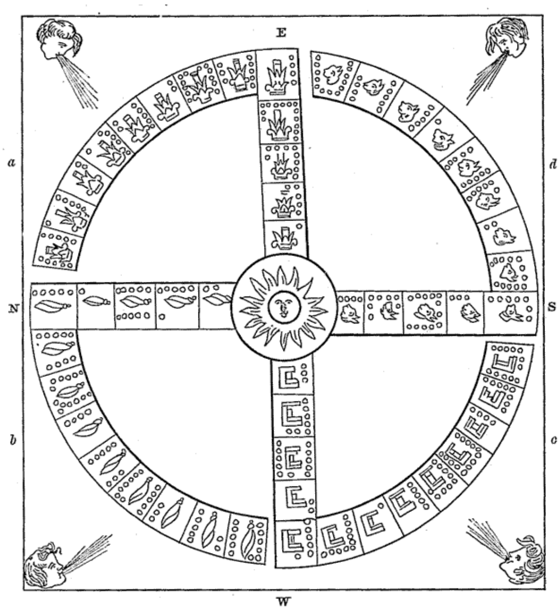
The ancient Maya may also have been interested in the geometry of time. When the Spaniards arrived in the Maya region, various sixteenth-century chroniclers reported that the Maya used “Katun Wheels,” or circular calendars, to count the passage of Katuns. Katun wheels are also mentioned and drawn in the Bacab manuscripts (Figure 307). The large body of information written about Katun wheels is beyond the scope of the simple observations I will present here. Suffice it to say that the Maya often recorded time in circular forms. The Katun stones, carved and set in the plazas of Classic Maya sites, are also generally circular.

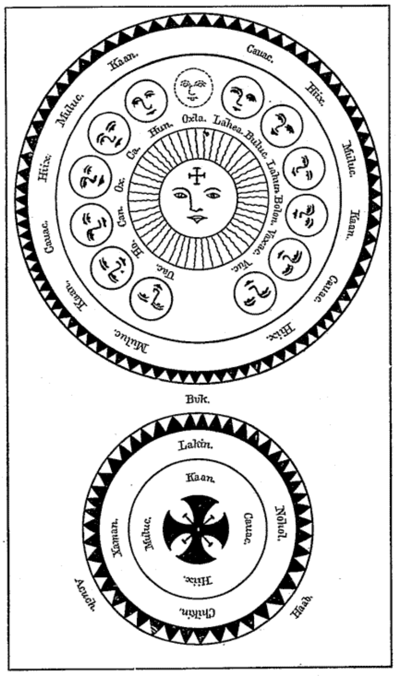
The observations diagramed in Figure 308 depend on the hypothetical arrangement of the 260-day Tzolkin into a circle of days. Roughly the first third of the Dresden Codex is concerned with prognostications assigned to fifty-two and sixty-five-day almanacs, along with longer almanacs that are subdivided into multiples of these. Several similar examples are also found in the Madrid and Paris Codices. If the Tzolkin is arranged as a circle of days and the completions of these fifty-two and sixty-five-day almanacs are plotted on this circle and straight lines are drawn to connect them, then a pentagon and a square are created respectively. If the lunar nodes are similarly plotted on this circle of time, then an equilateral triangle is produced. The days at which lunar and solar eclipses might occur (shown as red dots in Figure 308), some eighteen days to either side the lunar nodes, would also trisect the Tzolkin circle. Only a 260-day circle (or a 520-day circle that equals 2 x 260 and is redundant) would produce this pattern and grouping of lunar nodes and possible eclipse days. Plotting lunar nodes and possible eclipse days on a circle of 365 days or 360 days, for example, would produce apparently random and overlapping groupings of possible eclipse days.

The triangle, square, and pentagon represent the smallest set of polygons that express all of the square root (square root of one through five) and phi proportions that were and are apparently used by the Maya. The square and its diagonal express the square root of one and two proportions, the equilateral triangle expresses the square root of three proportion, and the pentagon expresses the phi proportion (see Figure 3). The square root of four rectangle may be viewed as a double square, and a square root of five rectangle may be viewed as a square and two phi rectangles. Thus, in a sense, they are redundant.

There are other calendrical and astronomical cycles that, if arranged as circles, would produce equilateral triangles, squares, and pentagons. One of them is the Venus-Solar period of 2,920 days (the least common multiple of the 365-day Haab and the 584-day Venus synodic period). If the number of days in this cycle is arranged as a circle, the Venus synodic periods would plot a pentagram and the Haabs would plot out an octagon. And, if a 360 day Tun were arranged as a circle, the equilateral triangle, the square, and the pentagon, their subdivisions, and multiples of these polygons, would all divide evenly. The circle is, and has been for thousands of years, almost universally subdivided by 360 degrees precisely because 360 is the least common multiple of these fundamental polygons and their subdivisions.

Figure 307
Katun Wheels



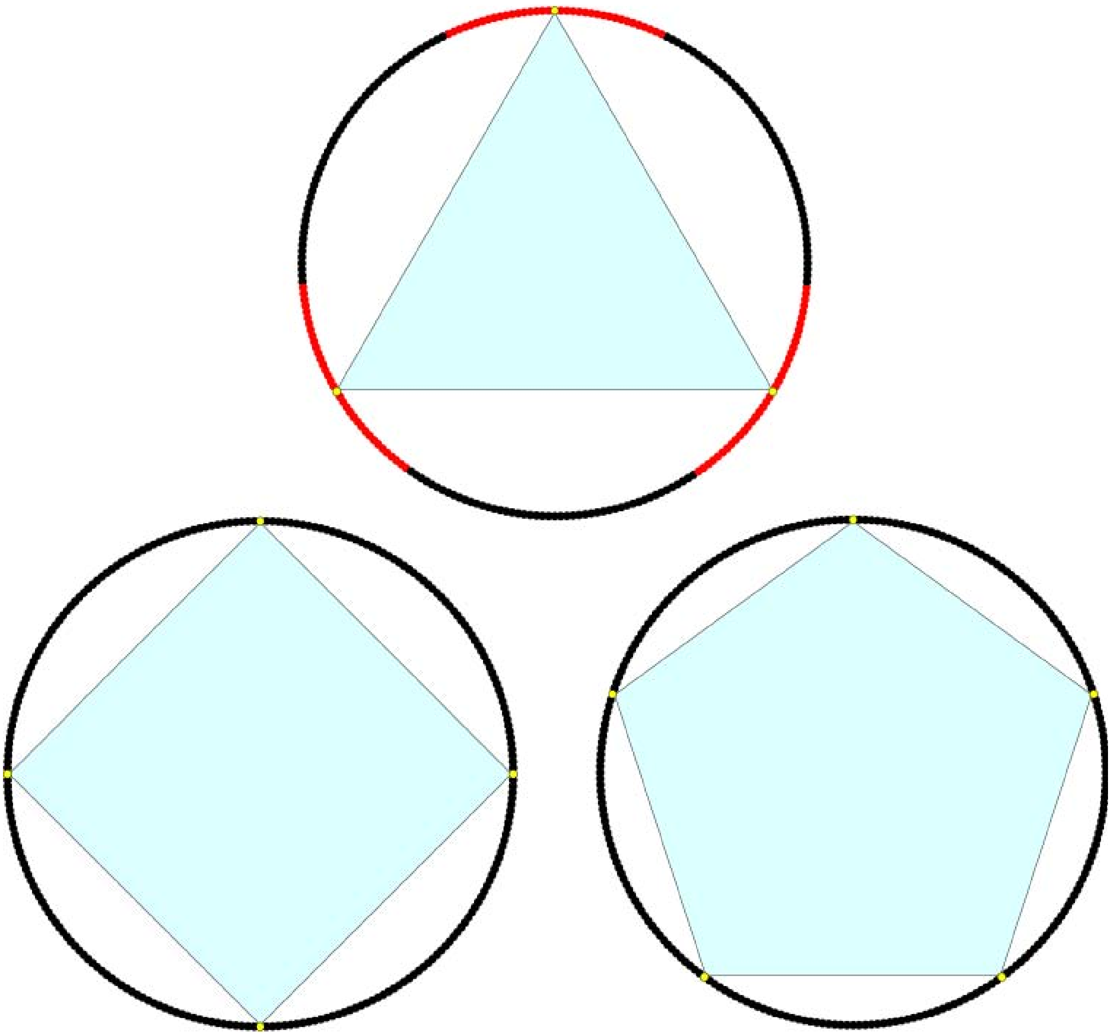


Three examples of “Katun Wheels” from Contact Period historical texts

Figure 308

Tzolkin Circle Divisions

Tzolkin Divided by Lunar Nodes



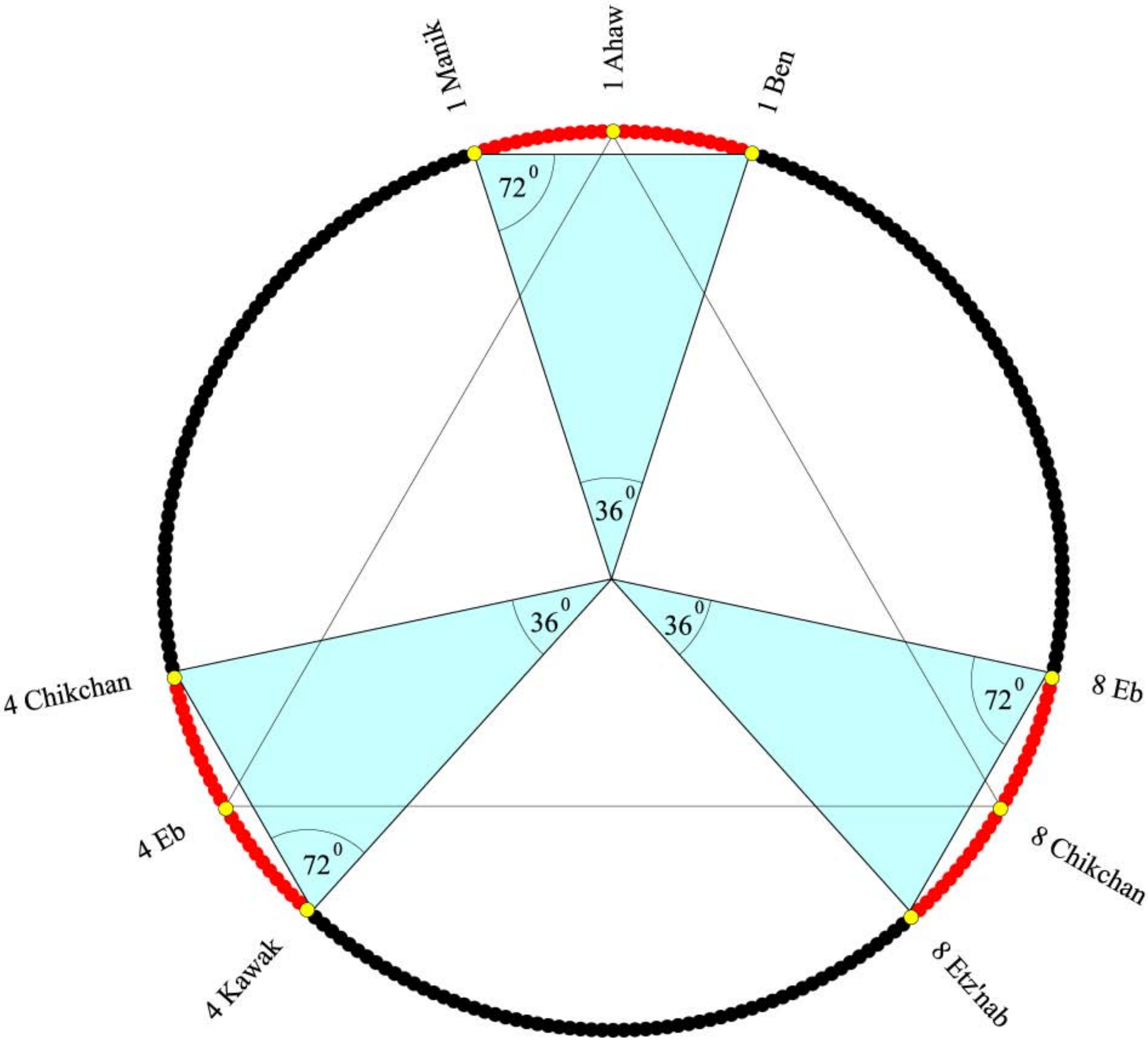
Tzolkin Divided by 65-Day Almanacs

Tzolkin Divided by 52-Day Almanacs

A Tzolkin circle divided by 173 1/3-day lunar nodes and sixty-five and fifty-two day almanacs produces an equilateral triangle, a square, and a pentagon, respectively.

Figure 309

Tzolkin Circle and Possible Lunar Eclipse Days

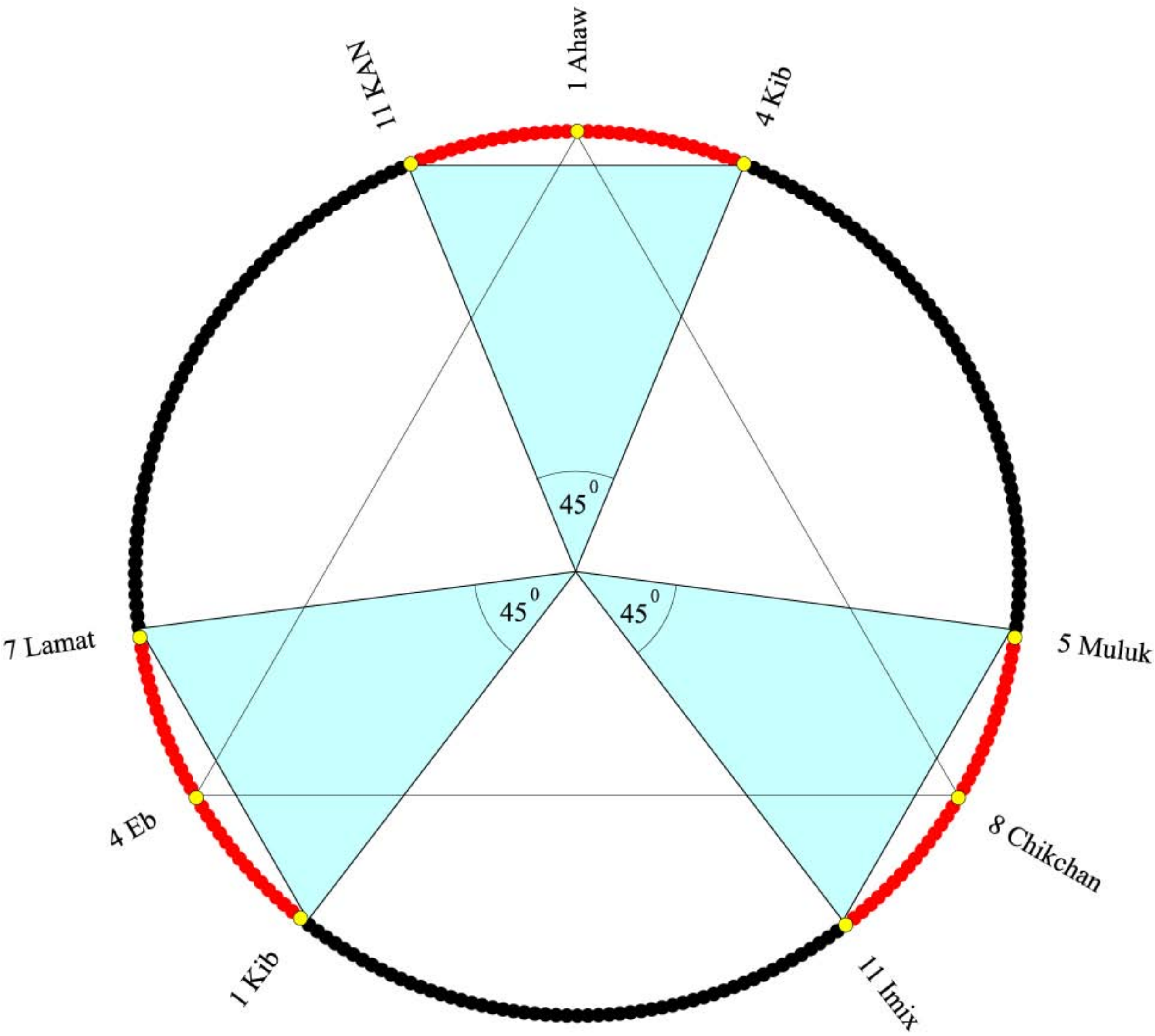


This Tzolkin circle of two hundred and sixty days shows the equidistantly spaced lunar nodes (1 Ahaw, 8 Chikchan and 4 Eb) at the vertices of an equilateral triangle. The twenty-six lunar eclipse possible days centered on the nodes (red dots) produce precise phi (or pentagram) triangles. The ratio of the base of these triangles to their slopes is one to phi.

On the 182 days represented by the black dots, lunar eclipses can not occur.

Figure 310

Tzolkin Circle and Possible Solar Eclipse Days



This Tzolkin Circle of 260 days show the equidistantly spaced lunar nodes (1 Ahaw, 8 Chikchan, and 4 Eb) at the vertices of an equilateral triangle. The thirty-six possible solar eclipse days centered on the nodes (red dots) produce nearly perfect forty-five degree triangles (the diagonal of a square).
On the 152 days represented by the black dots, solar eclipses cannot occur.

Stone henges, wood henges, and other ancient devices used to demarcate the rising and setting positions of the sun, the moon, and the planets are almost always arranged as arcs or circles. The most logical reason is that the easiest ways to obtain these data would produce circular instruments. I can think of no simpler method than to first find or make a flat surface with an unobstructed view to the east and west and to place a forked stick at eye-level height near the center of this surface. Next, attach a similar forked stick to this center post with a cord or string. This second forked stick would be somewhat shorter that the center stick; its height determined so that when the cord between them is pulled taut, a straight line between the crooks of the forks would terminate at the horizon. With one person at the center post and another pulling the second forked stick until the cord is taut, the rising or setting of the sun or moon on the horizon could be lined up between the crooks of the V- shaped forked sticks. The person at the center stick would direct his or her partner to move to the north or south until the alignment was obtained. A simple plumb bob attached with a string to the shaft of the shorter stick could be used to maintain a perpendicular position relative to the ground and thus maintain a consistent distance from the center stick while marking or staking the position of the alignment on the cleared ground surface.

After a year of measuring the rising and setting positions of the sun in this manner, the solstice positions would be defined at the ends of two arcs. To determine due east and west would be a simple matter of stretching a string or cord between the staked solstitial positions, halving the cord, marking its center, re-stretching the cord and stretching a second cord from the center post across this marked center point to a point on the arc of rising and setting positions. Turning a right angle at the center of the east/west line would determine due north and south. Using the original forked sticks and taut cord to connect the arcs would form a circle.

If Maya astronomers were using devices similar to the one described above to track the rising and setting positions of the sun on solstice, equinox, and zenith passages, and perhaps the rising and setting positions of the full moon, they would have produced easily recognizable polygonal expressions of the square roots of two and three and the phi proportions.

The observation circle seen in Figure 311 would be accurate to within a fraction of a degree throughout Maya latitudes. From the center post, the solstices rise and set at twenty-five degrees to either side of east and west, with a span of fifty degrees. The full moon rises and sets at the lunar standstills at thirty degrees to either side of east and west, with a span of sixty degrees, creating equilateral triangles (shaded blue). The maximum rising and setting positions of the five visible planets are bounded by the solstices and the lunar standstills. Within the observation circle the cardinal directions, north, south, east and west, inscribe a square; the lunar standstill positions and due north and south inscribe a hexagon.

The square, the equilateral triangles, and the hexagon are polygons that express the square root of two and three proportions. These polygons would be common to and fairly obvious in observation circles throughout Mesoamerica. What is missing here to complete the set of polygonal expressions of the square root and phi proportions in a Maya observation circle is the pentagon or phi proportion.

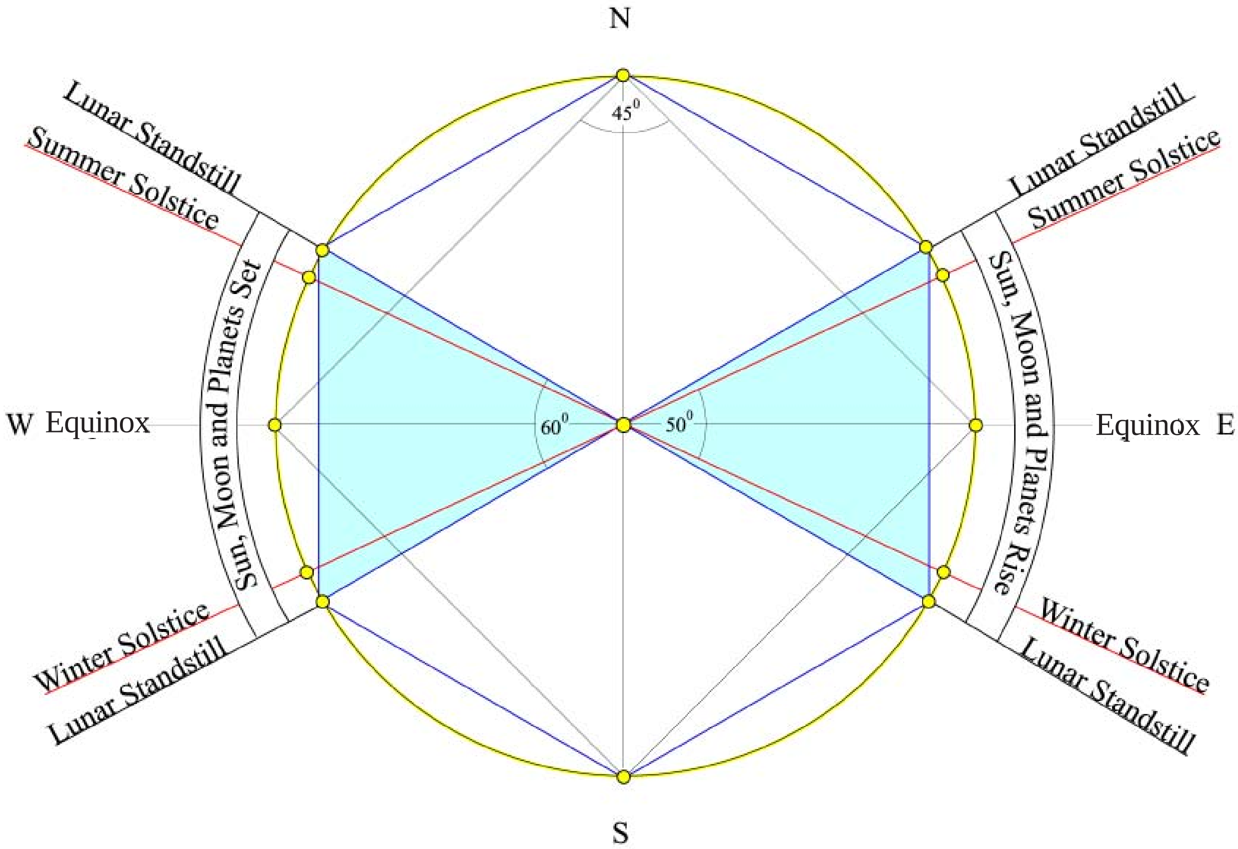
While conducting archaeo-astronomical investigations at Palenque with my friends and colleagues of the Maya Exploration Center (Dr. Ed Barnhart, Alonso Mendez, Carol Karasik and Moises Morales), we noticed that the rising and setting positions of the sun on the days of the zenith and nadir passages occur within a very small fraction of a degree, at eighteen degrees to either side of east and west, producing a span of thirty-six degrees. If these positions were demarcated with pegs on an observation circle and a string were attached between these two points and the center post, a pentagram or phi triangle would be produced. If the base of this triangle is one, then its slope is phi. Moreover, if these rising and setting positions of the zenith and nadir positions are then connected with strings to the north and south positions on the observation circle and back to the center post, four pentagon triangles of seventy-two degrees are created (see Figure 312).

After grasping the basic geometry inherent in these zenith and nadir positions at Palenque, I noted that Tikal and its sister site, Uaxactun, shared virtually the same latitude as Palenque. Thus, they would share the same pentagon and phi proportions relative to the rising and setting positions of the sun on the days of zenith and nadir passages.

I then considered the rising and setting positions of the sun at zenith and nadir passages at Copan and noticed that they occurred, again to within a small fraction of a degree, at fifteen degrees to either side of east and west. These positions on an observation circle, along with the due east and west positions, subdivide the equilateral triangles created by the lunar standstill positions into four evenly spaced positions of fifteen degrees each. Forty-five degrees (the diagonal of a square) separate the zenith and nadir positions and the lunar standstill positions (see Figure 313).

Figure 311

An Astronomical Observation Circle
at the Latitudes of the Maya World



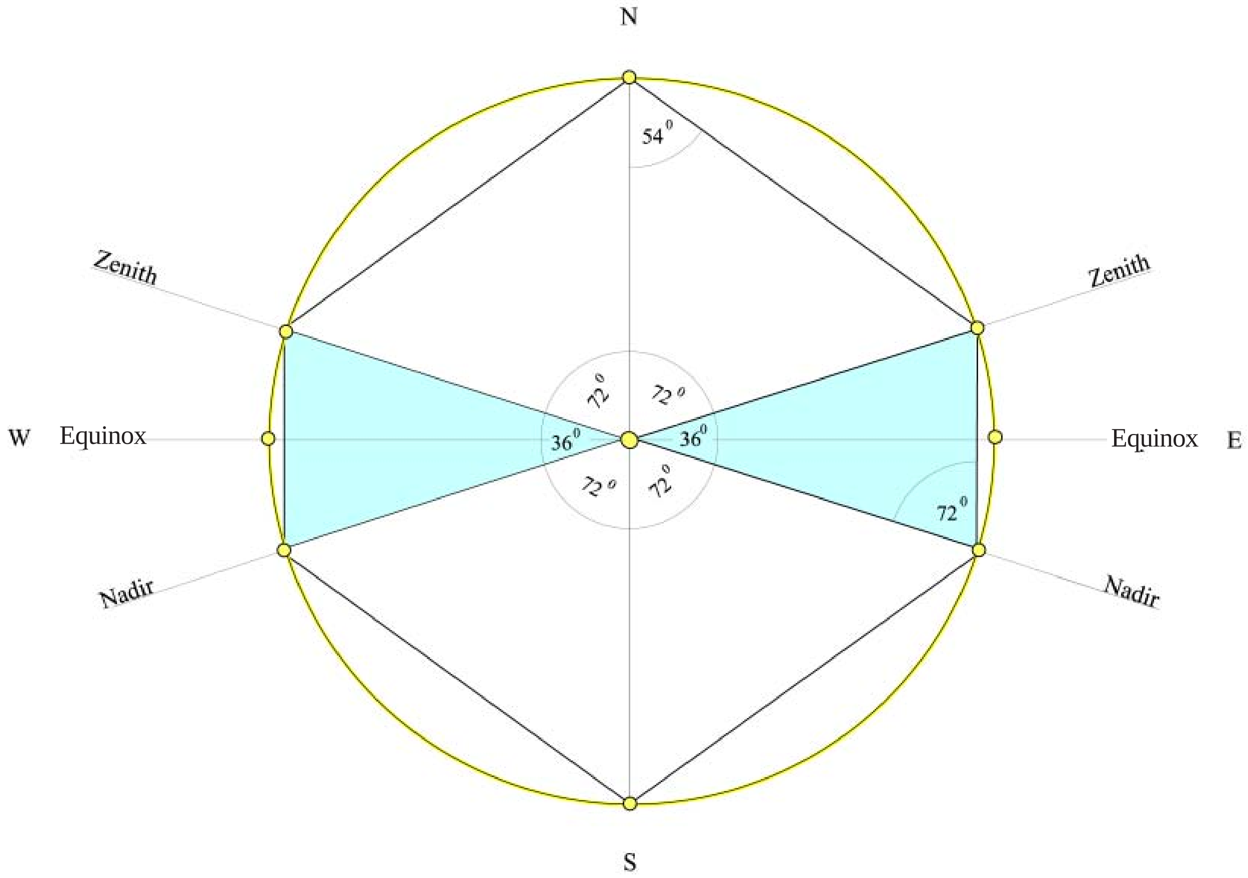
This general observation circle would be accurate to within a small fraction of a degree throughout the Maya latitudes. From the center post, the solstices rise and set at fifty degrees. The full moon rises and sets at the lunar standstills at sixty degrees.

The maximum rising and setting positions of the five planets visible to the naked eye are bounded by the solstices and the lunar standstills.

North, south, east and west inscribe a square within the observation circle and the lunar standstill positions, and north/south inscribe a hexagon within the observation circle.

Figure 312

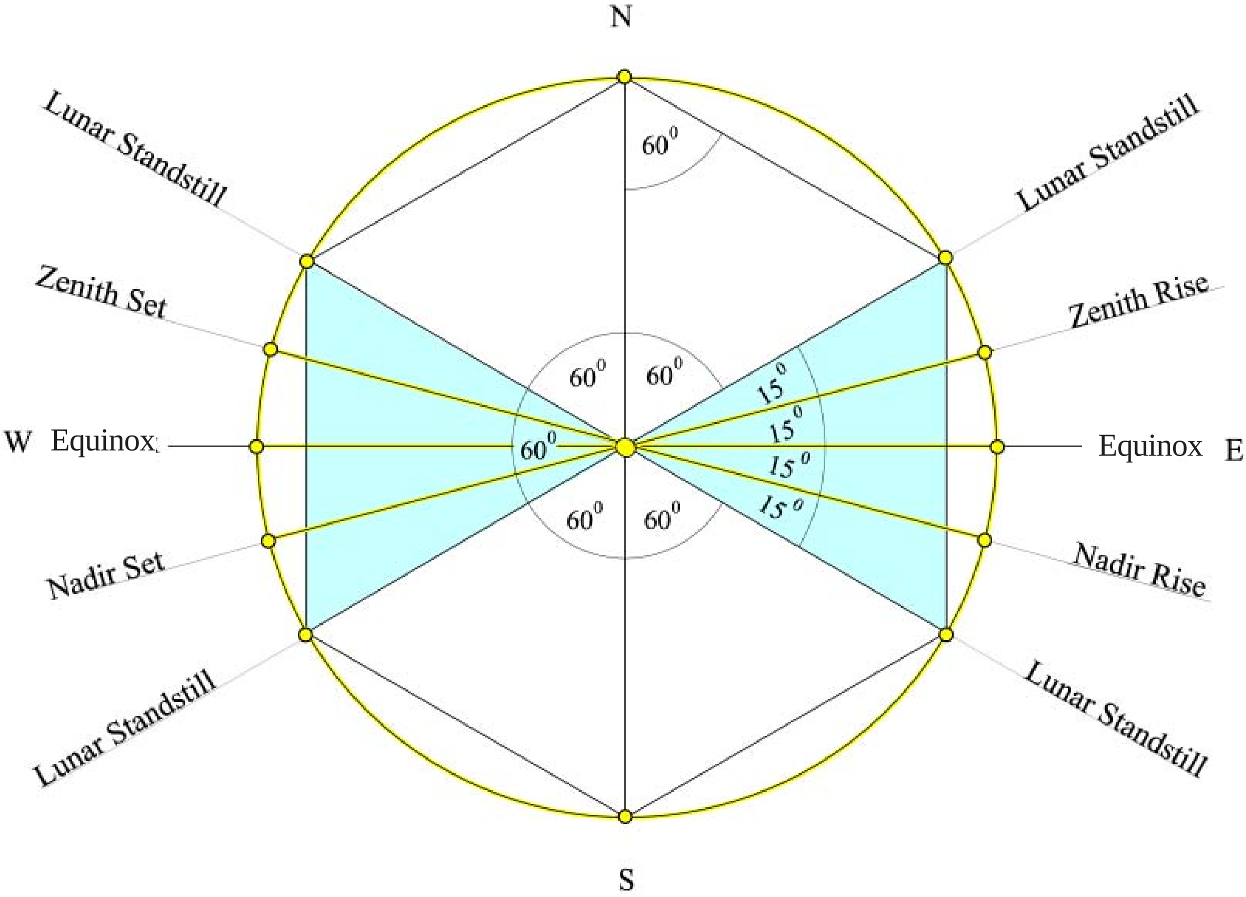
An Astronomical Observation Circle at Tikal and Palenque



From the center post, the zenith and nadir positions at Tikal and Palenque are separated by thirty-six-degree phi (or pentagram) triangles (shaded blue). Between the zenith positions and north is seventy-two degrees, forming two pentagon triangles. Between the nadir positions and south is also seventy-two degrees, forming two pentagon triangles.

Figure 313

An Astronomical Observation Circle at Copan



From the center post of an observation circle at Copan, the lunar standstills, zenith and nadir passages, and equinox positions on the horizon are separated by fifteen degrees. The nadir positions and the northern lunar standstill positions are separated by forty-five degrees, and the zenith positions and the southern lunar standstill positions are separated by forty-five degrees.

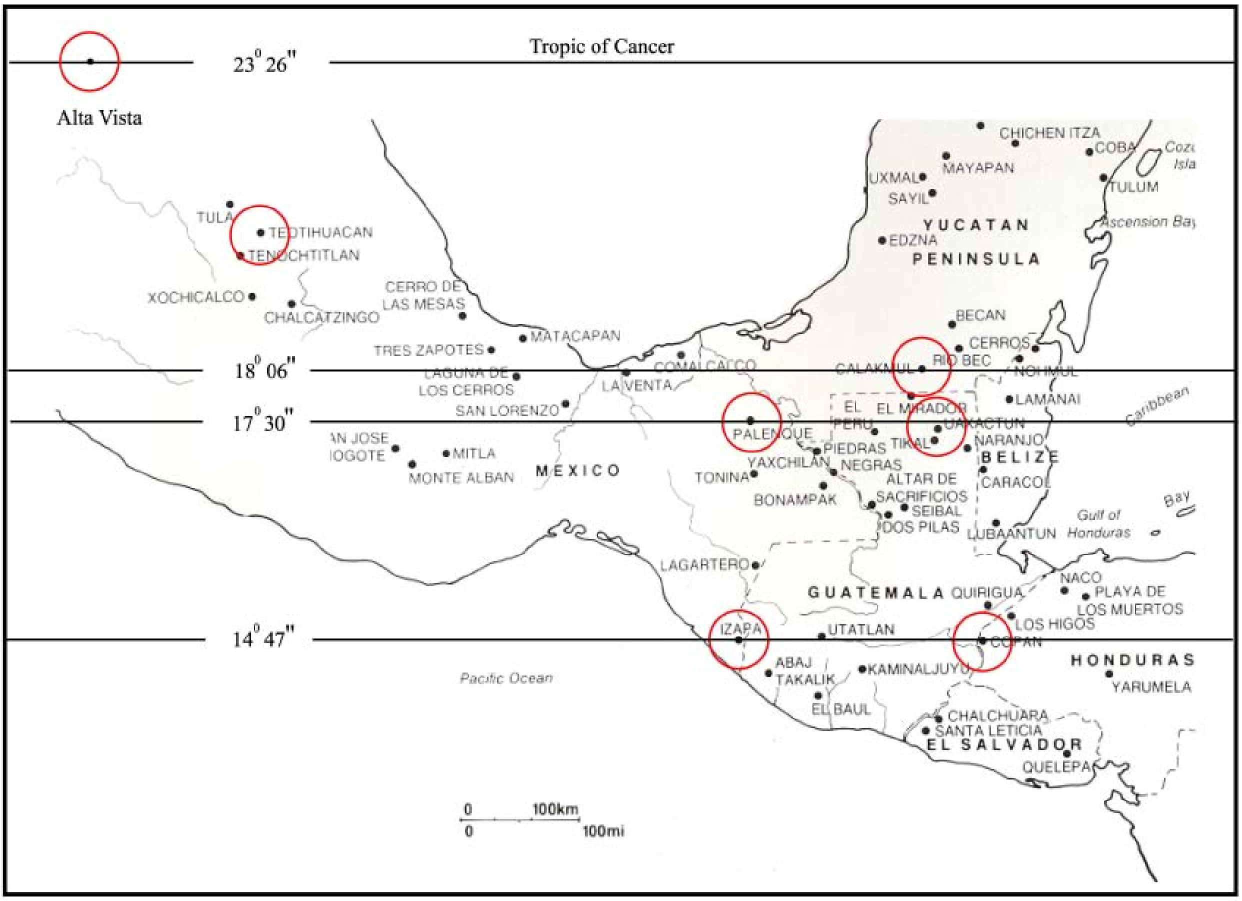
Malmstrom (1976) noticed that the two zenith passages at Copan and Izapa (both at 14.8 degrees latitude) neatly subdivide the solar year of 365 days into a 260-day (Tzolkin) period between zenith passages and winter solstice and 105 days between zenith passages and the summer solstice. Anderson, Morales, and Morales (1980), examine the apparent alignment of the Tower at the Palace in Palenque to the zenith sunset position at the latitude of Copan and Izapa. In their general discussion of the importance of zenith passage, they mention that the zenith passages bear a strong relationship to the corn cycle. In fact, the authors noted that some Maya communities still use a straight plumbed stick as a gnomon to identify the days of the zenith passage (by observing when the gnomon casts no shadow at noon); the determined April or May zenith passage heralds the corn planting season and the August zenith passage heralds the harvest. (On average, corn takes about 105 days to mature from planting to harvest.)

On the other hand, the 260-day cycle may be equated with human gestation. If a woman conceived on the August zenith, she would give birth on or very near the April zenith at the latitude of Copan. (The world wide average number of days of human gestation from conception to birth is 263 days.) The ideal zenith passages at Copan and Izapa — April 30 and August 13 (the day of Maya creation) — thus subdivide the solar year by the human gestation cycle and the corn cycle; a pattern that probably would have been noticed by the Maya and may well have been deemed elegant. When we are reminded that, in the Popol Vuh, the first human beings were made of corn, the intermeshing cycles become more resonant.

At this point in my investigations into the possible calendrical and geometrical significance of zenith and nadir passages and the proposed Maya or Mesoamerican observation circles, I recalled that the hieroglyphic passages on Stela A from Copan and Stela 10 from Seibal associate the four cardinal directions with the ceremonial centers of Tikal (east), Palenque (west), Copan (south), and Calakmul (north) (see Figure 314). I realized that the locations of three of these important Maya ceremonial centers may have been chosen, at least in part, because of their particular latitudes. That is, the observation circles at Tikal and Palenque would have produced phi and pentagon triangles, and these would be related to the geometrical and calendrical characteristics of the zenith and nadir passages as noted above for Copan.

Figure 314

Map of the Four Corner Cities of the Maya Region



Map with latitudes for the four “corner” cities of the Maya region: the southern corner (Copan), the northern corner (Calakmul), the eastern corner (Tikal), and the western corner (Palenque)

But Calakmul, to the north at 18.2 degrees latitude, did not seem to exhibit anything significant in relation to the zenith or nadir passages. Then Dr. Ed Barnhart sent me a map of Mesoamerica with the latitudes of one-day increments for zenith passages drawn across it. The April 12th and August 1st zenith passages noted on this map crossed right through the site of Calakmul. These dates fall exactly thirteen days earlier than the April 29th and August 13th zenith passages observed at Copan. (I later double-checked this on my Starry Night astronomy program and got the same result.)

I had just completed a diagram (Figure 315) to help explain a theory I was working on concerning idealized thirteen-day increments that separate the solstices, zenith passages, equinoxes and nadir passages at the latitude of Copan and Izapa (14.8 degrees). The increments, I suspected, bore some relationship to my proposed scheme for leap-year calculations. As is shown in Figure 315, the interval from the summer solstice to the zenith passage is 4 x 13 days and from the zenith position to the calendrical equinox is 3 x 13 days. The interval from the calendrical equinox to the calendrical nadir is also 3 x 13 days and from the calendrical nadir to the winter solstice is 4 x 13 days. The term “calendrical equinox” here refers to the midpoint in the count of days between the solstices, and the term “calendrical nadir” refers to an idealized nadir passage that is separated by the same number of days from the winter solstice as is zenith passage from the summer solstice. Both the calendrical equinox and the calendrical nadir would occur four days after the actual equinox and nadir as the sun moves towards the winter solstice and four days before when the sun is moving towards the summer solstice. This is because the sun tarries eight days longer between the equinoxes and the winter solstice than it does between the equinoxes and the summer solstice, since the earth's orbit is further away from the sun during the winter.

Using the proposed thirteen-day arrangement, all of the principal solar events would occur on the same day number of the Tzolkin in a given year and would advance one day each year until the need to add a leap day would ruin the scheme.

What I am proposing here is an entirely theoretical but very accurate and simple method for adjusting this scheme to account for the difference between the canonical 365-day Haab and the tropical year of 365.2422… days without the need to make changes in the Haab calendar. This scheme is based on Lounsbury's (1976) observation that the Maya probably let the 365-day year drift through the seasons until it accumulated 365 days' worth of drift every twenty-nine Calendar Rounds (of fifty-two Haabs and seventy-three Tzolkins each) and would equate 1,508 Haabs of 365 days to 1,507 tropical years of 365.2422… days. Lounsbury also provides evidence that the Maya had kept track of the number of Maya "leap days" that had accumulated between given dates. To accomplish this would have been a simple matter of accounting for one day of tropical year drift every 1,508 days.

This type of tropical year calculation would be necessary for the Maya to be able to predict which day a given solar station would occur. The simplest way to apply this proposed Maya “leap year” calculation to the Haab calendar would be to begin counting multiples of 365 days between solstices (winter or summer) and also maintain a continuous count of 1,508-day cycles. Every time a 1,508-day cycle was completed during a given 365-day year, the Maya would add a day, so to speak, at the end of that year and begin the day count for the next solar year with the solstice occurring on the 366th day. Because this particular tropical year calculation is virtually as accurate as our present best estimates, the proposed scheme could be used for many thousands of years without the need to adjust it.

For all latitudes in the Maya region, the solstices and the calendrical equinoxes would occur on the same day number of the Tzolkin cycle and would advance by one every year until a 1,508 day-cycle was passed, and then the day number would advance by two. For most of the Maya area, the zenith and nadir passages would occur on Tzolkin day numbers different from those of the solstices and calendrical equinoxes, depending on the latitude of a given site. But at 14.8 degrees latitude (Copan) and 18.2 degrees latitude (Calakmul), and only at these latitudes, all of these solar events (solstices, zenith passages, calendrical equinoxes and calendrical nadir passages) would occur on the same Tzolkin day number (see Figures 315 and 316).

Figure 315

A Count of Days at Copan

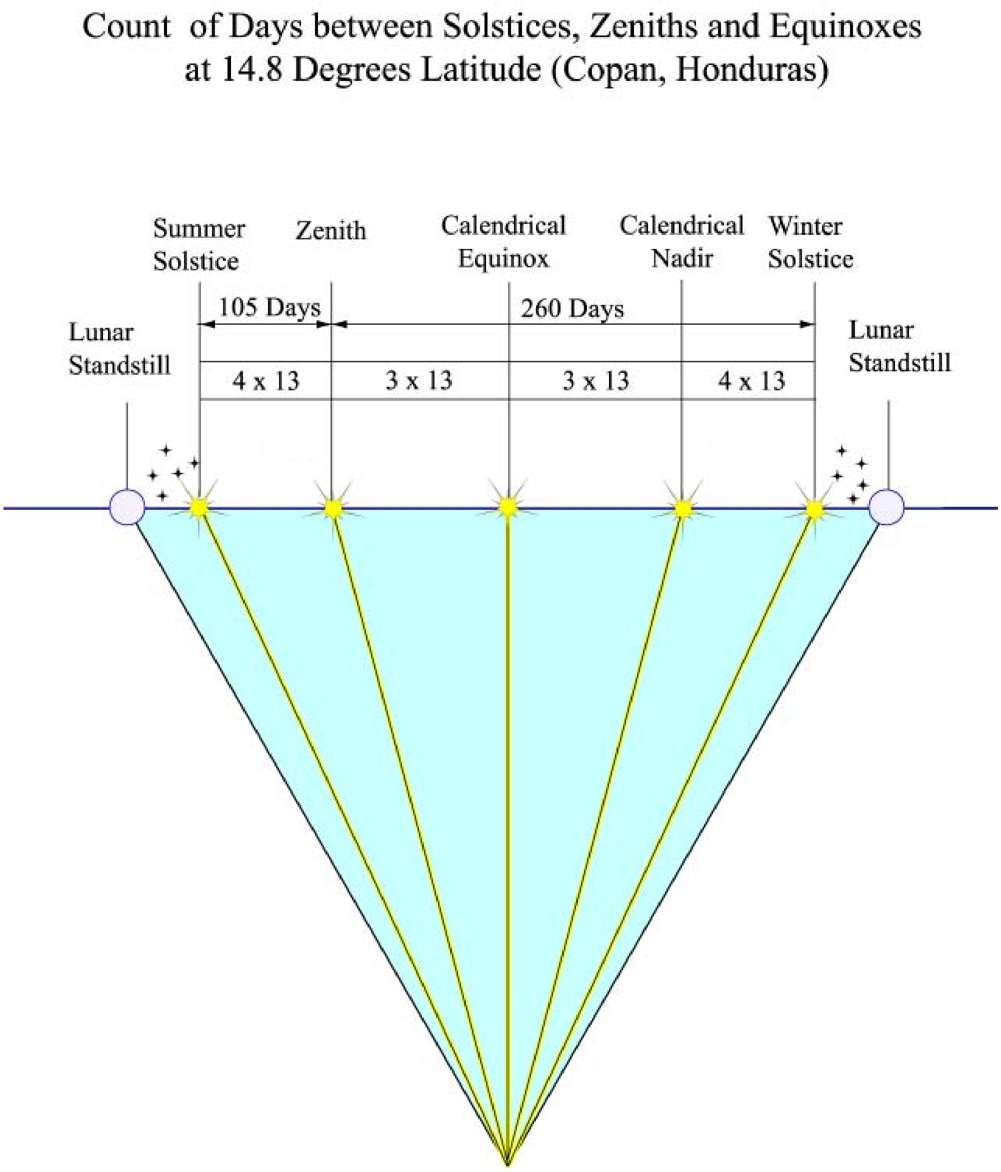
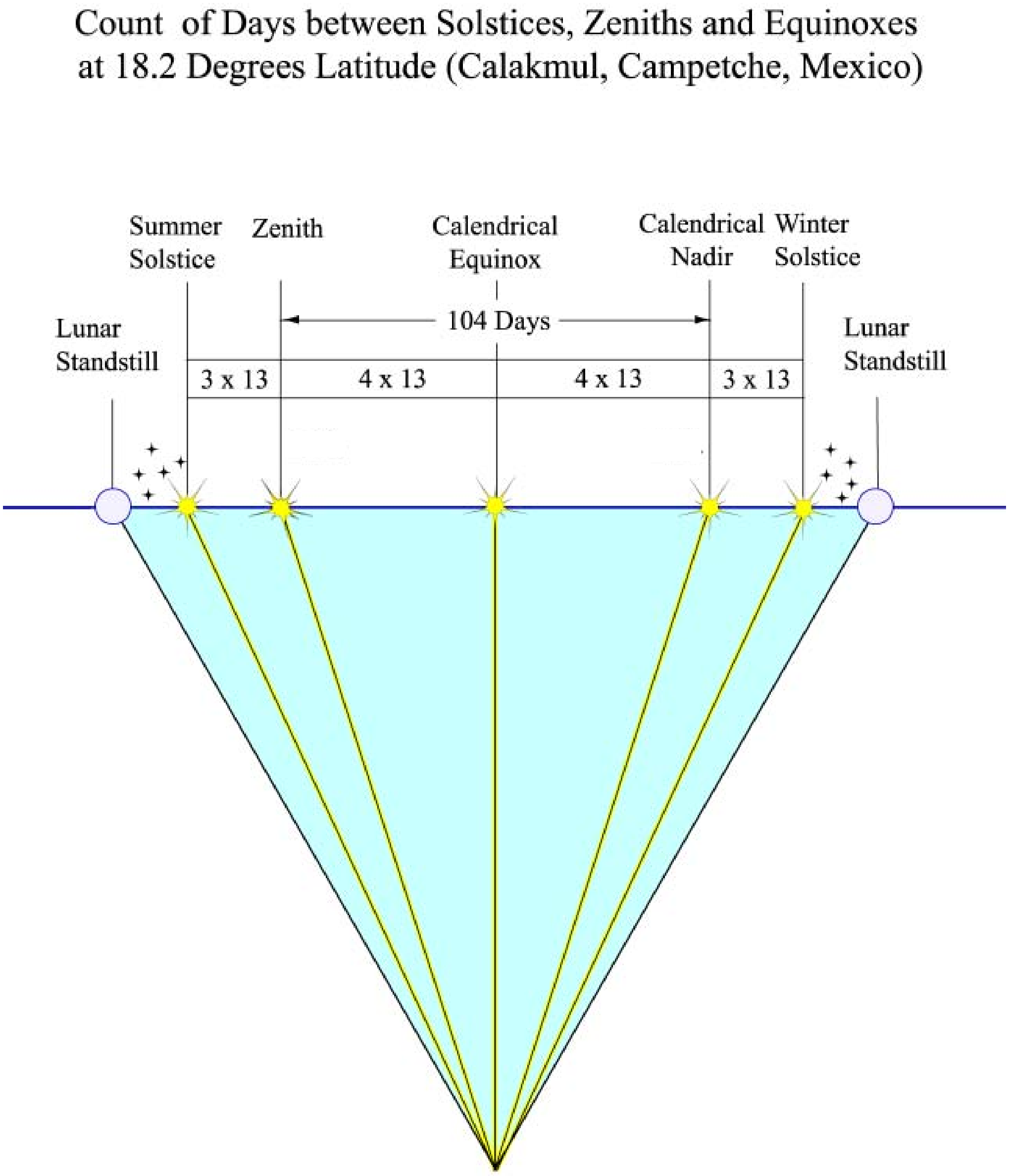


Figure 316

A Count of Days at Calakmul



Regarding this proposed “leap year” scheme, I pointed out in my Master’s thesis (Powell, 1996) that 1,508 days is equal to thirteen whole day approximations of the Mercury synodic period of 116 days (13 x 116 = 1,508) and that the complete tropical year drift cycle of twenty-nine Calendar Rounds, or 1,508 Haabs, is the least common multiple of the 116-day Mercury synodic period and the Calendar Round. Several interesting numerological observations concerning this proposed tropical year drift calculation were also noted in my Master’s thesis. One example is that one twenty-day month, or Uinal, worth of drift equals the Tzolkin times the Mercury synodic period (20 x 1,508 = 116 x 260).

The idea that multiples of the Mercury synodic period could be used with extreme accuracy to calculate and to keep track of the tropical year relative to the Haab and the Tzolkin possesses a certain astronomical poetry. More than the other four planets visible to the naked eye, Mercury appears to both hound and herald the sun. Mercury is never more than about twenty-three degrees from the sun’s rising or setting position on the horizon. It rises, shortly before sunrise as morning star and shortly after sunset as evening star, as if tied to the sun by a short leash.

My rather belabored point here is that the four major Maya ceremonial centers associated with the cardinal directions may have been regarded as the four corners of the Maya region because of their unique latitudes relative to both the calendrical and geometrical positions of the zenith and nadir passages. The eastern and western corners share the only latitude that would produce phi and pentagon triangles in an observation circle, and the northern and southern corners are located precisely at the only latitudes where all of the major solar events occur on the same day numbers of the Tzolkin cycle and whose zenith and nadir passages are separated by exactly thirteen days.

Outside the Maya area, there is evidence to suggest that the Teotihuacanos may also have been interested in changes in the calendrical and geometrical positions of the zenith passages relative to changes in latitude. The most obvious evidence is that most of the architecture at Teotihuacan is aligned in a grid whose east/west axis points to the sunrises and sunsets on April 29th and August 13th, the same rise and set days for the zenith passages at Copan, as discussed above.

In ways that remain unclear, the Teotihuacanos had a major influence on a number of far-flung foreign ceremonial centers, most of which occupy key latitudes related to this discussion. One of these is the ceremonial center of Alta Vista, well to the north of Teotihuacan and located within a few kilometers of the Tropic of Cancer, which is the northern limit of the occurrence of zenith passage and where zenith passage takes place only once a year, at the summer solstice (Aveni, 1992).

Moving south, the Teotihuacanos arrived at Tikal in A.D. 378 and quickly intermarried with the royal bloodline there. They then sent an emissary to rule over Uaxactun, the sister site of Tikal. At Uaxactun we find an early and clearly functional E Group solar observatory. From the vantage point of a central pyramid, three smaller temples arranged on a raised platform to the east demarcate the rising positions of the sun at the solstices and the equinoxes. Alonso Mendez and I noticed independently that the outer temples also mark the rising sun at the zenith and nadir passages. The same has proved true for the E Group structures at Dzibilchaltun and Ek Balam in the Yucatan. At Tikal, which is about a quarter of a degree south of the latitude of Palenque, the zenith and nadir positions of the sun on the horizon are slightly short of creating a precise thirty-six-degree phi triangle on an observation circle. But the latitude of Uaxactun, some twenty kilometers to the north of Tikal, shares the same geometrically ideal latitude with Palenque to within a few minutes of a degree. Also at Uaxactun, a pristine pecked cross, orientated to within a fraction of a degree to the zenith sunrise there, is found inscribed on the floor of Structure A-V (Aveni, Hartung, and Buckingham, 1978). Virtually identical pecked crosses at Teotihuacan might also have functioned as benchmarks to lay out the orientations of the architecture to the Copan zenith sunrise date.

By the early fifth century, a foreigner called Yax K’uk Mo arrived at the southern “corner” of the Maya region, Copan. He is depicted as a Teotihuacan warrior and established a royal bloodline there with a local queen. A decade or so later, the royal bloodline of the eastern “corner,” Palenque was also established, perhaps by a Teotihuacan emissary as well. The three “corners” of the Maya region, Palenque, Tikal, and Copan, remained close allies throughout the Classic period and, interestingly, were almost constantly at war with the city at the northern “corner,” Calakmul. In these historical events we may be seeing the Maya carefully choosing four principal ceremonial centers associated with the cardinal directions and their unique latitudes relative to the ideal geometrical and calendrical zenith and nadir positions.

CHAPTER 10

CONCLUSIONS

For thousands of years, the Maya, their predecessors, and their Mesoamerican neighbors have produced architectural and iconographic models of the cosmos based largely on symbolic reenactments of the primordial creation (Freidel, Schele, and Parker 1993).

Recent examinations of these artifacts, their associated texts, and relevant ethnographic data have served to catalyze decades of research from within the fields of ethnography, linguistics, epigraphy, archaeology, iconography, and art history. These indigenous expressions of creation myths range from the sublime to the vernacular and are expressed in a variety of contexts. Some of the most exalted and dramatic examples are now shown to be cyclically enacted in the very movements of the heavens (Freidel, Schele, and Parker, 1993). Shaman kings and queens claimed religious and political charter by reenacting creation events in public ceremonies and by establishing themselves as living conduits between the gods, ancestors, and the human realm.

Grand cosmogramic architectural schemes were commissioned to serve as foci for these supernatural communications. Upon these structures, superbly talented artists and scribes employed their crafts to further elucidate themes of creation and to record the actions of shaman kings, queens, and the elite who brought these mythic images and events to life. In the hamlets and villages surrounding ceremonial centers, communal shamans incorporated cosmograms into their ceremonies by using measuring cords to define ritual space in symbolic reenactments of the primordial creation (Powell, 1993). Today, performances of house-building, Cha Chac, and other ceremonies in modern Maya communities vividly express and preserve ancient cosmological beliefs. Even vernacular houses and milpas are imbued with profound cosmological symbolism (Breedlove and Laughlin, 1993).

The preceding discussion of the physical manifestations of Mesoamerican cosmology strongly supports the hypothesis that the Maya sought to emulate and incorporate the geometries and proportions that were observed in nature and alluded to in their creation myths into the layout and design of their ceremonial and vernacular works of art and architecture.

By focusing on geometrical formulae encoded in individual Maya structures and works of art embedded with grand cosmological schema, new layers of cosmological interpretation can be examined and added to the whole. For example, the set of repeated proportions uncovered in this study are also the principal proportions found in nature.

The shapes of flowers and shells; the human form; the cardinal directions and the square Maya world; Maya concepts of time (circular calendars); their place in the universe (astronomical observation circles); the architecture of their physical environment (the shapes of their houses, milpas, and temples); and their works of art all share the proportions inherent in three simple geometrical forms — the equilateral triangle, the square, and the pentagon. These three regular polygons, with their square root of two, square root of three, and phi rectangular expressions, provide an underlying structure that unites the Maya cosmos. Viewed in this way, Maya geometry might be compared to the modern concept of $E = MC_2$.

The myriad expressions of Maya geometry are subtle. Like contrived distance numbers that unite historical and mythological dates via the least common multiples of various astronomical and calendrical cycles, they are not advertised. You have to do the math to understand which astronomical and calendrical cycles are being used and you have to do the geometry to understand which proportions and formulae are being expressed. Almost certainly, intentional asymmetries are incorporated into ancient art and architecture, just as today intentional errors are incorporated into the complex designs of Maya textiles (Walter F. Morris, 2009, personal communication). And continuing with the analogy to distance numbers, it is possible that some of the suspiciously common mathematical “errors” found in Maya hieroglyphic texts were also intentional.

We are told in the Popol Vuh (Tedlock 1985:165-167) that when the gods made human beings from corn the first men and women were so like the gods that they saw everything under the sky perfectly. This worried the gods, so they decided to shorten their eyesight. “They were blinded as the face of the mirror is breathed upon.” Though the Maya went to great lengths to penetrate the mathematical and geometrical patterns of their universe and to emulate and incorporate these patterns into their own creations, it may have been deemed presumptuous and unwise to attempt to do so blatantly or flawlessly.

In this light, I will share what may be a geometrical formula cleverly hidden in the telling of the creation story in the Popol Vuh (Tedlock 1985:72).

> There is the original book and ancient writing, but he who reads it and ponders it hides his face. It takes a long performance and account to complete the emergence of all the sky-earth:
> the fourfold siding, the fourfold cornering,
> measuring, fourfold staking,
> halving the cord, stretching the cord,

in the sky, on the earth,

the four sides, the four corners,

as is said,

by the Maker, Modeler,

mother-father of life, of human kind…"

This passage may be viewed as a concise formula for staking out a phi rectangle with a cord. The "fourfold siding, the fourfold cornering, measuring, fourfold staking," is presumed to describe the staking out of a square, the shape that must be drawn or staked out first to create any of the square root or phi rectangles. The next two phrases, "halving the cord, stretching the cord in the sky, on the earth," would refer to the next steps taken to create a phi rectangle; that is, "halving the cord" at the base of the square and "stretching the cord" from the center of the base of the square to an upper corner of the square "in the sky," then downward and parallel to the base of the square "on the earth." The next phrase, "the four sides, the four corners," would refer to staking out the sides and corners to inscribe this measure and complete the phi rectangle.

That the phi proportion would be used to describe the making and modeling of the heavens and the earth is particularly apt. Phi is the most common proportion observed in nature. Most flowers are classified by botanists as five somes with five petals or multiples of five petals. Pentagonal arrangements of seeds in the cross-sections of fruits are common. The phi equiangular spiral is readily observed in seashells and snail shells and in the growth spirals of various plants. Recall the Yucatec Maya word for belly button, “tzuk,” or division place, which divides the human form by the phi proportion. And, when the intrinsic subdivisions of the phi rectangles and triangles are considered, it is clear that phi is by far the most dynamic of the formal set of dynamic proportions.

I conclude by comparing the contents of this dissertation to a uinic, the Yucatec Maya unit of measure and the word for human being. The feet are the ethnographical descriptions of the phi, square root, and Pythagorean rectangles that are used by the modern Maya to design and lay out their houses. Interpretations regarding the meaning of Maya geometry rests on the words once spoken to the shaman Don Guadalupe by his mentor and grandfather: “The shapes of the flowers are in our houses.”

The fact that the Maya use these proportions today provides the solid footing for the subsequent geometrical analyses of Pre-Columbian artifacts. And a Maya shaman who uses a measuring cord to lay out square root and phi-proportioned houses, and who then says that the shapes of the flowers are in them, serves as a foundation for subsequent philosophical interpretations that suggest that the Maya were, and still are, consciously emulating and incorporating the observed shapes of nature into the designs of their art and architecture.

The legs and body are represented by the numerous geometrical analyses of Pre-Columbian art, architecture, and books. The illustrations show how the ancient Maya were using geometry, exemplify some of the general rules that were followed in the designs of buildings and artifacts, and present a clear explanation of the principles of perfect and virtually perfect formulae.

The head and brains are the final chapters that explore how Maya concepts of geometry, space, and time can be integrated into a profound understanding of the cosmos.

The heart represents the hundreds of generations of Maya who had the courage and capacity to investigate and learn the mathematical and geometrical principles used by their "Maker, Modeler, mother-father of life," to create their universe, and who have left us at least a glimpse of how it works.

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