Applied Geometry of the Śulba Sūtras

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Abstract

The Śulba Sūtras, part of the Vedic literature of India, describe many geometrical properties and constructions such as the classical relationship

$$a^2 + b^2 = c^2$$

between the sides of a right-angle triangle and arithmetical formulas such as calculating the square root of two accurate to five decimal places. Although this article presents some of these constructions, its main purpose is to show how to consider each of the main Śulba Sūtras as a finely crafted, integrated manual for the construction of citis or ceremonial platforms. Certain key words, however, suggest that the applications go far beyond this.

1 Introduction

The Śulba Sūtras are part of the Vedic literature, an enormous body of work consisting of thousands of books covering hundreds of thousands of pages. This section starts with a brief review of the position of the Śulba Sūtras within this literature and then describes some of its significant features. Section 2 is a discussion of some of the geometry found in the Śulba Sūtras while Section 3 is the same for arithmetical constructions. Section 4 looks at some of the applications to the building of citis and Section 5 integrates the different sections and discusses the significance of the applied geometry of the Śulba Sūtras. I am grateful to Ken Hince for assistance with the diagrams and to Tom Egenes for helping me avoid some of the pitfalls of Sanskrit transliteration and translation.

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1.1 Vedic Literature and the Śulba Sūtras

To understand the geometry of the Śulba Sūtras and their applications, it is helpful first to understand their place in Vedic knowledge as expressed through the Vedic literature.

Maharishi Mahesh Yogi explains that the Vedic literature is in forty parts consisting of the four Vedas plus six sections of six parts each. These sections are the Vedas, the Vedāṅgas, the Upāṅgas, the Upa-Vedas, the Brāhmaṇas, and the Prātishākhyas. Each of these parts "expresses a specific quality of consciousness" [4, p. 144]. This means that often we have to look beyond the surface meanings of many of the texts to find their deeper significance.

The Śulba Sūtras form part of the Kalpa Sūtras which in turn are a part of the Vedāṅgas. There are four main Śulba Sūtras, the Baudhāyana, the Āpastamba, the Mānava, and the Kātyāyana, and a number of smaller ones. One of the meanings of śulba is "string, cord or rope." The general formats of the main Śulba Sūtras are the same; each starts with sections on geometrical and arithmetical constructions and ends with details of how to build citis which, for the moment, we interpret as ceremonial platforms or altars. The measurements for the geometrical constructions are performed by drawing arcs with different radii and centers using a cord or śulba.

There are numerous translations and references for the Śulba Sūtras. The two that form the basis of this article are [7] (which contains the full texts of the above four Śulba Sūtras in Sanskrit) and [9] (which is a commentary on and English translation of the Baudhāyana Śulba Sūtra). Another useful book is [8] since it contains both a transliteration of the four main Śulba Sūtras into the Roman alphabet and an English translation.

It is timely to be looking at some of the mathematics contained in the Vedic literature because of the renewed understanding brought about by Maharishi of the practical benefits to modern life of this ancient Vedic knowledge. Details and further references can be found in [5].

1.2 Features of the Śulba Sūtras

For me, there are three outstanding features of the Śulba Sūtras: the wholeness and consistency of their geometrical results and constructions, the elegance and beauty of the citis, and the indication that the Sūtras have a much deeper purpose.

1.2.1 Integrated wholeness of the Śulba Sūtras

When each of the main Śulba Sūtras is viewed as a whole, instead of a collection of parts, then a striking level of unity and efficiency becomes apparent. There are exactly the right geometrical constructions to the precise degree of accuracy necessary for the artisans to build the citis. Nothing is redundant. This point is nicely made by David Henderson [2] who argues that the units of measurement used easily lead to the accuracy of the diagonal of one of the main bricks of "roughly one-thousandth of the thickness of a sesame seed."

There is also a remarkable degree of internal consistency such as the way that the 'square to circle,' 'circle to square' and 'square root of two' constructions fit together with an accuracy of 0.0003%. (Details are given in Section 5.) A related discussion in the literature is whether or not the authors of the Sūtras knew their construction of the root of two was an approximation.[1] Viewing the Sūtras as utilitarian construction manuals suggested that they knew that they were describing an approximation but that they achieved what they set out to do, namely to provide the first terms of an expansion of the root of two sufficient to ensure the reciprocity of the 'square to circle' and 'circle to square' constructions.

If mathematicians were asked to write such a manual, it is likely that they would do two things, firstly give the construction procedures to level of accuracy appropriate for the actual constructions, and secondly, for their own enjoyment, show to other mathematical readers that they really understood that they were dealing with approximations. Both these features are observed in the Śulba Sūtras.

[1]This discussion hinges on the meaning of *viśeṣa*, which [1] and others take to mean, in this context, a small excess quantity or difference. See also [8, p. 168].

1.2.2 Beauty of the citis

Each of the citis are low platforms consisting of layers of carefully shaped and arranged bricks. Some are quite simple shapes such as a square or a rhombus while others are much more involved such as a falcon in flight with curved wings, a chariot wheel complete with spokes, or a tortoise with extended head and legs. These latter designs are particularly beautiful and elegant depictions of powerful and archetypal symbols, the falcon as the great bird that can soar to heaven, the wheel as the 'wheel of life,' and the tortoise as the representative of stability and perseverance.

1.2.3 Deeper significance

Sanskrit is a rich language full of subtle nuances. Words can have quite different meanings because of their context and, in any case, frequently there is no reasonable English equivalent.

There are a number of key terms in the Śulba Sūtras which, because of their etymology and phonetics, suggest that there is a much deeper significance to the Sūtras. One is the word *citi* introduced above. In the context of the Śulba Sūtras, the usual translation is a type of ceremonial platform but it is close to the word *cit* which means consciousness. Another is *vedi* which is usually translated as the place or area of ground on which the citi is constructed. But since the word *veda* means "pure knowledge, complete knowledge" [4, p. 3], vedi also means an enlightened person, a person "who possesses Veda."

A third is *puruṣa* which is usually translated as a unit of measurement obtained by the height of a man with upstretched arms (Mānava Śulba Sūtra IV, 5) or as 120 *aṅgulas* (Baudhāyana Śulba Sūtra I, 1–21), a measurement based on sizes of certain grains.

However, in [3] *puruṣa* is defined as "the uninvolved witnessing quality of intelligence, the unified ... self-referral state of intelligence at the basis of all creativity" (p. 109). Thus we could easily infer that a more expanded role of the Śulba Sūtras is as a description of consciousness. Further discussion of this point is given in Section 5.

When these and other examples are combined with the general direction of all the Vedic literature towards describing "qualities of consciousness," we are led to the conclusion that the Śulba Sūtras are describing something much beyond procedures for building brick platforms, no matter how far-reaching their purpose. This theme is referred to again in the concluding section. In this article

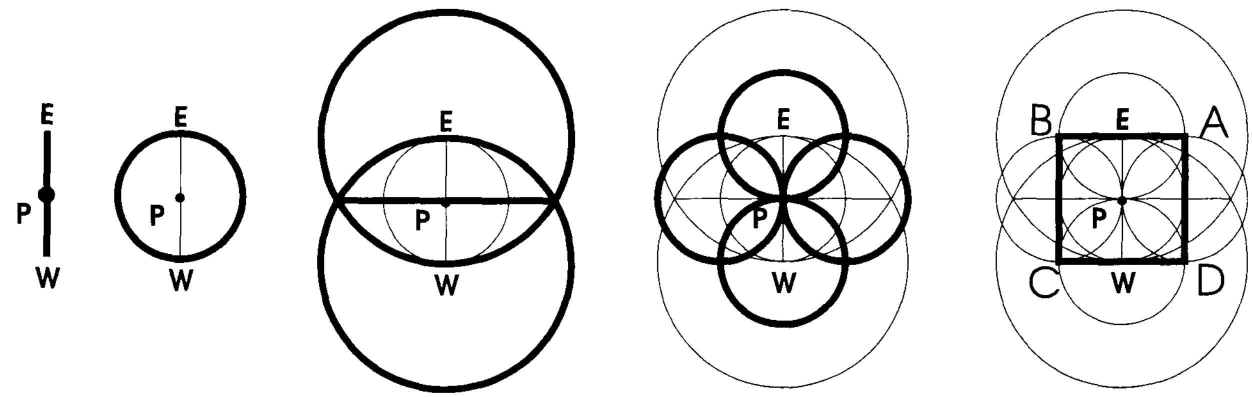

Figure 1: Steps for the construction of a square.

the focus is on the geometrical content of the text, but because of the range of meanings of the key terms, they will generally be left in their original (but transliterated) form. In a later article, I hope to develop some of these deeper themes of the Śulba Sūtras.

2 Geometry

Most of the geometric procedures described in the Śulba Sūtras start with the laying out of a *prācī* which is a line in the east-west direction. This line is then incorporated into the final geometric objects or constructions, generally as a center line or line of symmetry. This section describes some of the main geometric constructions given in the Sūtras.

2.1 Construction of a square with a side of given length

From verses I, 22–28 of the Baudhāyana Śulba Sūtra (BSS)[2], the procedure is to start with a prācī and a center point (line EPW in Figure 1) and, by describing circles with certain centers and radii, construct a square $ABCD$ in which E and W are the midpoints of AB and CD. The steps of the construction are displayed in Figure 1.

2.2 Theorem on the square of the diagonal

Verse I, 48 of BSS states:

> The diagonal of a rectangle produces both (areas) which its length and breadth produce separately.

There appears to be no direct mention of areas in this verse. When, however, it is combined with the subsequent one,

> This is seen in rectangles with sides three and four, twelve and five, fifteen and eight, seven and twenty-four, twelve and thirty-five, fifteen and thirty-six,

it is clear that it is an equivalent statement to the theorem named in the west after Pythagoras, namely that in a right-angle triangle with sides a, b and c (c the hypotenuse), $a^2 + b^2 = c^2$. Note also that all possible pairs (a, b) are given (except one) which (i) allow an integer solution of $a^2 + b^2 = c^2$ with $1 \leq a \leq 12$ and $a \leq b$, and (ii) are coprime. (The interested reader might like to check this.) This accounts for five of the listed pairs, the last pair $(15, 36)$ being derivable from the earlier pair $(5, 12)$.

Further evidence that Baudhāyana had a clear understanding of this result and its usefulness is provided by the next two constructions.

2.3 A square equal to the sum of two unequal squares

Verse I, 50 of BSS describes the construction of a square with area equal to the sum of the areas of two unequal squares. Suppose that the two given squares are $ABCD$ and $EFGH$ with $AB > EF$. Mark off points J, K on AB and DC with $AJ = DK = EF$ as shown in Figure 2. Then the line AK is the side of a square with area equal to the sum of the areas of $ABCD$ and $EFGH$.

[2] There are two main numberings of the verses of the BSS, one used by Thibaut [9] and one attributed to A. Bürk in [8] and used there. In this article we follow the numbering used by Thibaut.

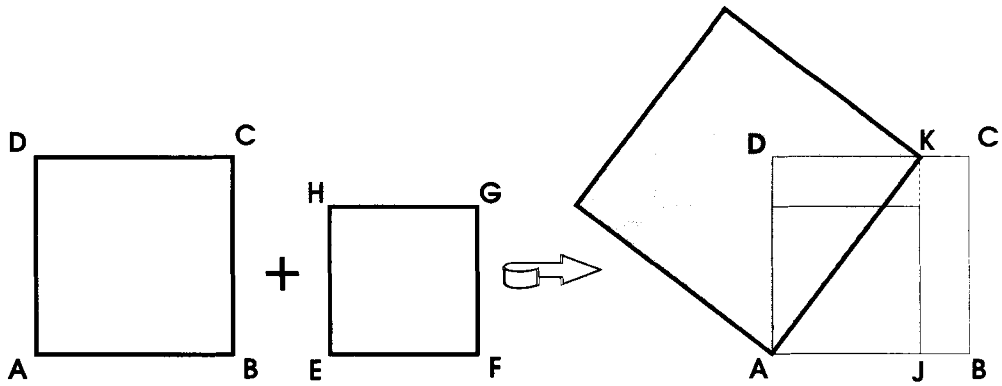

Figure 2: Sum of two squares.

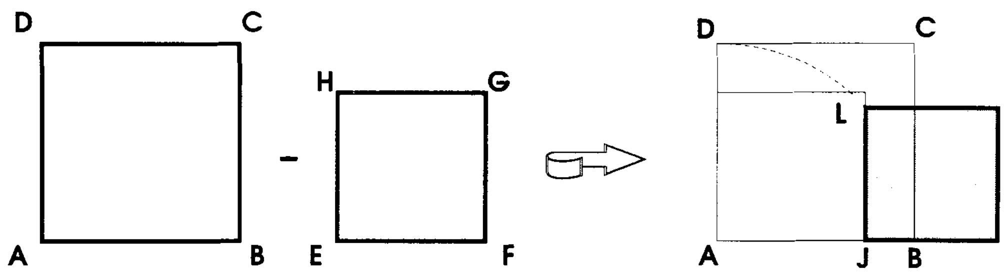

Figure 3: Difference of two squares.

2.4 A square equal to the difference of two squares

The subsequent verse (I, 51) describes the construction of a square with area equal to the difference of two unequal squares. With the notation the same as the preceding example, form an arc DL with center A as shown in Figure 3. Then JL is the side of the required square.

This follows from the facts that $(AJ)^2 + (JL)^2 = (AL)^2$, $AL = AB$, and $AJ = EF$, so that $(JL)^2 = (AB)^2 - (EF)^2$.

2.5 Converting a rectangle into a square

The method of converting a rectangle into a square with the same area described in BSS I, 54 makes use of the previous construction. Start with a rectangle $ABCD$ with $AB > CD$ as shown in Figure 4 and form a square $AEFD$. The excess portion is cut into equal halves and one half is placed on the side of the square. This gives two squares, a larger one $AGJC'$ and a smaller one $FHJB'$; the required square is the difference of these two squares. In Figure 4 the side of this square is GL, where L is determined by $EL = EB'$.

To see this, denote the sides of the rectangle by $AB = a$ and $AD = b$. Then the side GL satisfies

$$(GL)^2 = (EL)^2 - (EG)^2 = (EB')^2 - (EG)^2$$
$$= \left(b + \frac{a-b}{2}\right)^2 - \left(\frac{a-b}{2}\right)^2 = ab$$

and so the area of the shaded square equals the area of the initial rectangle. as required.

Datta [1] suggests that the steps in this construction of marking off a square, dividing the excess, and rearranging the parts could be the basis of the method described later in the BSS I, 61 (see 3.1) of finding the square root of two. By repeating these steps, he shows that the successive approximations to the square root of two described in I, 61 are obtained. If this is the procedure they used, then this is another example of the tightly knit methods and logic of the Sūtras.

2.6 Converting a square into a circle

Verse I, 58 describes the procedure for constructing a circle with area approximately equal to that of a given square. Start with a square $ABCD$ as in

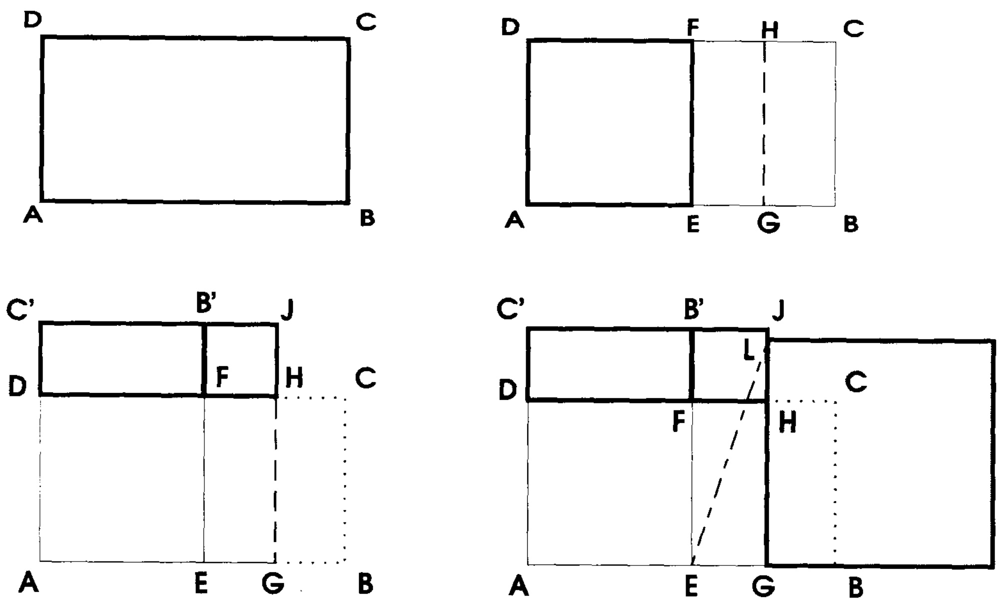

Figure 4: Steps in converting a rectangle to a square.

Figure 5 with center O. Draw an arc DG with center O so that OG is parallel to AD. Suppose that OG intersects DC at the point F. Let H be a point 1/3 of the distance from F to G. Then OH is the radius of the required circle.

To see what is going on here, let $2a$ be the length of the side of the square $ABCD$ and r the radius of the constructed circle. Then

$$\begin{aligned} r &= OH = OF + FH \\ &= OF + \frac{1}{3}(OG - OF) \\ &= a + \frac{1}{3}(a\sqrt{2} - a) \end{aligned}$$

and hence

$$r = \frac{a}{3}\left(2 + \sqrt{2}\right). \tag{1}$$

If we substitute the values $\pi = 3.141593$ and $\sqrt{2} = 1.414214$ we get that the area of the constructed circle is

$$\text{Area} = \pi r^2 = 4.069011\ldots \times a^2$$

which is within about 1.7% of the correct value of 4.

In the next verse Baudhāyana describes how to go in the opposite direction, namely from a circle to a square.

2.7 Converting a circle into a square

Thibaut's translation of Verse I, 59 is:

> If you wish to turn a circle into a square, divide the diameter into eight parts and one of these eight parts into twenty-nine parts; of these twenty-nine parts remove twenty-eight and moreover the sixth part (of the one part left) less the eighth part (of the sixth part).

In modern mathematical notation, starting with a circle of diameter d, the length of the side of the corresponding square is:

$$\text{length} = d - \frac{d}{8} + \frac{d}{8 \times 29}$$

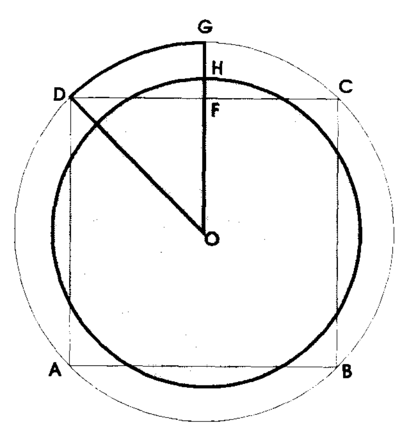

Figure 5: Converting a square to a circle.

$$- \frac{d}{8 \times 29}\left(\frac{1}{6} - \frac{1}{6 \times 8}\right) = \frac{9785}{11136} \times d. \qquad (2)$$

Taking $\pi = 3.141593$ as before, we get

$$\frac{\text{Area of square}}{\text{Area of circle}} = \left(\frac{9785}{11136}\right)^2 d^2 \Big/ \frac{\pi}{4} d^2 = 0.983045\ldots.$$

The correct value is 1 and so, just as in the previous section, the result is accurate to approximately 1.7%. The precision of these accuracies will be looked at in more detail in Section 5 below.

3 Arithmetic

We shall first consider the example in the Baudhāyana Śulba Sūtra of the calculation of the square root of two and then look at what could be the underlying principle.

3.1 The square root of two

Verse I, 61 of BSS writes:

> Increase the measure by a third and this (third) again by its own fourth less its thirty-fourth part; this is the (length of) the diagonal of a square (whose side is the measure).

Using modern notation, the assertion is that if the length of the side of the original square is a, then the length of its diagonal is

$$\begin{aligned} \text{diagonal} &= \left(1 + \frac{1}{3} + \frac{1}{3 \times 4} - \frac{1}{3 \times 4 \times 34}\right) \times a \\ &= \frac{577}{408} \times a \\ &= 1.414215\ldots \times a \end{aligned}$$

3.2 Discussion

Given that $\sqrt{2} = 1.414213\ldots$, it is not surprising that many commentators have proposed ways that explain the underlying method of achieving such an accurate result. The common thread passing through most of the explanations is to start with the initial approximation

$$\sqrt{2} \approx 1 + \frac{1}{3}$$

and then sequentially estimate and reduce the size of the errors. Let x_1 be the first error so that

$$\sqrt{2} = 1 + \frac{1}{3} + x_1.$$

Squaring both sides and neglecting terms in x_1^2 gives $x_1 = \frac{1}{3 \times 4}$. Now define the next approximation x_2 by

$$\sqrt{2} = 1 + \frac{1}{3} + \frac{1}{3 \times 4} + x_2.$$

Repeating the preceding step gives

$$x_2 = -\frac{1}{3 \times 4 \times 34}$$

which yields the approximation given in the BSS just described.

Another repetition of this step gives the approximation:

$$\begin{aligned} \sqrt{2} &\approx 1 + \frac{1}{3} + \frac{1}{3 \times 4} - \frac{1}{3 \times 4 \times 34} \\ &\quad - \frac{1}{3 \times 4 \times 34 \times 2 \times 577} \\ &= \frac{665857}{40832} = 1.414213562374\ldots \end{aligned}$$

which is accurate to 13 decimal places.

Another approach has been suggested by [1] which maintains the geometric flavor of the Sūtras. The idea is to use the first steps of the method described in 2.5 to convert a rectangle of size 2×1 into a square of side $\sqrt{2}$. If these steps are repeated the above sequence of approximations is obtained.

Henderson [2] shows just how natural the procedure is and how it can be generalized to a large class of real numbers. He also observes that same sequential approximation to $\sqrt{2}$ can be obtained by applying Newton's method for finding the roots of an algebraic equation. Take $f(x) = x^2 - 2$ with an initial approximation of $x_0 = 4/3$ for its positive root. Newton's method is that the successive approximations are given by

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} = \frac{x_n}{2} - \frac{1}{x_n}.$$

It is easily checked that this gives the approximations described in the Śulba Sūtras.

4 Applications to the construction of citis

In this section we look at the applications of the geometrical and arithmetical constructions of the

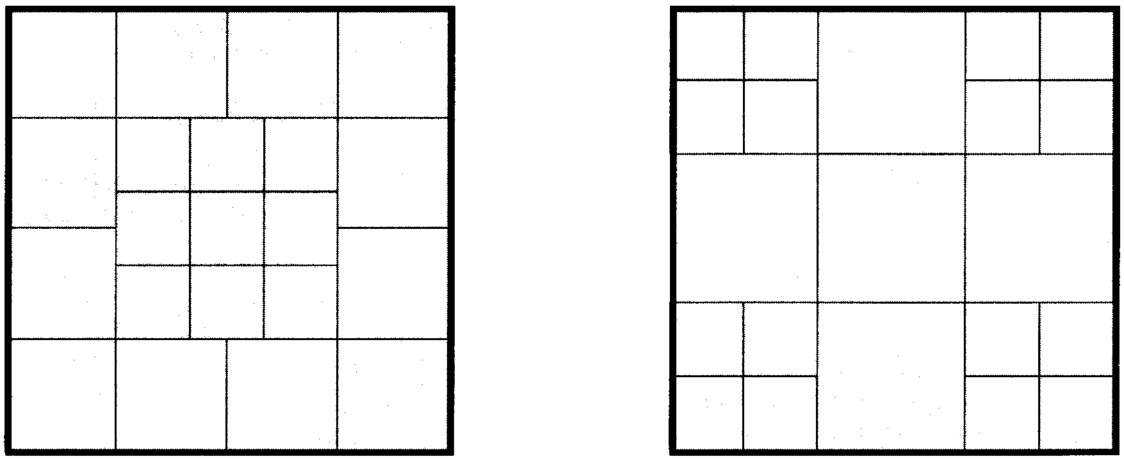

Figure 6: Possible configuration of the gārhaptya citi for the odd and even layers.

Śulba Sūtras to the construction of citis. As stated in Section 1, this is just the first level of applications of the Sūtras.[3]

4.1 General introduction

Each of the citis is constructed from five layers of bricks, the first, third and fifth layers being of the same design, as are the second and fourth. Quite a lengthy sequence of units is used; the two that are referred to the most are the *aṅgula* and the *puruṣa* which were discussed in 1.2.3 above. They are approximately 0.75 inches and 7 feet 6 inches with 120 *aṅgulas* equal to a *puruṣa.*

The heights of each layer are 6.4 *aṅgulas* which is about 4.8 inches and the successive layers are built so that no joins lie along each other. This last requirement is sometimes difficult to achieve and adds to the aesthetics of the finished product as much as to its strength. Generally each layer has 200 bricks with the exception of the gārhaptya citi which has 21 bricks in each layer.

For each design (with the exception of the gārhaptya citi), the citi is first constructed with an area of 7.5 square puruṣas, then with 8.5 square puruṣas, and so on up to 101.5 square puruṣas (BSS II, 1–6). Verse II, 12 explains how these increases in size are to be brought about. If the original citi of 7.5 sq. puruṣas is to be increased by q sq. puruṣas, a square with area one sq. puruṣa has substituted for it a square with area $1 + (2q/15)$ sq. puruṣas. The sides of this square form the unit of the new construction replacing the original puruṣa. Hence the area of the enlarged citi is

$$7.5 \times \left(\sqrt{1 + \frac{2q}{15}}\right)^2 = 7.5 + q \text{ sq. puruṣas,}$$

as required.

Many of the designs described in the BSS have a number of variations. For simplicity not all the variations will be described and we shall choose one of them usually without commenting that there are other possibilities.

4.2 The gārhaptya citi

This citi, described in BSS II, 66–69, is one vyāyāma square and consists of 21 bricks on each level. (A vyāyāma is 96 *aṅgulas* or about 6 feet; BSS I, 21). Three types of bricks are used: one-sixth, one-fourth and one-third of a vyāyāma. The odd layers consist of 9 bricks of the first type and 12 of the second, while the even layers consist of 6 bricks of the third type and 16 of the first. One possible arrangement is shown in Figure 6.

4.3 The śyena citi

This is a particularly beautiful depiction of a falcon (*śyena*) in flight, its construction steps being described in BSS III, 62–104. See Figures 7 and 8.

4.4 The rathacakra citi

This citi is in the shape of a chariot wheel (*rathacakra*) with nave (or center), spokes and felly (or rim), its construction being described in BSS III, 187–214. It requires seven types of bricks for the

[3]Photographs from Kerala, India of citis and associated ceremonies are contained in [6].

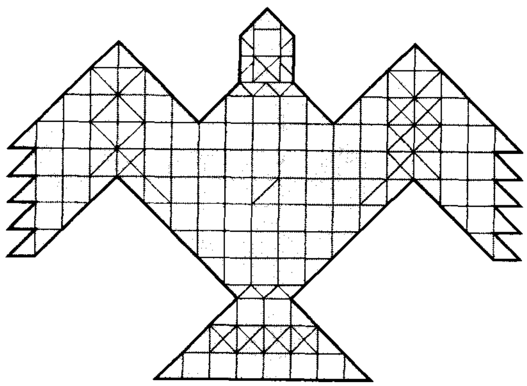

Figure 7: The śyena citi: layers 1, 3, and 5.

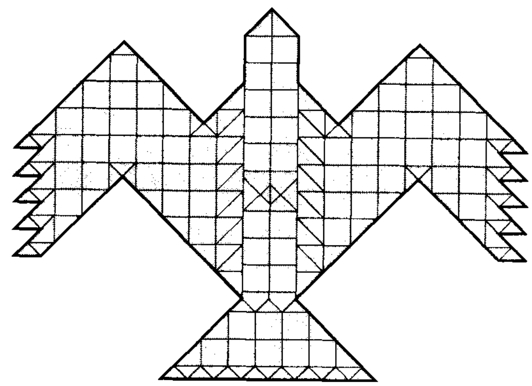

Figure 8: The śyena citi: layers 2 and 4.

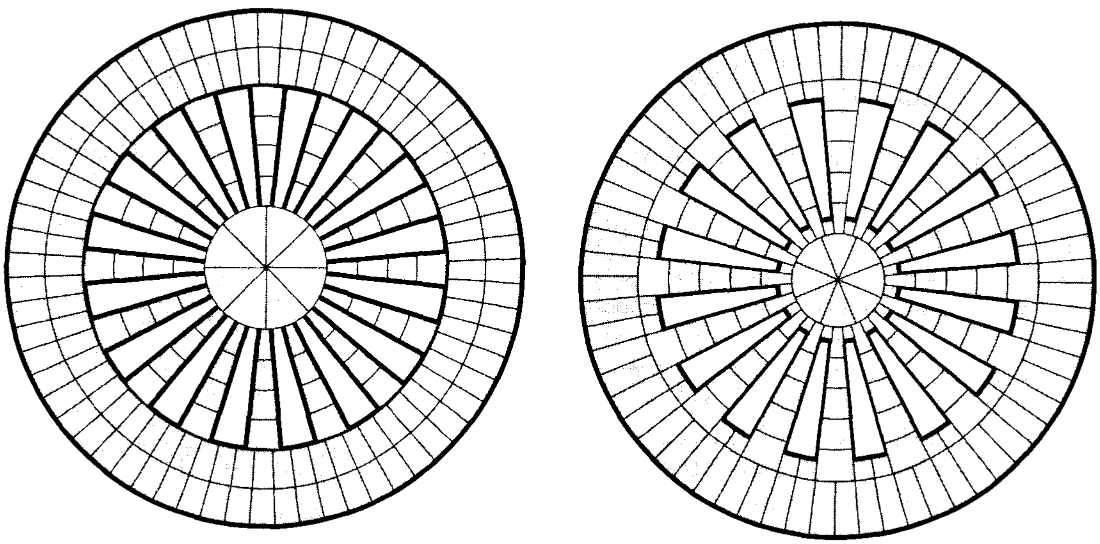

Figure 9: Design of the rathacakra citi.

odd layers and nine types for the even layers. There seems to be some flexibility about the final design, Figure 9 being one possibility.

The initial calculations for determining the different parts of the wheel are in terms of square bricks each of area 1/30 square puruṣas. Since the final area is required to be 7.5 square puruṣas, the number of bricks is $7.5 \times 30 = 225$. The nave of the wheel consists of 16 of these bricks, the spokes 64 and the rim 145, making 225 in all.

The spaces between the spokes are equal in area to the spokes and so, if these spaces are included, the overall area is $225 + 64 = 289$ bricks. (Notice that these numbers satisfy $15^2 + 8^2 = 17^2$ and are one right-angle-triangle triples described above.) Hence the radius of the outer rim of the wheel is equal to the radius of a circle equal in area to a square of side 17 bricks so the methods described in subsection 2.6 could be used to construct this circle. Similarly, the inner radius of the rim is the radius of a circle equal in area to a square with side the square root of $16 + 64 + 64 = 144$, namely 12. Finally, the radius of the nave comes from a square of side 4.

4.5 The kūrma citi

Another fascinating series of constructions are in the form of a tortoise (*kūrma*). In the BSS there are two types of these constructions, one is described as having twisted limbs (*vakrāṅgāśca*) and the other as having rounded limbs (*parimaṇḍalāśca*). Figures 10

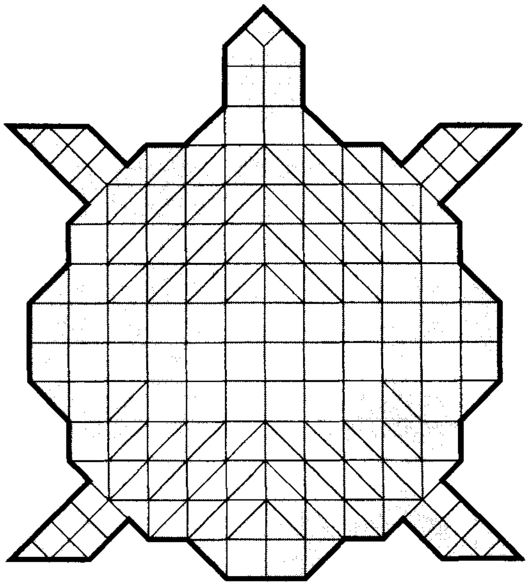

Figure 10: The kūrma citi: layers 1, 3, and 5.

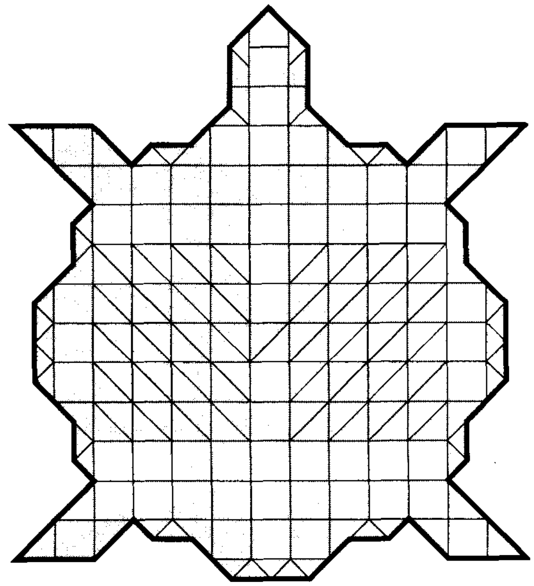

Figure 11: The kūrma citi: layers 2 and 4.

and 11 depict the first type.

The construction for the odd layers starts with a square of side 300 aṅgulas and then the four corners are removed by isosceles triangles with equal sides of 30 aṅgulas. Head, legs, sides and tail are now added with the result shown in Figure 10.

For the even layers, the starting step is a square of side 270 aṅgulas which is offset from the basic square for the odd layers by 15 aṅgulas. The plan of the final construction is shown in Figure 11.

5 Discussion

Having listed some of the constructions of the Śulba Sūtras, we are now in a position to look at how they fit together. A key example is the duality of the 'circle-square' constructions. As explained in 3.1, verse I, 61 of BSS gives a value of the square root of two as $\frac{577}{408}$. Using this value, there is a remarkable duality between the 'square to circle' and 'circle to square' results.

Suppose that we start with a square of side 2. By equation (1), the diameter d of the corresponding circle is

$$\begin{aligned} d &= \frac{2}{3}\left(2+\sqrt{2}\right) \\ &= \frac{2}{3}\left(2+\frac{577}{408}\right). \end{aligned}$$

Now use equation (2) to get the side of the square as

$$\begin{aligned} \text{side} &= \frac{2}{3}\left(2+\frac{577}{408}\right)\frac{9785}{11136} \\ &= \frac{13,630,505}{6,815,232} \\ &= 2+\frac{41}{6,815,232} = 2.000006, \end{aligned}$$

an accuracy of 0.0003%.

A second example is the use made of the 'difference of two squares' construction to construct a square with area equal to a given rectangle. Also, as explained in Section 2, construction may well form the basis of the method of finding the root of two.

Other examples are the conversion of squares to circles for the construction of citis in circular shapes such as the rathacakra citi, or the methods used to increase the size of the citis by scaling up all the dimensions. These and other similar results show the level of integration and completeness of the body of results in the Śulba Sūtras.

In the opening section, several examples were given of key words in the Śulba Sūtras that either had alternative deeper meanings or were related to such words. The study of the meaning of words in Sanskrit is a large and technical field founded on the work of Pāṇini. Because of the technical nature of the area, drawing the specific conclusion that the Sūtras were also intended to be dealing with the field of consciousness would require considerably more work. But there is an example that explicitly connects the construction of the citis with consciousness, namely Verse II, 81 of BSS. It reads that after having constructed a citi for the third time, then a *chandaścit* is to be constructed. The word *chandas* means mantra or mantras which are the 'structures of pure knowledge, the sounds of the Veda' [4, p. 3]. Commentators interpret this verse as indicating that the fourth and later constructions are to be carried out on the level of consciousness with mantras replacing the actual bricks. (See, for example, [8, 9].)

As with all the Vedic literature, the Śulba Sūtras can be read and interpreted on many levels. At the very least, they provide a fascinating chapter in the growth of geometrical and arithmetical knowledge and its application to the design and construction of complex brick platforms. But there are many indications, some of which have been pointed out above, that they are also a description, or perhaps a map, of the structure and qualities of the field of consciousness.

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