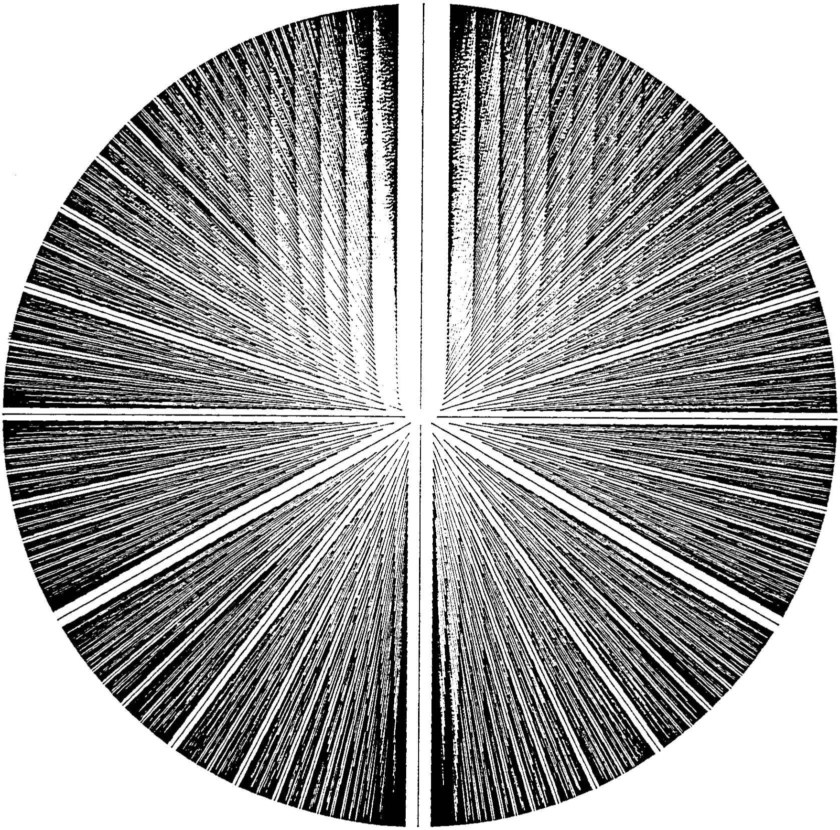
Images from Harmonics



Catalog of the Exhibition

Peter Neubäcker

The Exhibition “Images from Harmonics”

In studying harmonics, I have continually felt the need to share the fascination that this domain holds for me with others who might also be disposed to such a fascination, but who for various reasons have not yet found it readily accessible.

At the heart of the matter lies the fact that the path to harmonics leads mainly through hearing; in this sense it is a matter of exploring the structures in nature and spirit that draw upon musical elements. But in presenting these elements, images also frequently appear which give some idea of the fascinating nature of their content without it being necessary to understand the presented material in all its ramifications.

This exhibition is therefore intended to show a small selection from various areas of study, as far as these things can be rendered visually. From the aforesaid it is obvious that in this, the most important part of harmonics cannot be imparted - that can only be achieved through listening and intensive individual work - nor is harmonics as a whole represented. In many cases, the images were selected merely from an aesthetic point of view.

The exhibition can be divided broadly into two domains: one consisting of illustrations reproduced from books on harmonics from widely varied subject areas, the other of previously unpublished images and objects, the product of my own work. In this booklet, all the images reproduced are supplemented with commentary on their content, and also partly on their background within their respective domains of harmonics.

In the selection of images and their formats, a further harmonic element emerged that had not been planned earlier: each format, i.e. the image’s ratio of width to height, corresponds to a musical interval - and in the process of formatting it became apparent that all twelve intervals in the octave could be represented. The respective formats are indicated for the images, along with the corresponding musical intervals. Where no interval is indicated, the image is either obviously round or square - corresponding musically to the prime interval - or else it is in DIN format, the ratio $1 : \sqrt{2}$, which represents a tempered tritone in music. This booklet is also in DIN format, which gives an idea of how the various intervals in the picture formats “fit” with the tritone.

My heartfelt thanks to the “Freies Musikzentrum München“ for their support in the realization of this exhibition.

Peter Neubäcker München, November 1988

Translated from the German by Ariel Godwin (2006)

Image 1

The Monochord as a Model of the World

Robert Fludd (1574-1637)

Format 4:5 (Major Third)

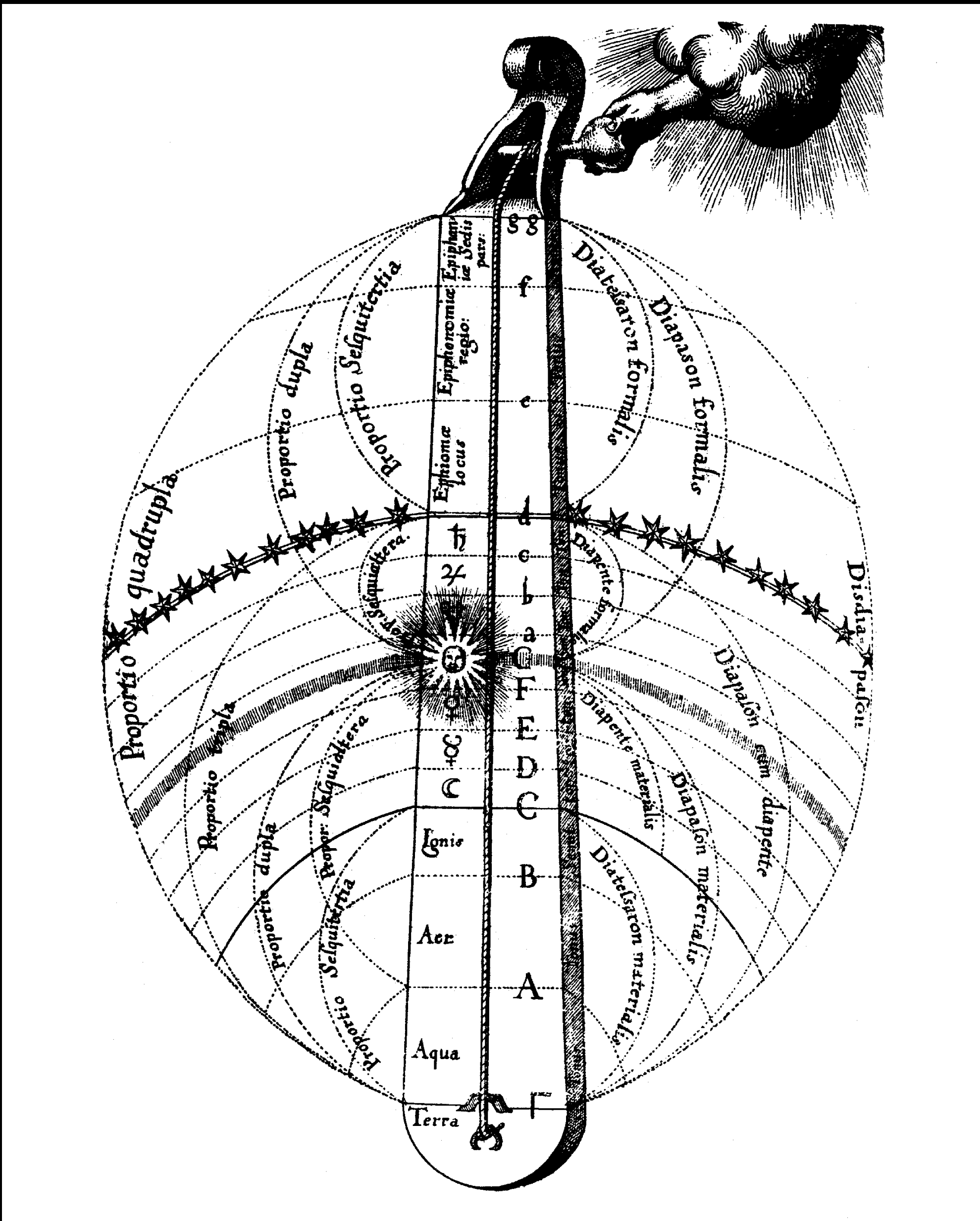
This image symbolizes an idea fundamental for harmonics: that the entire structure of the universe is musical, and that this structure can be expressed on the monochord. This idea, in Western culture, began around 500 B.C. with Pythagoras, who introduced the monochord as a musical measuring instrument.

The image shows the assignment of tones on the monochord, first to the traditional four elements, then to the seven planetary spheres, then to the heavenly domains. Significant here is the implication that the Creator (in the sense of the Greek Demiurge), who tunes the monochord from above, knows the ratios of the monochord as they were established before all creation, and transfers them into the domain of hearing by adjusting the string - thus making them perceptible in the realm of material creation.

But at the same time, the image symbolizes the end of an era of harmonic thought: from the time of Pythagoras up to the Renaissance, the concepts of the universal harmonic order and the harmony of the spheres were current for all thinkers, but hardly anyone questioned how these ideas were realized in empirical nature. Robert Fludd was a hermetic philosopher of the old school; he drew his ideas above all from tradition, and when people today attempt to reconstruct the conditions depicted in these images, they appear relatively arbitrary and interchangeable.

Robert Fludd's contemporary, the great harmonist and astronomer Johannes Kepler, was the first to forge the path now followed by modern harmonics: starting with the ancient idea of the harmony of the spheres, he sought to discover, in the movements of the planets, the places in empirical nature where this harmony was realized. In his Harmonices Mundi he presents the results of his studies, and concludes by contrasting his methods of observation with those of Fludd, of whom he says:

"One can see that he is satisfied with incomprehensible picture-puzzles of reality, whereas I begin by bringing things of nature, shrouded in darkness, right out into the bright light of knowledge..."



Proportio quadrupla
Proportio dupla
Proportio Sesquitertia
Proportio Sesquialtera
Proportio tripla
Proportio dupla
Propor: Sesquialtera
Proportio Sesquitertia
Proportio Sesquitertia
Diatessaron formalis
Diapason formalis
Diapente formalis
Disdiapason
Diapason cum diapente
Diapente materialis
Diapason materialis
Diatessaron materialis
Empyrean
Epiphaniae Sedis pars
Epiphenomiae regio
Ephionae locus
Ignis
Aer
Aqua
Terra
g
f
e
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c
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G
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Γ

Image 2

The Platonic Solids as a Model for the Solar System

from: “Mysterium Cosmographicum“ by Johannes Kepler (1571-1630)

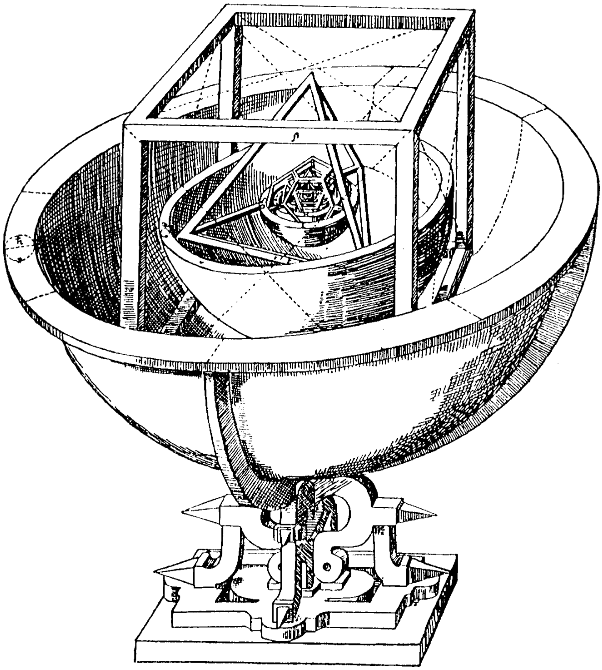
Format 5:9 (Minor Seventh)

Johannes Kepler was convinced from his youth onwards that the harmonic order of the universe could be proved empirically. His investigation of this was ground-breaking for the modern approach to natural science: according to his Laws of Celestial Mechanics, still applied today, a planet’s distance from the sun can be calculated from its observed orbital period - but this does not explain why it has exactly this distance for this orbital period. This was precisely the question that interested Kepler, and this type of observation is typical for the harmonic way of thinking: it is a matter of the identification of morphological correspondences, which comprise a “physiognomic” statement for the observer. His first approach to this subject was geometric:

When one attempts to assemble regular three-dimensional solids from equilateral polygons, it becomes apparent that only five such solids are possible. From equilateral triangles, three solids can be built: the tetrahedron from four, the octahedron from eight, and the icosahedron from twenty triangles. From squares, only the cube can be built, and finally the pentagon-dodecahedron can be built from twelve pentagons. These five solids were first described by Plato, and are therefore called the “Platonic solids”- they can be seen in Image 3. Now each solid can be placed inside a sphere, so that all the corners of the solid touch the surface of the sphere on the inside; and another sphere can be placed inside each solid, so that the sphere’s surface is tangent to the center of each surface of the solid from the inside.

Kepler then discovered that when these solids are placed inside each other so that the outer sphere around one solid is also the inner sphere inside the next, the radii of the spheres accurately correspond to the distances between the planets. This is illustrated as a model in Image 2, the lower figure being a magnification of the central part of the upper figure.

Admittedly, the outer planets (the first of which, Uranus, was discovered two hundred years after Kepler’s time) have no place in this model, since only five such solids are geometrically possible; and Kepler himself was not completely satisfied with the precision of the ratios. But this discovery encouraged him to look further wherever he could for clues to the harmonic structure of the universe.



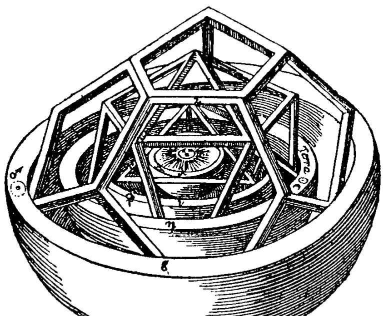


Image 3

The Platonic Solids and their Relationships

from: “Harmonices Mundi” by Johannes Kepler (1571-1630)

Format 8:9 (Whole-Tone)

In 1619, Johannes Kepler published “Harmonices Mundi - the Five Books of Universal Harmonics“, which he considered his own most significant work, since it described the musical harmony of the solar system which he had spent his life seeking and had finally found.

In the first two books of this work, he derives qualities of numbers from geometric investigations of the ways in which the regular solids can be arranged. Image 3 comes from the second book; here he puts the regular figures together into three-dimensional bodies. The Platonic solids appear again, here connected to the elements, fire, water, air, and earth, and to the cosmos. A few figures derived from these solids also appear, such as the star-solids first described by Kepler.

In the third book, he transfers the number qualities found in the first two books into the audible domain, using the monochord, and develops an all-embracing musical theory. In the fourth book he applies the insights gained in geometry and music to astrological observations.

In the fifth book, the astronomical part, he presents his great harmonic discovery: the realization of musical harmonies through the movements of the planets. After first musically examining the distances and orbital periods of the planets from various viewpoints, and obtaining no satisfying results, he then examines the angular velocities of the planets, as an observer on the sun would perceive them, and establishes that these velocities, at the perihelion and aphelion of each planet, when positioned on the monochord, yield the most beautiful musical harmonies - for each individual planet as well as for the various planets compared with one another. This result also applies to the outer planets discovered later, as calculated in modern times; it appears to be a universally valid principle of the structure of our solar system.

Astonished at his discovery, Kepler writes: “...Whether people read this book now or later does not matter. It can wait a hundred years for the readers, if God Himself waited six thousand years for His work to be seen...”

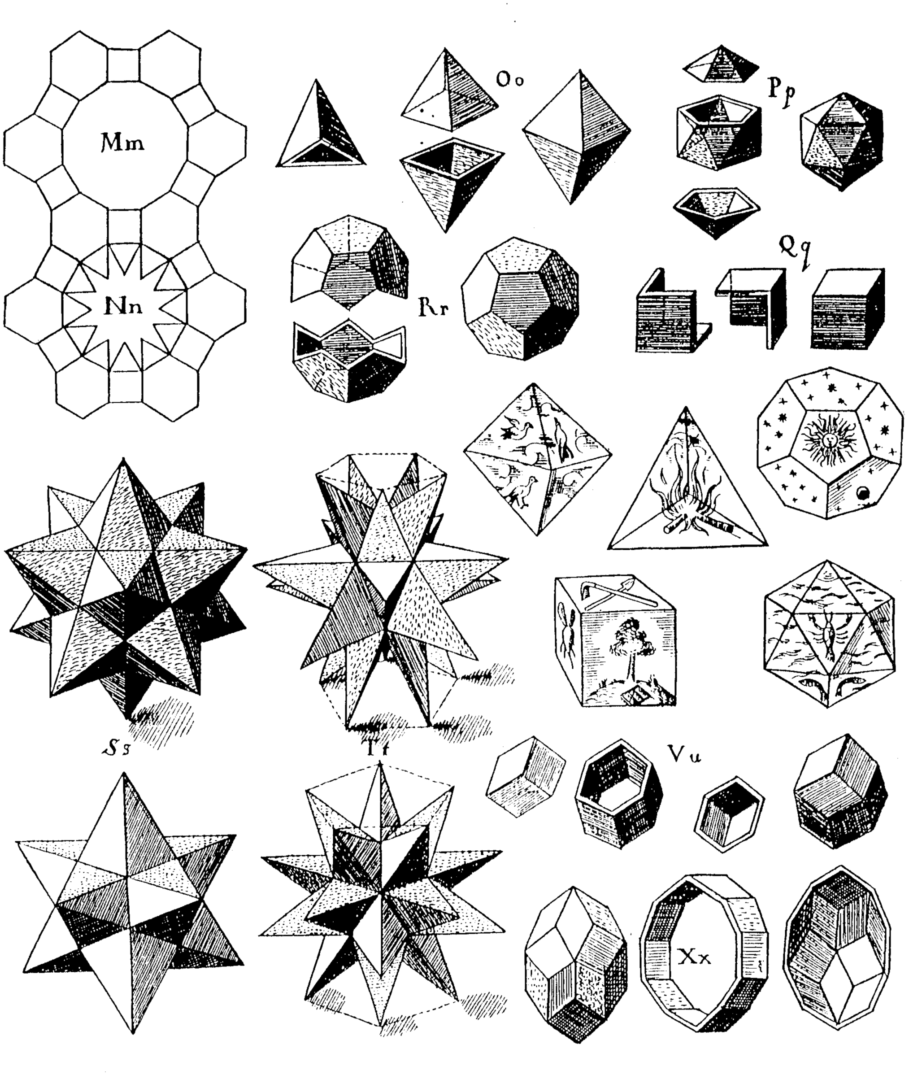

Mm
Nn
Oo
Pp
Qq
Rr
Ss
Tt
Vu
Xx

Image 4

Variations in the Forms of Snowflakes

from: “Atlas der Krystalformen” by Victor Goldschmidt[1] (1853-1933)

In his observations of geometric structures in nature, Johannes Kepler also studied snowflakes and their six-pointed forms, and presented his ideas on this in the small volume “A New Year’s Gift, or On Six-Cornered Snow”. Here, in addition to causal “scientific” considerations, he makes more morphological observations, such as comparing the six-pointed form of the snowflake and the six-petaled flowers of the lily family, and contrasting these with the five-petaled blossoms of fruit trees-and seeks to draw conclusions on the natural qualities of these plants thus revealed.

Image 4, however, is not from Kepler, but is much more recent: it is from Victor Goldschmidt’s “Atlas der Krystalformen”. Just as Kepler is famous as an astronomer, but hardly anyone knows that the results of his studies emerged in the course of a search for the musical harmony of the universe, so the “Atlas der Krystalformen” is known as the standard work of this crystallographer, but it is little known that Goldschmidt studied the connections between crystallography and harmonics: in his book “Über Harmonie und Complication”, he investigates analogous phenomena in music, colors, and various domains of nature, on the basis of a fundamental law he developed from crystallography.

It was above all the works of Victor Goldschmidt, along with those of Kepler and Thimus, that inspired Hans Kayser’s work - Kayser reestablished harmonics as we know it today and presented it in a new form for our times.

The commentary on Images 17 and 18 also relates to Goldschmidt’s system of “complication.”

[1] Translator’s note: This was Victor Mordechai Goldschmidt, not to be confused with Victor Moritz Goldschmidt (1888-1947), another chemist and crystallographer.

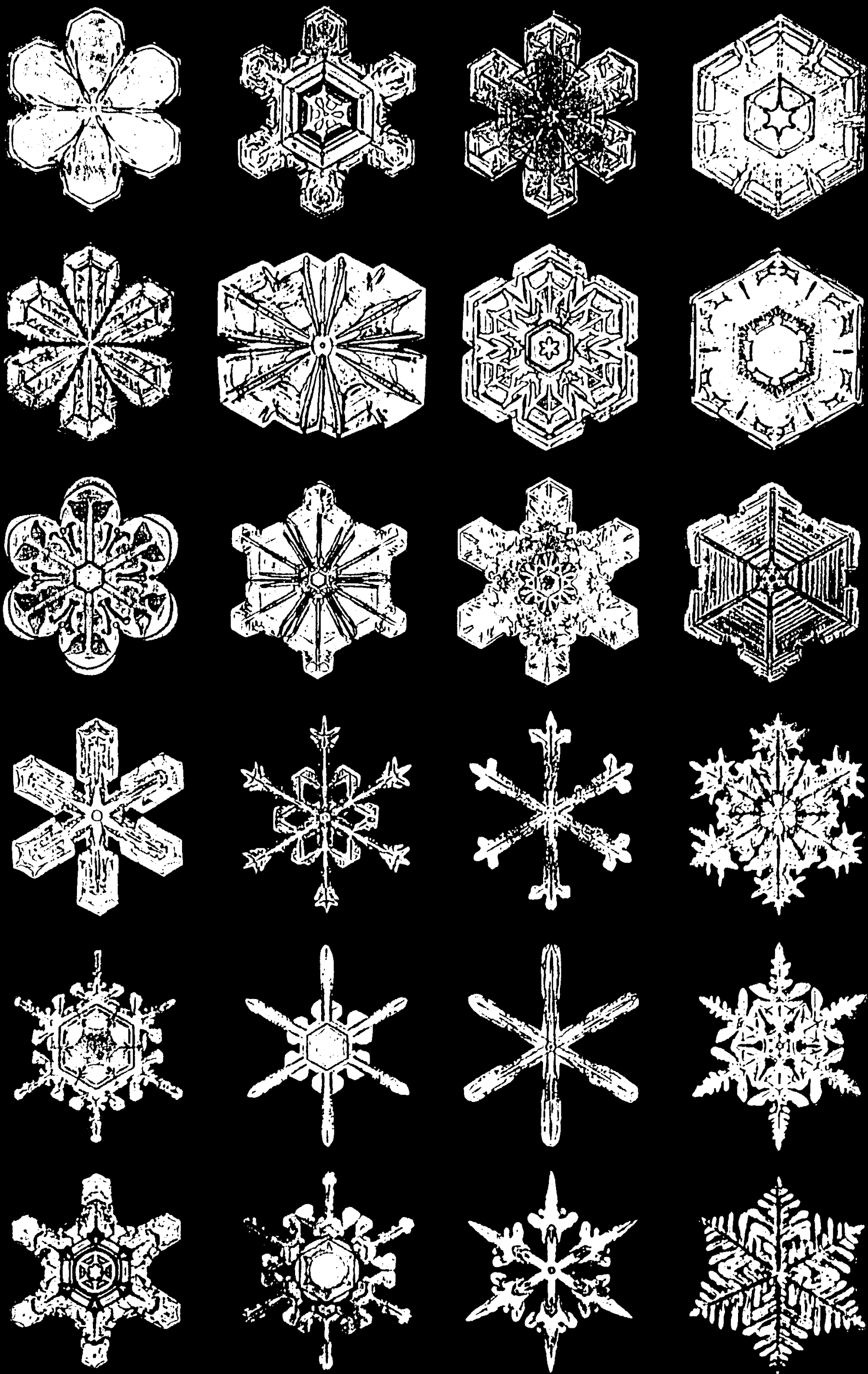


Image 5

The Number Five in the Plant Kingdom
from: “Urzahl und Gebärde” by Hugo Kükelhaus

Die Zahl Fünf im Lauf der Venus
from: “Rhythmen der Sterne” by Joachim Schultz

Format 2:3 (Fifth)

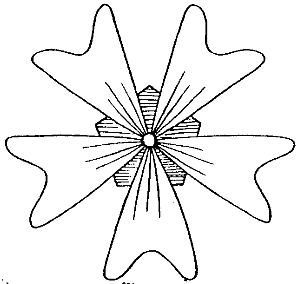
Once one has formed a qualitative sensory rapport with the numbers in harmonics through studying their interval qualities, one can look further in nature to see which things express themselves morphologically through which numbers. On the one hand, the structural numbers of things in nature become the expression of the natural quality that is represented through these numbers, and on the other hand the qualitative image of the number itself deepens when one sees which natural qualities serve for the expression of which numbers.

The previous image was an example for the number six, which corresponds harmonically to the fifth as the octave of three. Other examples of the six include the structure of members of the lily family in the plant kingdom, honeycombs in the animal kingdom, and the orbit of the planet Mercury (relative to Earth) in the solar system.

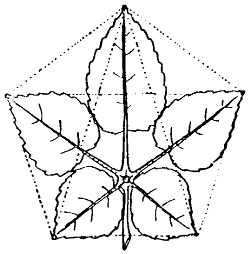
Images 5 and 6 show examples of the number five. Five corresponds harmonically to the thirds - located between four and six, it generates the minor and major thirds. It is a characteristic of the five that it is not present as a structural number in the mineral domain; it only starts appearing in the plant kingdom. The number five is exhibited by plants in the rose family, hence by all significant fruit-bearing plants.

In the solar system, Venus manifests itself through the five, forming an almost complete pentagon in its conjunctions with the Sun in the zodiac - the illustration shows the path of Venus in its progressively decreasing and increasing distance from the earth, at the center, over the course of eight years. The ratio of 5 conjunctions in 8 years yields the ratio of the minor sixth, which in turn is an inversion of the major third. The ratio 5 : 8 is also an approximation of the Golden Section, which for its part is manifested completely in the five-pointed star through the progressive division of the sectional lengths of the star.

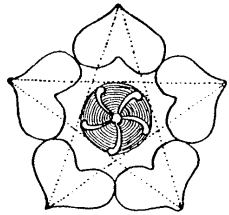
The commentary on Images 8 and 17 also relates to the Golden Section and its role in harmonics.



Malve



Himbeerblatt



Glockenblume

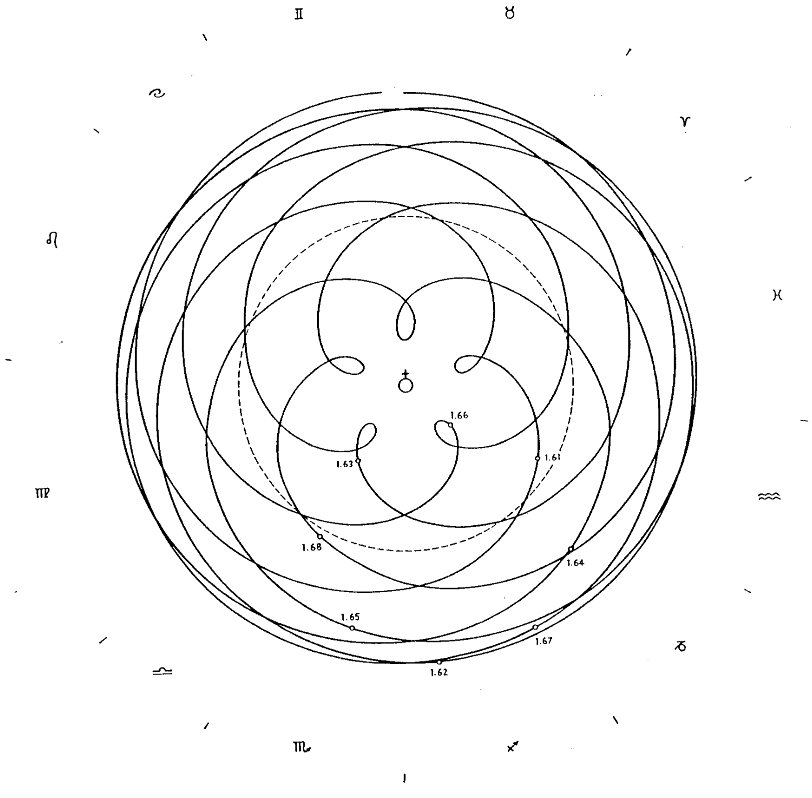


Image 6

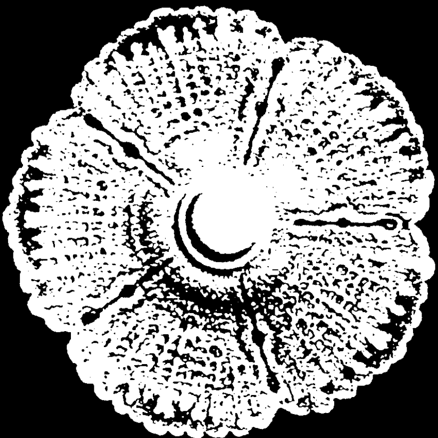
The Number Five in the Animal Kingdom
from: “Urzahl und Gebärde” by Hugo Kükelhaus

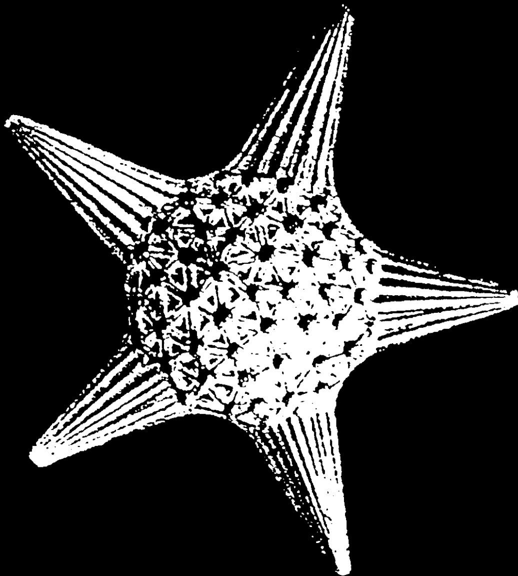
This illustration, like the upper part of the preceding one, comes from “Urzahl und Gebärde” by Hugo Kükelhaus. This book is not, strictly speaking, a book on harmonics, since Kükelhaus hardly discusses the tone-ratios of the numbers; but in his method of number observation he is completely in the spirit of Pythagoras and Kepler, and much of the material in his book contributes to a qualitative observation of numbers.

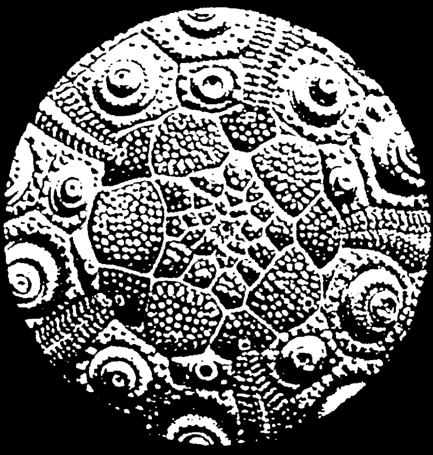
The image shows sea creatures: sea urchins and a starfish. In the sea, the “cradle of life,” the number five appears here morphologically in a most pronounced and unequivocal manner, the foreshadowing of the five fingers on the human hand, with which Creation takes hold of the world. Paleontology has also proved that the ancestors of all mammals today definitely had five toes, and that only through later specialization, such as in the horse, only one was left.

Kükelhaus writes of this picture:

“Thus it might appear as a friendly accordance that crystals know nothing of the number five and remain in the forms of the triangles, squares, and hexagons that resound through their domain. Since they do not reproduce, they cannot break through their world into the wonder of the fruit, into the realm of the Mothers. The formal equilibrium, the static symmetry of crystals, changes in plants and animals to an inwardly manipulated changeable equilibrium, a “dynamic symmetry,” whose archetype is the pentagon with its unequal constant division. The silence of the sea’s depths, the eternal night, and the great pressure of the water weave together thoughts of sacrificial freedom from the memory of the distant wisdom of the stars, and send their messages as floating and flying star-pentagons into the higher levels of the world of light...”







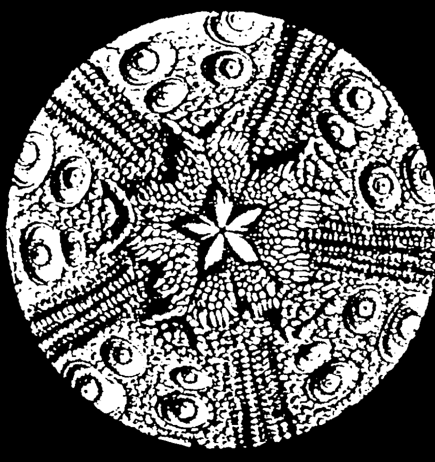


Image 7

Numeric Structures in the Forms of Plants
from: “Harmonia Plantarum” by Hans Kayser (1891-1964)

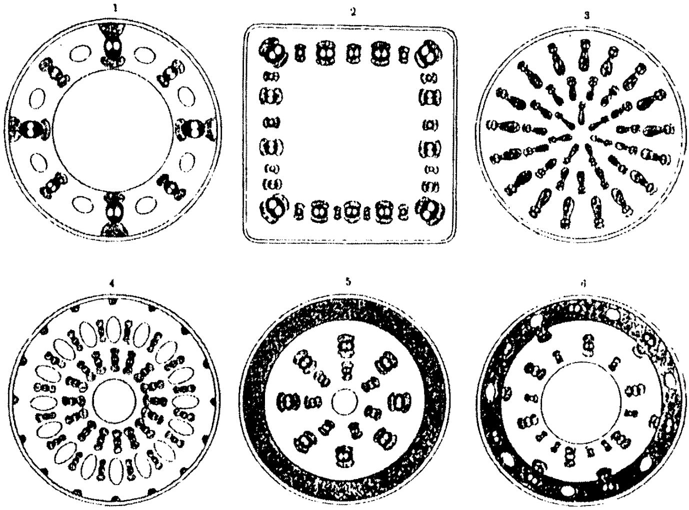
Format 5:8 (Minor Sixth)

Looking at trees or flowers, the visible outer forms give no hint of structuring through numbers. Outward appearances emphasize the polar opposite of this structure: living organic growth in communion and competition with all of surrounding nature.

But looking more deeply into the hidden structures, one will continually see an order here, based on numbers and symmetry - as manifold in its implementation as corresponds to the diversity of the plants. Image 7 shows sections of stems and stalks of various plants. Here one can see pure structures of four and eight, corresponding harmonically to the principle of the octave, and structures of five and six, corresponding to the principles of the third and fifth; but there are also higher numbers. Here one must decide cautiously whether the plant always manifests itself through the same number - only in this case would that number truly be a characteristic structural number. In cases of higher numbers, they often vary from one individual to another in the same species. Here a “ranking” of the numbers emerges: the smaller the number, the more “power of character” it contains; the larger the number, the more its character is mixed with that of its neighbors.

This image, like the three following ones, is from the work of Hans Kayser. Harmonics as we know it today was largely reestablished for modern times by Kayser. Inspired by the works of Johannes Kepler, Albert von Thimus, and Victor Goldschmidt, he performed his own harmonic investigations in many domains, and published his results in several books - here we will only mention “Orpheus - vom Klang der Welt”, “Der hörende Mensch, “Grundriß eines Systems der harmonikalen Wertformen”, “Harmonia Plantarum”, and most importantly “Lehrbuch der Harmonik” - unfortunately these works are all out of print today.[2] “Akroasis - die Lehre von der Harmonik der Welt” can still be found in bookstores; in this book Kayser offers an overview of the various domains and conclusions of harmonics.

[2] Translator’s note: respectively, Orpheus, The Sound of the World, The Hearing Human, Outline of a System of Harmonic Value-Forms, The Harmony of Plants, and Textbook of Harmonics. An English translation of Kayser’s Textbook of Harmonics was published by the Sacred Science Institute in 2006, and the S.S.I. offers facsimile editions of some of Kayser’s works.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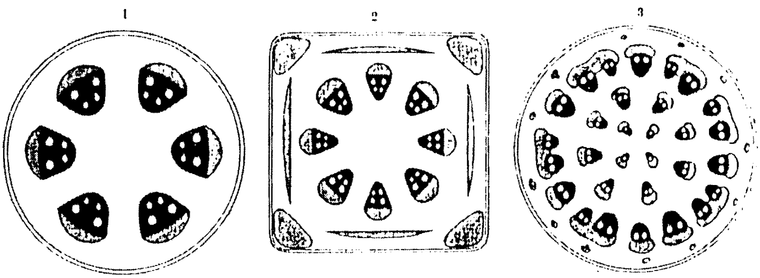
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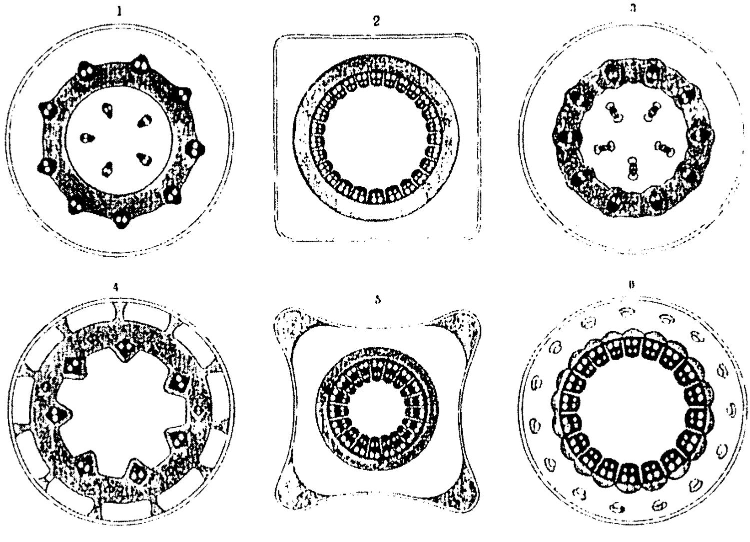
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Image 8

Numeric Structures of Leaf Positions

from: “Harmonia Plantarum“ by Hans Kayser (1891-1964)

Format 3:4 (Fourth)

In examining the positions of leaves on various plants, it can be established that for the respective species, definite laws of repeating leaf positions exist, depending on the number of revolutions around the stem. In the simplest case, the next leaf is opposite the preceding one; here the ratio of revolution to number would be 1 : 2. In other cases ratios appear such as one revolution for three leaves, or two revolutions for five leaves, i.e. the ratios 1 : 3 or 2 : 5, corresponding harmonically to a duodecimal or to an octave plus a third, respectively. Placing together the ratios that emerge produces the “main series of leaf position numbers”:

1:2 1:3 2:5 3:8 5:13 8:21 13:34 21:55 ...

Observing these numbers more closely, one sees that the series is constructed so that on each side, each number is the sum of the two preceding numbers, thus forming the series:

1 2 3 5 8 13 21 34 55 89 144 ...

This series is known as the “Fibonacci Sequence”; at the beginning, the ratios of each pair of neighboring numbers are musical intervals; later they approach the “Golden Section,” a ratio that meets the requirement that the smaller part has the same ratio to the large part that the larger part has to the sum of both parts. This ratio is irrational and is not a harmonic proportion - but to the ear’s perception, the ratio 8 : 13 approaches the Golden Section so closely that in listening, hardly any change of the interval can be detected. Since, furthermore, the number eight is an octave of the root note, the number 13 can be viewed among the smaller numbers as a “representative” of the idea of the Golden Section in the rational domain.

Here a remarkable polarity emerges between the idea of the Golden Section on the one hand and the small whole numbers on the other hand. Kayser writes: “There are no leaf position series in nature, only leaf positions!” - meaning that the realization always occurs through the harmonic numbers; but the idea of the Golden Section somehow emerges behind this. The commentary on Image 17 gives another example of this polarity.

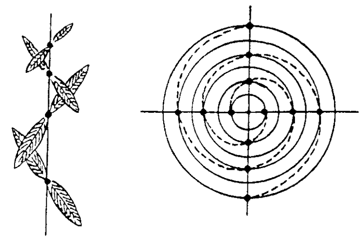


Abb. 65

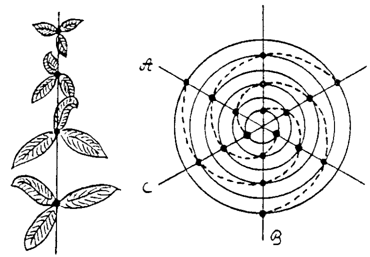


Abb. 66

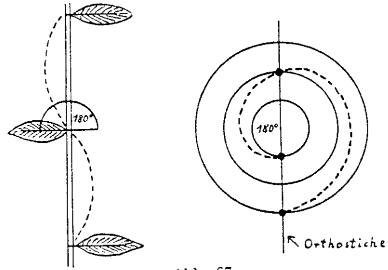


Abb. 67

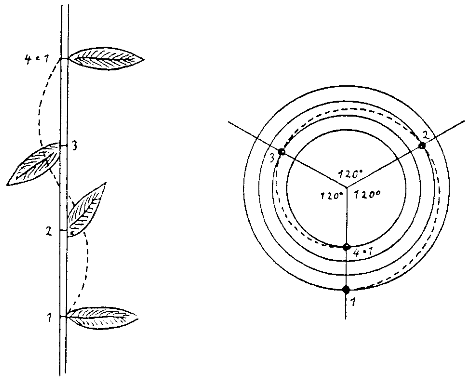


Abb. 68

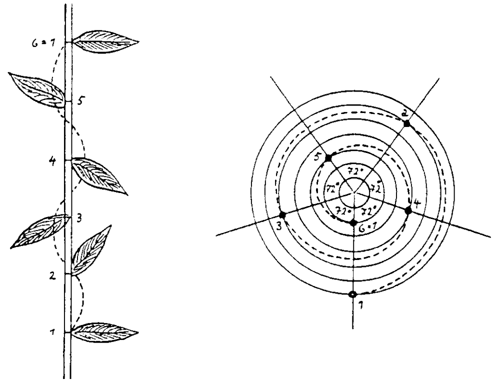


Abb. 69

Image 9

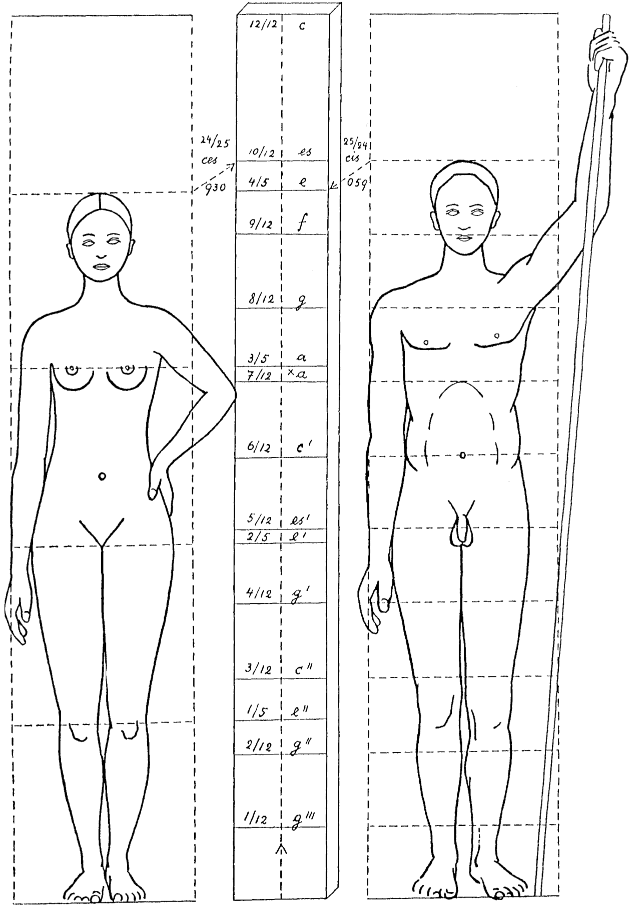
The Musical Measure of the Human Body

from: “Lehrbuch der Harmonik” by Hans Kayser (1891-1964)

Format 5:6 (Minor Third)

Kayser writes, regarding these illustrations: “The search for a rational understanding of the structure of the human form is ancient. Archaic imagery is so strictly uniform that, even if we did not know of a ‘canon,’ we could assume that one or many very probably existed for this early epoch in art. ... The ancient sculptors sought to elucidate the factual qualities of these models, which they continuously created anew in thousandfold modifications; this is shown in proportion grids for reliefs found in Egyptian tombs, but above all in the legacy of the famous ‘Canon of Polyclitus,’ of which we admittedly know nothing specific, but which must have been authoritative to the highest degree for the Greek sculptors. Almost all significant artists in the Italian and German Renaissance studied proportions very earnestly, especially those of the human form.”

Regarding the left side of the image: “I show the two figures (woman and man) from Wyneken’s table 5, but for clarification I include the tonal ratios of the monochord. ... As one can see, Wyneken uses the same measuring unit for man and woman, but divides the space of the man (string-lengths) into third-ratios, and that of the woman into fifth-ratios. Thus the top of the man’s head (measured from below) is 5/6 of the unit, that of the woman 4/5 of the unit. ... To me, the most interesting thing in this regard is the aspect of the tone evaluation of the male-female ratio 5/6 : 4/5. ... This (by string-length) is the ratio of the minor third 5/6 e to the major third 4/5 e! ... In my Harmonia Plantarum (pp. 201-206) I attempted to show that the third, whether major or minor, is in itself not only the ‘sex-tone’ in the musical sense, since it defines the major and minor triads, but also has the capability of interpreting the problem of sexuality in a completely new way, as the ‘pentadic’ (fifth-) ratio in the morphological-metaphysical sense. Man and woman, therefore, have on average the ratio of two third intervals in their pure size-ratios, i.e. their pure external measurements hint at the inner sexual relationship of the two sexes!”



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8/12 g
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5/12 es'
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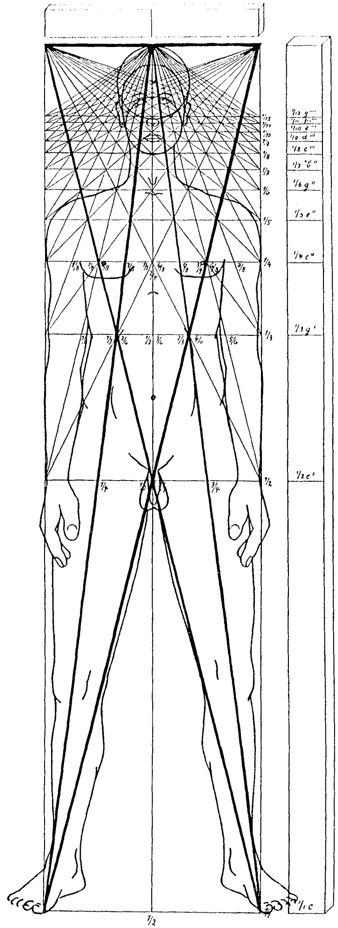


1/6 g''
1/5 e''
1/4 c''
1/3 g'
1/2 c'
1/1 c
1/2

Image 10

Emergence of Three Types of Architecture

from: “Lehrbuch der Harmonik” by Hans Kayser (1891-1964)

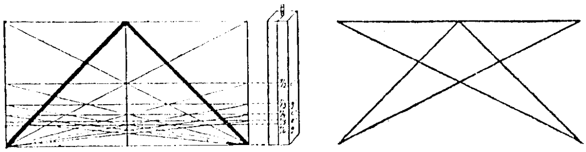
Format 1:2 (Octave)

This image, like the right side of the preceding one, comes from a “harmonic division canon”: If one takes any rectangle and draws diagonals in it, then divides the rectangle in two through the intersection point of the diagonals and draws more diagonals in the newly created rectangles, proceeding this way at one’s discretion, the intersection points always produce harmonic intervals. This division canon was discovered by Hans Kayser in medieval architecture.

Regarding Image 10, Kayser writes: “This harmonic analysis not only gives us an idea of how the ancient architects, who were doubtless familiar with this ‘rational segment division,’ may have proceeded in their designs. It also gives us something far more important: an inner characterization for these three styles in relation to each other. The lengthening of the monochord - the increasing of the octave space - can be observed psychically as an enlargement of the psychical configuration space. Whereas in the Egyptian aspect, the tone-lines still adhere to the ‘earthly’ and allow only the pyramid as a prototypical form, Romanesque architecture introduces the ‘tower’ and thereby allows for basilical symmetry. In the Gothic aspect, this tower is of commanding importance and pulls all other forms upwards along with it, reaching the maximal expansion, the greatest possible upsurge of the architectonic measure as a symbol for the relationship of the earthly and the human to the divine.

“Naturally, for these three types of building, one cannot measure ‘after the fact’ whether the pyramid’s angles are exactly in tune, whether the ratio of church and roof in the Gothic aspect is exactly ‘right,’ and so forth. As for all ‘sound-images,’ it is not immediately essential to make special individual measurement analyses, but instead to produce the evolution of the model - here of three architectural styles - from a unified idea.”

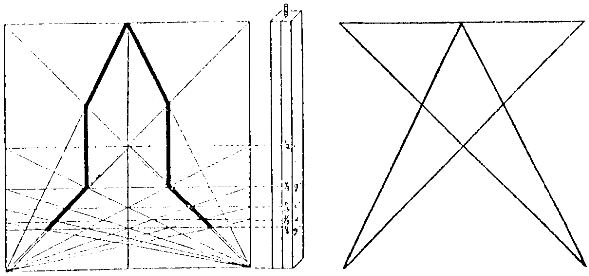
1. Oktave



Ägyptisch

(1 : $^1/_2$)

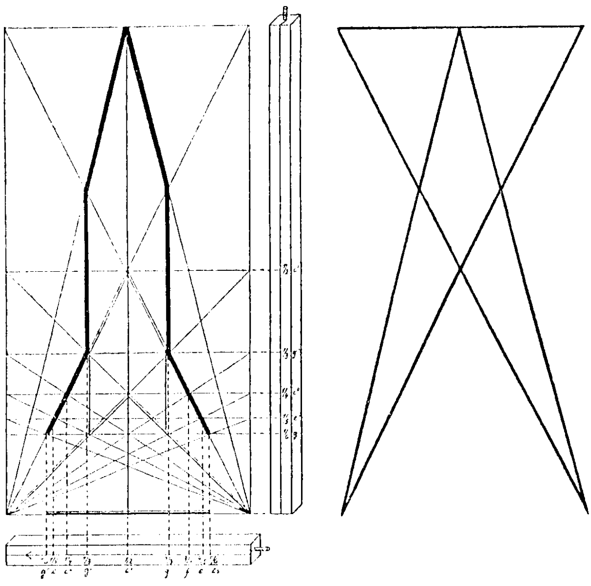
2. Oktave



Romanisch

(1 : 1)

3. Oktave



Gotisch

(1 : $^2/_1$)

Entstehung dreier Stilarten
aus demselben harmonikalen Teilungskanon durch Oktavpotenzierung,
also durch Erweiterung des seelischen Raumes

Image 11

The Proportions of the Temple of Athena at Paestum

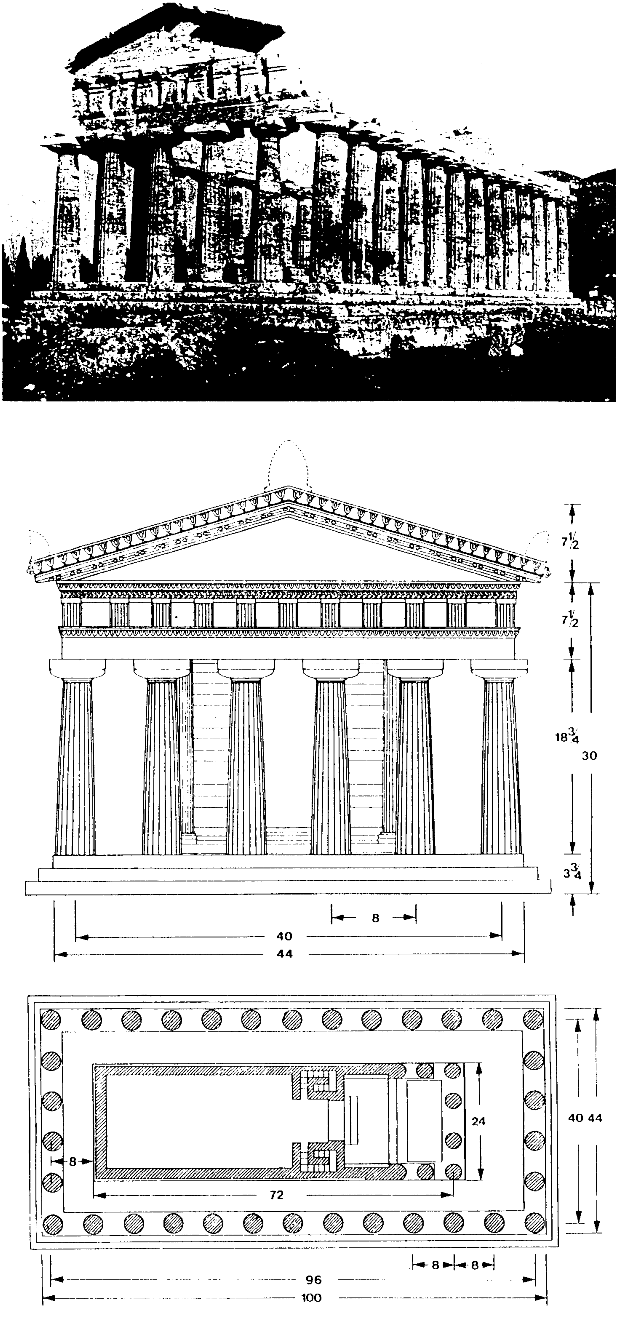
from: "Architektur und Harmonie" by Paul v. Naredi-Rainer

Format 8:15 (Major Seventh)

Whereas the previous image relates to ideal observations on archetypes of fundamental architectonic principles, this and the next image examine existing buildings concretely in terms of the harmonic measures at their basis. The book Architektur und Harmonie by Paul v. Naredi-Rainer contains many such investigations. Regarding this image he writes:

"The flourishing of the Pythagorean school, approximately contemporary with the construction of the Temple of Athena, makes it seem possible that Pythagorean speculations on numbers were reflected in the Temple's architectonic expression. The numbers of the Temple's axial measurements, 40 x 96, can be derived from the numbers of the Tetraktys (1 2 3 4), which the Pythagoreans held sacred: division by 4, the measure of half the span, yields 10 as the sum and 24 as the product of the Tetraktys numbers (40:4 = 10 , 10 = 1+2+3+4 ; 96:4 = 24 , 24 = 1x2x3x4).

"This already points to the central meaning that is attributed to the reciprocal correspondence of numbers and tones in Pythagorean thought. Hans Kayser, who revived Pythagorean harmonics in our century, performed a harmonic analysis of the Temple of Paestum, translating the rational number ratios into musical intervals. Inspired by the romantic idea of viewing architecture as 'frozen music,' Kayser perceives numbers and proportions not as 'purely intellectual quantities,' but as 'tones, intervals, and melodic types, "nomoi," which are identical to the forms in our psyche and express them in many direct ways as mere numbers and measurements.' Amidst all the fascination emerging from this harmonic approach, the question must be asked of whether the ancient architects were as familiar with the philosophical and mathematical theories of their time as, for example, the humanistically educated architects of the Renaissance - Kayser, in any case, is convinced that 'where the architect knew nothing of harmonic proportions, he still applied them, simply because instinct and feeling led him to arrange his plans and ideas according to these proportions.'"



Athenatempel in Paestum (um 510 v. Chr.)

Image 12

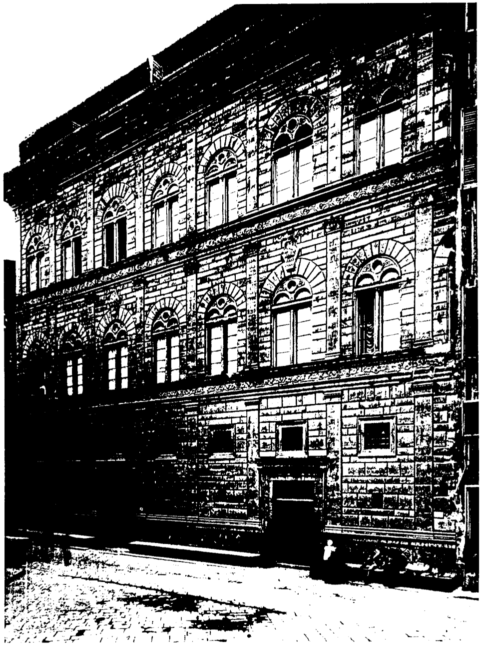
Musical Intervals in the Façade of the Palazzo Rucellai

from: Architektur und Harmonie by Paul v. Naredi-Rainer

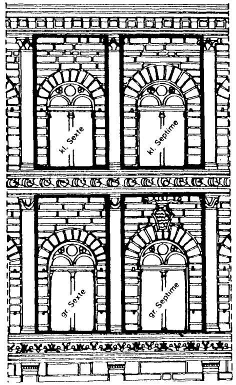
Format 9:10 (Whole-Tone)

Regarding this image, Naredi-Rainer writes: "The architectural aesthetics of the Renaissance are saturated with the Pythagorean-Platonic idea that the harmony of the cosmos is built upon musical number ratios; its first and most significant verbalization was in Alberti's tract on architecture, De re aedificatoria libri decem, which was written in the mid 15th century but first printed in Florence in 1485. The fundamental aesthetic principle, the 'concinnitas,' is manifested according to Alberti in certain numbers and proportions, which appear most clearly in music. Thus one should obtain the entire law of the relationship from the musicians, who know these numbers best.

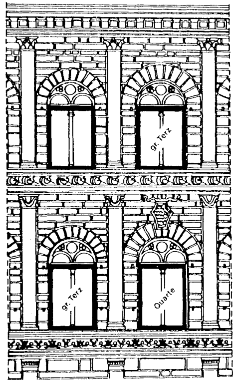
"... The three-story façade of the Palazzo Rucellai, subdivided into pilasters and cornices, is now 7 axes across, but was originally conceived upon 5 axes. The measuring unit, the Florentine braccio equal to 58.3 cm, is visible in the width of the finely carved pilasters, which together with the fascia organize the wall surface into a grid-patterned arrangement and frame the 'outer panels' thus formed. However, this grid is in no way uniform, but is subtly varied through the differences in story height and axis width: the middle axis accentuated by the doorway is wider than the other axes by the ratio of a whole-tone ($5\,^2/_5 : 4\,^4/_5 = 9 : 8$). Subtracting this difference in width of $^3/_5$ braccia from the total width (30 $^3/_5$ braccia) yields the round measure of 30 braccia, whose ratio to the total height is that of a minor third (36 : 30 = 6 : 5). The complementary interval to the minor third, completing the proportion of the façade panels, is the major sixth. It determines the proportion of the central panel on the 1st story above the ground floor, engraved with a coat of arms, the Piano Nobile ($9 : 5\,^2/_5 = 5 : 3$). The outer panels of the remaining axes have the proportions of the major seventh on the 1st upper story, the minor seventh in the 2nd story, while the outer panels of the middle axes on the 2nd story represent the ratio of a major sixth, which is constructed in a quasi-mirror image relationship from the ratio of the height of the doorway to its width (including the doorframe). The high rectangles of the window apertures repeat the form of the outer panels; they yield the ratios of the fourth and major third."



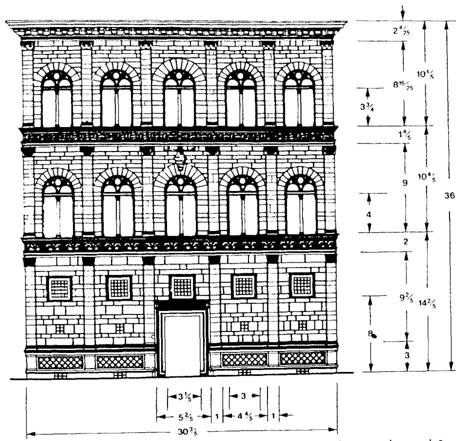
70 *Fassade des Palazzo Rucellai in Florenz (Leon Battista Alberti, begonnen 1455)*



72 *Palazzo Rucellai in Florenz, Fassadenausschnitt, Proportionsschema der von Pilastern und Gesimsen gerahmten 'Schauflächen'*



73 *Palazzo Rucellai in Florenz, Fassadenausschnitt, Proportionsschema der Fensteröffnungen*



71 *Palazzo Rucellai in Florenz (vgl. Abb. 63), Rekonstruktion der Fassade mit 5 Achsen, nach Sanpaolesi, Maßangaben in florentinischen bracci à 58.3 cm*

Image 13

Musical Number Ratios in Geometry

from: “Wege zur Harmonik” by Rudolf Stössel

Format 3:5 (Major Sixth)

Just as Johannes Kepler saw the foundations of harmonics in geometry, which describes the fundamental laws of space, so Rudolf Stössel also begins with geometry in his work, and finds many ratios of small whole numbers there, which form the building blocks for musical intervals. Regarding the illustrations he writes:

“Cicero, as he told us, found an old crumbling gravestone in the undergrowth, the carving on which indicated that it was the gravestone of Archimedes.

“Figure 4 reproduces the drawing and shows the two primal geometric forms, the square and the circle in their closest relationship, and also the symmetrical form of the isosceles triangle. If this image is rotated around its axis of symmetry, three bodies emerge, a cone, a sphere, and a cylinder. Their volumes have the exact ratio 1 : 2 : 3. These primal forms correspond to a proportion of the three smallest whole numbers, to the beginning of the number series, and to mathematics generally. But they also correspond to the tones c c’ g’, i.e. the intervals of the octave and fifth, the strongest consonances, also the primal consonances, so to speak, and to the duodecimal.”

The two middle rows in the illustration mostly speak for themselves - they show the many forms in which ratios of small whole numbers emerge from the combination of the primal forms of the square and circle. Regarding the lower figure, Stössel writes:

“I now wish to combine elementary forms once again, an equilateral triangle with its inscribed circle (Fig. 11). We place the compass in the top corner and draw an arc through the lower corners. This gives us a large sector. Now we construct the inner circle of the sector, which touches its radii and the arc. I will leave it to the reader to construct its center and calculate my result. In any case, the ratio of the area of the smaller circle to the greater one, and then to the sector, is 3 : 4 : 6.

“Here we already see the fifth 6/4 = 3/2, the fourth 4/3, and the octave 2/1. Now we multiply the numbers by 2, getting 6 : 8 : 12, and add the small sector with the dashed arc, then we have four areas with the ratio numbers 6 8 9 12 - the Harmonia Perfecta Maxima.”

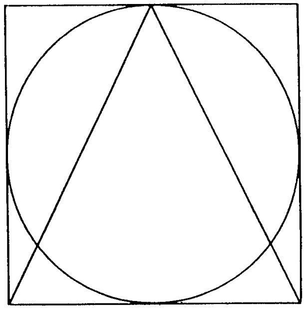


Fig. 4 Grabstein des Archimedes

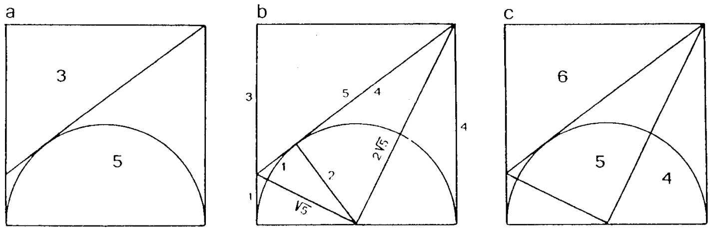


Fig. 6 a und c: Flächenverhältnisse, b: Streckenverhältnisse[2]

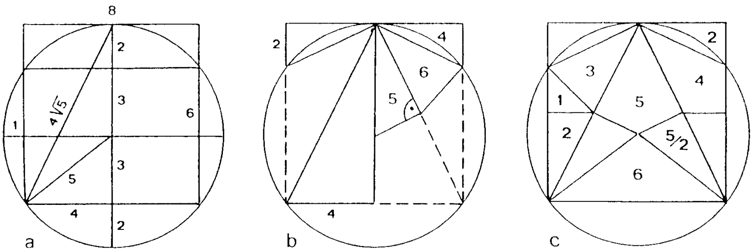


Fig. 10 a: Streckenverhältnisse, b und c: Flächenverhältnisse

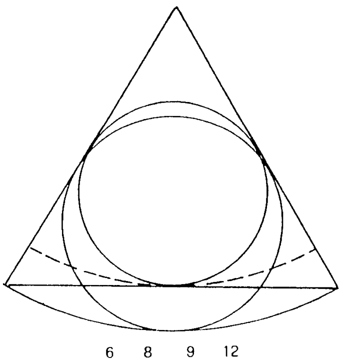


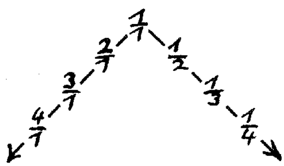
Fig. 11 Flächenverhältnis:
Harmonia Perfecta Maxima

Image 14

The Lambdoma as a Model

Pattern developed by Albert von Thimus (1806-1878)
Model drafted and constructed by Peter Neubäcker

In 1868, Baron Albert von Thimus published his two-volume book Die harmonikale Symbolik des Alterthums, in which he develops, among other things, the Lambdoma - he considered it to be the rediscovery of a Pythagorean scheme, but it was more likely a new construction in the Pythagorean spirit. From ancient times we only know of the Lambdoma's basic form, from which it got its name: when all whole numbers expressed as fractions, and their reciprocals, are arranged in two basal series in the form of the Greek letter Lambda Λ, this image emerges:



These basal series simultaneously represent the overtone series and its reflection, the undertone series. If the space between them is filled out in terms of the overtone series for each undertone, or vice versa, the result is the complete diagram, which can be continued in both directions without limit. Image 14 is from Kayser's Lehrbuch der Harmonik, and goes up to index 16; the model exhibited goes up to index 32.

This diagram exhibits various peculiarities which are highly interesting both musically and symbolically. One of these peculiarities is that all fractions that have the same tone-value are connected by a straight line, and all these lines meet at a point which is actually outside the diagram: the point 0/0. These lines are called equal-tone lines; the string in the model represents one of these equal-tone lines in each position. If the string is positioned, for example, at the value 2/3, it can be seen also to pass through the values 4/6, 6/9, 8/12, etc., which all produce a fifth. At the same time the bridge on the monochord marks off precisely these proportions, so that the ratio 2/3 is audible as a fifth (the upper part of the string represents the interval in each case). The second string represents the base tone, always remaining the same. In this manner all numeric proportions can be made audible in the model.

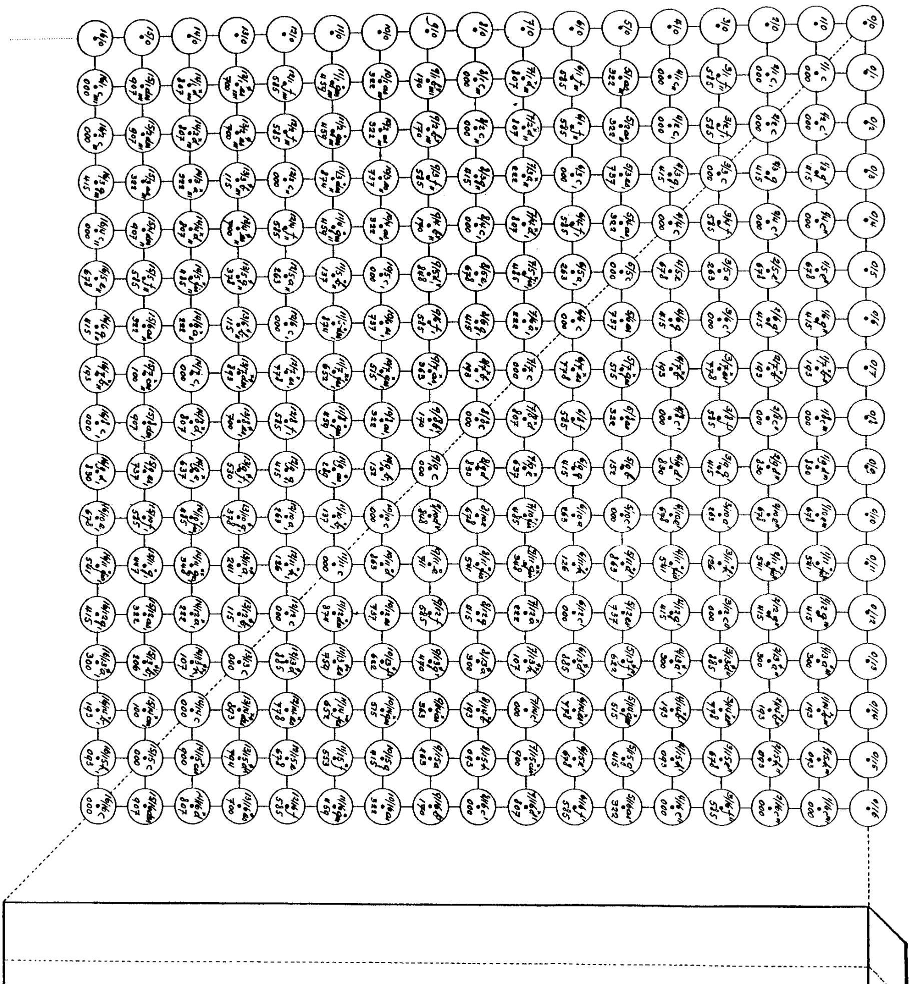


Image 15

Spatial Logarithmic Illustration of the Lambdoma
by Rudolf Stössel
Displayed at the exhibition “Phenomena” in Zurich, 1984
Photo by Dieter Trüstedt

The structures of the Lambdoma can be further clarified by illustrating the values that exist in each field in the third dimension. For example, the values 1/1, 2/1, 3/1, and 4/1 represent the single, doubled, tripled, and quadrupled string lengths of the monochord, which can then be built in a model as column heights reaching upwards.

But the model thus emerging does not correspond to the auditory experience, since for the ear the differences in tones become continually smaller toward the higher numbers; in other words, the hearing of the intervals takes place logarithmically. The model shown here takes these conditions into account.

The outline of this model is quadratic, like the Lambdoma itself, and the diagonals of the Lambdoma run directly through the middle of the image at a constant height. Upon these diagonals lie the values 1/1, 2/2, 3/3, etc., i.e. all values that are identical to the base tone. This line is called the “generator-tone line.” Starting from it, all tone-values go upwards and downwards as logarithmic curves.

The Swiss harmonist Rudolf Stössel studied the three-dimensional illustration of the Lambdoma from various points of view and hearing, built several models, and published the results in his paper Harmonikale Modelle zur Veranschaulichung der Ober- und Untertonreihe. At the 1984 exhibition “Phenomena” in Zurich, an especially large version of this model was displayed, built specially for the exhibition; this model is shown here.

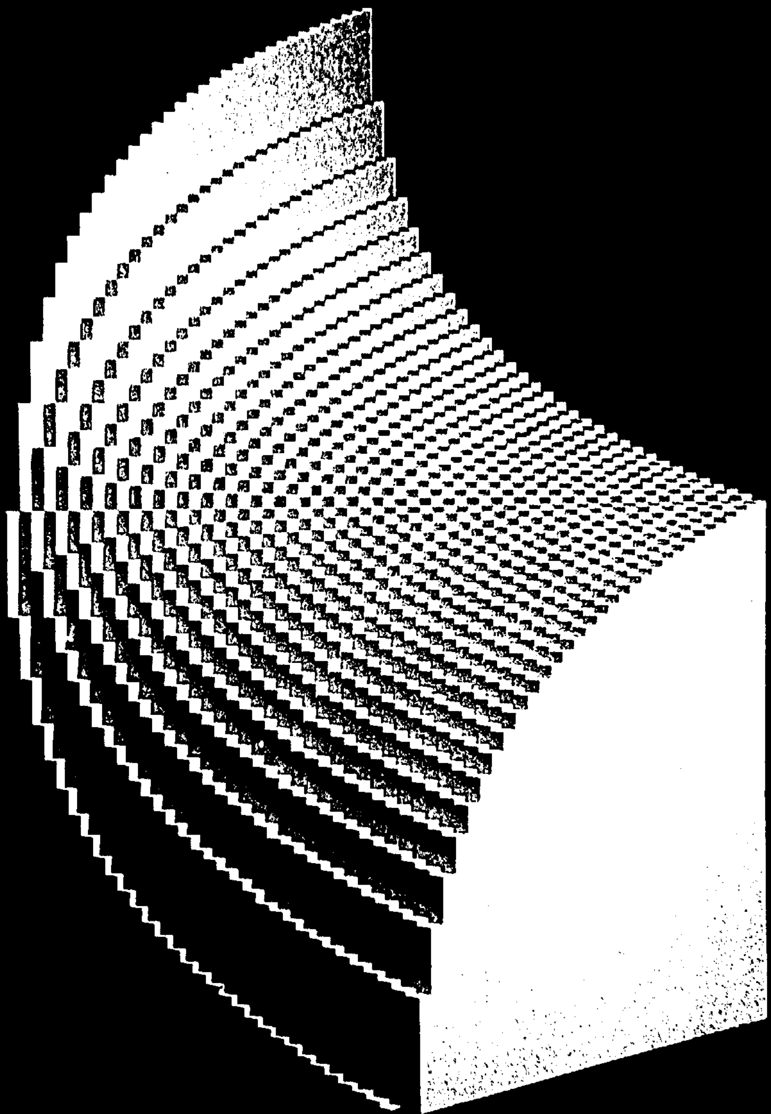


Image 16

The Tones of the Lambdoma Represented as Tubular Bells
by Dieter and Ulrike Trüstedt
Displayed at the exhibition "Phenomena" in Zurich, 1984
Photo by Dieter Trüstedt

The musician and physicist Dieter Trüstedt, from Munich, explored several possibilities of using the structures of the Lambdoma as a musical instrument. Among them was a framework in the form of the Lambdoma with index 12 x 12, upon which the intervals that corresponded to the proportions of the Lambdoma could be electronically generated through striking.

Another instrument was based on the ch'in, a Chinese stringed instrument. 13 strings are tuned in the progression of the undertone series; the overtones for each of these undertones are generated by striking the strings so that the flageolet tones sound, which are then electronically amplified. Thus the tones of the Lambdoma can be made audible up to index 13.

Dieter Trüstedt built a third instrument for the "Phenomena" exhibition - this is shown in Image 16. Here, the tones of the Lambdoma are represented by 12 x 12 tubular bells of brass; the longest bell is over 2 meters long. The lengths of the tubular bells do not correspond to the numbers of the Lambdoma, since the vibrations of bars and bells obey different laws from those of strings.

Another form of the Lambdoma as a musical instrument was realized by Peter Neubäcker in a computer program: here the Lambdoma is shown on the screen up to a given index - the Lambdoma tone to which the player points on the screen is then sent to a synthesizer. In this way all tones of the Lambdoma are available, throughout the entire auditory domain.

The most charming thing about playing with the Lambdoma is the fact that musically related tones are always nearby, in contrast with a piano keyboard, on which the available tones are simply laid out in a row. Also, in the Lambdoma all the tones are available in their pure tuning, and thus also the intervals that are formed from the higher prime numbers, such as 7, 11, 13, etc., which cannot be achieved at all in our traditional music.

Image 17

Development of the Surfaces of Crystals Represented in the Lambdoma

Above: up to 7 complication steps

Below: up to 12 complication steps

Conceived and drawn by Peter Neubäcker

In the construction of the surfaces of crystals, laws can be established which can also be expressed in the form of musical intervals in the realm of harmonics. For example, if a coordinate axis is drawn through the simplest crystal form, the cube, one surface is parallel to an axis, the other perpendicular to it. This can be expressed in the ratios $0/1 = 0$ for the perpendicular, $1/0 = \infty$ for the parallel surfaces. These two surfaces are described as primary surfaces. The further appearing surfaces are arranged so that they form an angle between the two primary surfaces - thus only certain angles are possible, namely those which always mark off whole numbers from both axes. So the next possible surface would be that which forms an angle of 45 degrees to the primary surfaces; since it marks off equal parts from both axes, it is described as 1/1 - other possible surfaces include 1/2, 2/3, etc. If the axes of the crystal system are now imagined as two monochord strings placed at a right angles to each other, then the crystal surfaces always mark off consonant intervals, and the more consonant the corresponding interval, the more likely the surface is to occur.

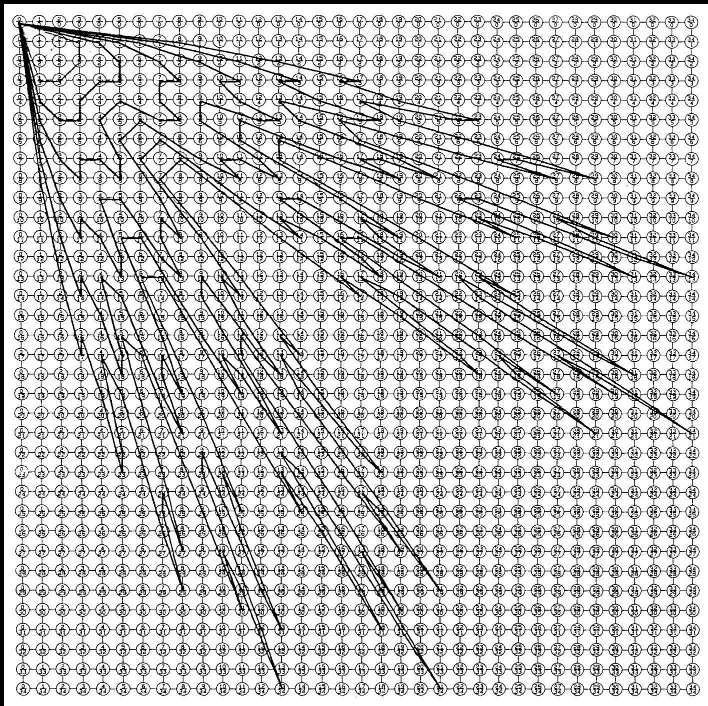
The crystallographer Victor Goldschmidt developed another law which determines the probability of crystal surfaces - he calls it the Law of Complication. Thus the following "normal series" ("Normalreihe") appear:

Primärflächen:	A	·	·	·	·	·	·	·	B	
$N_0 =$	0	·	·	·	·	·	·	·	∞	= Normalreihe 0.
1. Complication:	A	·	·	·	C	·	·	·	B	
$N_1 =$	0	·	·	·	1	·	·	·	∞	= Normalreihe 1.
2. Complication:	A	·	D	·	C	·	E	·	B	
$N_2 =$	0	·	$\frac{1}{2}$	·	1	·	2	·	∞	= Normalreihe 2.
3. Complication:	A	F	D	G	C	H	E	I	B	
$N_3 =$	0	$\frac{1}{3}$	$\frac{1}{2}$	$\frac{2}{3}$	1	$\frac{3}{2}$	2	3	∞	= Normalreihe 3.

u. s. w.

The links between the series that follow in each case then emerge through the adding of the denominator and numerator of the neighboring fractions. In nature the emerging surfaces only go beyond the 3rd normal series in rare cases. But the idea of this law of surface construction can be continued further, leading to the images shown.

The Lambdoma can be understood directly as a coordinate axis in the cubic crystal system (also for other crystal systems, when the angle of the axis is changed) -



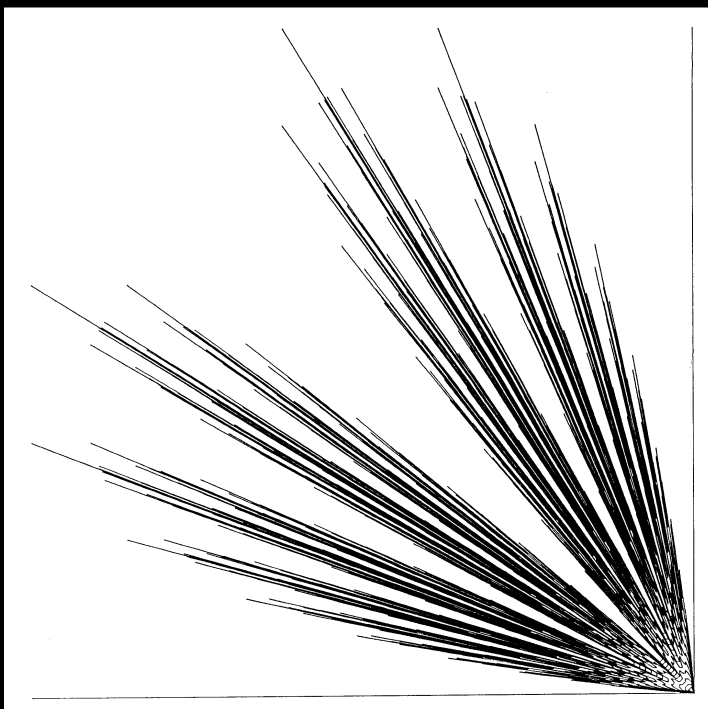


Image 18

Surface Development of Crystals Represented in the Lambdoma

The Four Quadrants in the Cubic System

Conceived and drawn by Peter Neubäcker

(text continued:)

- an equal-tone line going from point 0/0 to a certain tone-value is then identical to the "surface norms," i.e. the lines perpendicular to the respective crystal surfaces. According to Goldschmidt, the "direction of the particle energy that constructs the surfaces."

The tone-values lying nearest to the origin of the Lambdoma thus represent the most commonly appearing crystal surfaces, and the ones further out represent the less common - but not arbitrarily chosen tone-values, instead those belonging to the next normal series in each case, according to Goldschmidt's Law of Complication. In the illustrations here, all tone-values or surface indices newly appearing in a normal series are joined by a line - which can be seen most plainly in the first image. These images can thus be understood as a direct illustration of Goldschmidt's Law of Complication, or better yet, as the illustration of the tendency of this law, because such high degrees of complication only appear rarely in nature.

The similarity in form to certain minerals that grow in a radiant pattern (for example antimony) is evident, especially when the lower picture of Image 17 is continued into the spatial realm. It is theoretically conceivable that a causal connection exists in the sense of crystallography, in addition to the morphological.

It is interesting to consider which variations from one degree of complication to the next emerge in the Lambdoma in this illustration. A process of inversion can be clearly seen here: precisely where the surface development at the lower degrees reaches farthest into the Lambdoma, gaps emerge at the higher degrees; so the greatest gap in the image is at 1/1-that is the direction in which the first surface emerges. For all further degrees the same phenomenon can be observed.

Following each of the newly emerging numbers and their directions in the Lambdoma, it becomes apparent that they are the numbers of the Fibonacci Sequence: the ray reaching the farthest tends to the direction of the Golden Section, the next is identical in its direction to that of the "main series of leaf position numbers" described in Image 8. But because mostly only the lower degrees of complication appear in nature, here we once again see the polarity of an ideal tendency towards the Golden Section on the one hand and the realization through the harmonic intervals on the other hand.

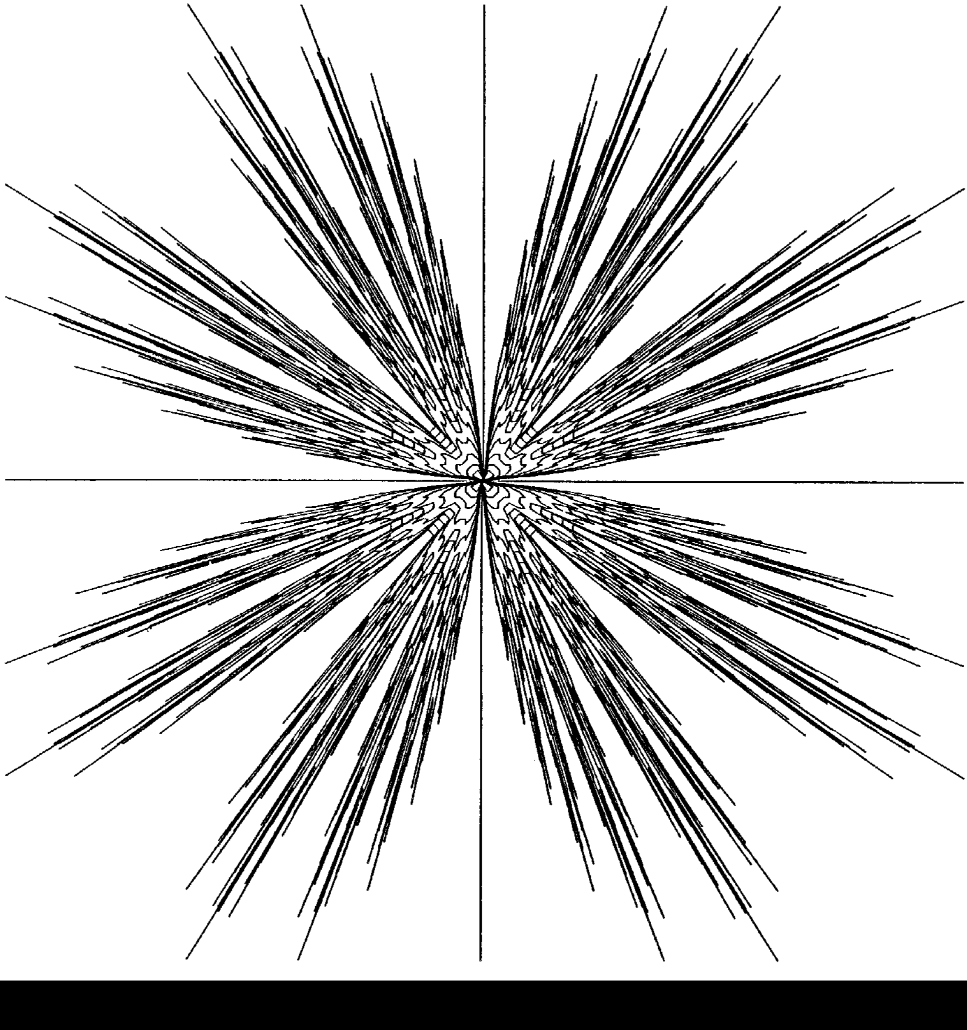


Image 19

Model:
The Distribution of All Rational Numbers
or Musical Intervals on the Monochord
Conceived and constructed by Peter Neubäcker

In order to investigate how all fractions, or rational numbers, are distributed, one can divide segments of a unit of length - here the monochord string - progressively by all the fractions, and mark the place of the division with a line. For example, one could begin with the number 2 and mark half of the length, then 1/3 and 2/3, then 1/4, 2/4, 3/4, then 1/5, 2/5, 3/5, 4/5, and so forth. The result is represented by the drawing, placed under the monochord - here all fractions are plotted up to index 100. For clarification, the lines are of different lengths: the smaller the number, the longer the line that indicates the ratio produced by it.

In this process a very interesting structure is revealed: the smaller the number from which the interval in question is constructed, the more space is left to it from the following ratios - so the degree of consonance of the interval can be determined directly from the width of the gaps that appear in the illustration.

The string itself also conforms to this pattern in its vibrations: the wider the in-between space in the drawing, the more pronounced is the vibration point at this location, responsible for producing the respective overtone. This can be tested by moving the rubber disc back and forth on the string and plucking the string as near as possible to one of the two nuts; the rubber disc stops the string from vibrating at the place in question, and thus sets a nodal point so that the corresponding overtone will sound. Thus one can establish that there are some overtones at almost every point, which sound with greater or lesser strength - but coming near to one of the large gaps in the drawing, one enters the "catchment area" of a small number, and the corresponding stronger tone overwhelms the others, which are manifested through higher numbers. One could say that the numbers have an "attractive power," which is stronger when the number is smaller.

Besides the musical properties, a plethora of other applications lie hidden in this illustration - Rudolf Stössel examined these structures more closely - with concepts appearing such as symmetry, isolation, reproduction, the rule of addition, the product law, and connections with the Fibonacci Sequence and its relatives, i.e. the Golden Section, the Lambdoma, and crystals.

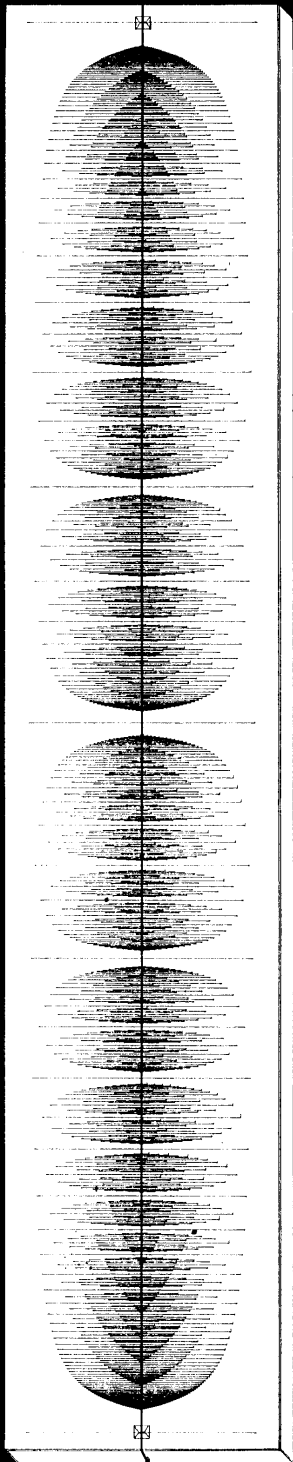


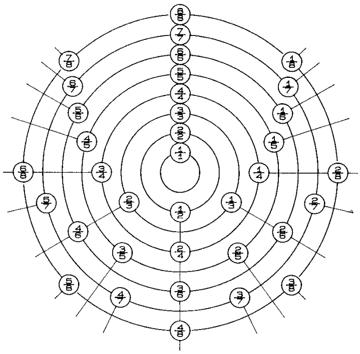
Image 20

The Distribution of All Rational Numbers

or Musical Intervals in Circular Form

Conceived and drawn by Peter Neubäcker

Not only can the arrangement of all fractions or musical intervals be illustrated in quadratic form, as with the model of the Lambdoma (Image 14) - a circular arrangement is also possible. For this, the entire circle is defined as the unit; the fractions are marked off as fractional parts of the whole round angle. Fractions with the same numerator are on the same circle-each circle therefore corresponds to a perpendicular line in the quadratic Lambdoma. Up to index 8, this arrangement emerges:



Tones with the same value, i.e. fractions that can be cancelled down, are always on the same line, which leads outwards radially. These "equal-tone lines" are also plotted here. Image 20 shows only these equal-tone lines, each drawn from their origin outwards - here for all fractions up to index 100.

While the values with the same numerator - corresponding to the undertone series - always lie upon concentric circles, the values with the same denominator form spirals, corresponding to the overtone series, and all finally run parallel to the generator-tone line.

Here the same phenomenon can be observed as was seen in the positioning of the fractions on the monochord (Image 19): the fractions are distributed unequally in favor of the smaller numbers or the more consonant intervals. This can be thought of as a hierarchy in the world of numbers, or an "aristocracy of small numbers."

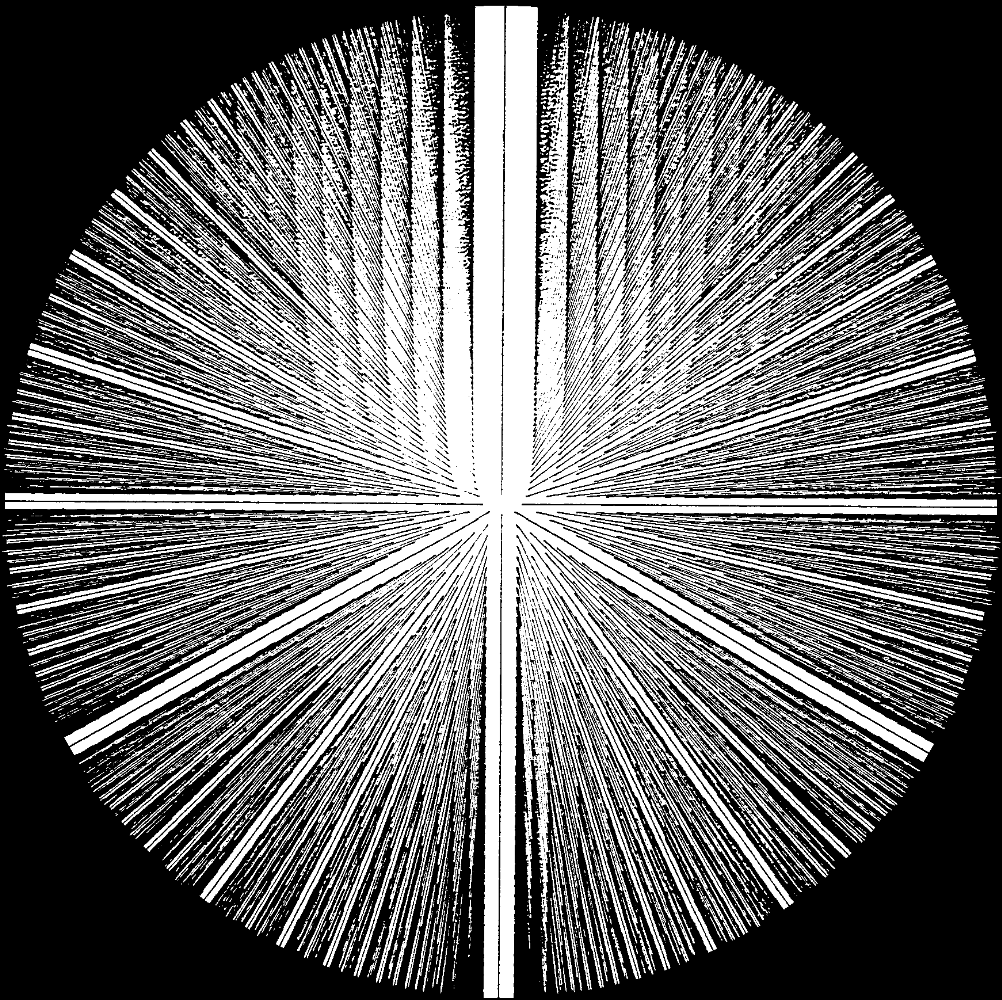


Image 21

The Distribution of All Rational Numbers
Above: Image 20 reflected inwards
Below: magnification of the inner space
Conceived and drawn by Peter Neubäcker

The round Lambdoma illustration actually only contains half of the values of the quadratic Lambdoma, namely all true fractions in the domain between zero and one. Their reciprocals, which would correspond in the model in Image 19 to a lengthening of the unit string, are only present latently: if the circles in the round illustration are understood as "bent strings," then all values above one can be represented in this way. The circumference of the second circle is exactly twice that of the unit circle, the third three times as long, and so on. Thus, for example, the equal-tone line 1/3 marks off the length 4/3 on the fourth circle, the length 5/3 on the fifth, etc. In this representation, the true fractions are realized by means of the angles, and their reciprocal values through the circle arcs.

Image 21 shows another way to illustrate the reciprocal values in the round Lambdoma: through reflection on the unit circle. These images must be imagined as tiny circles at the center of Image 20: All rays going out from Image 20 go inwards here, and their origin corresponds, in each case, to the reciprocal value of the point on the outside.

The lower image shows a large magnification of the upper one, and moreover the fractions here are not represented by rays (equal-tone lines) but only by points; all reducible fractions are thereby discarded. An infinite number of values go inwards-these images can be seen as a symbol of the potential endlessness in the spiritual domain, which is expressed as true unending multitude after transcending the unit circle in Image 20.

22

(no image here)

Three-Dimensional Model of the Distribution of Fractions
Corresponds to the Scheme of Image 20
Model conceived and built by Peter Neubäcker

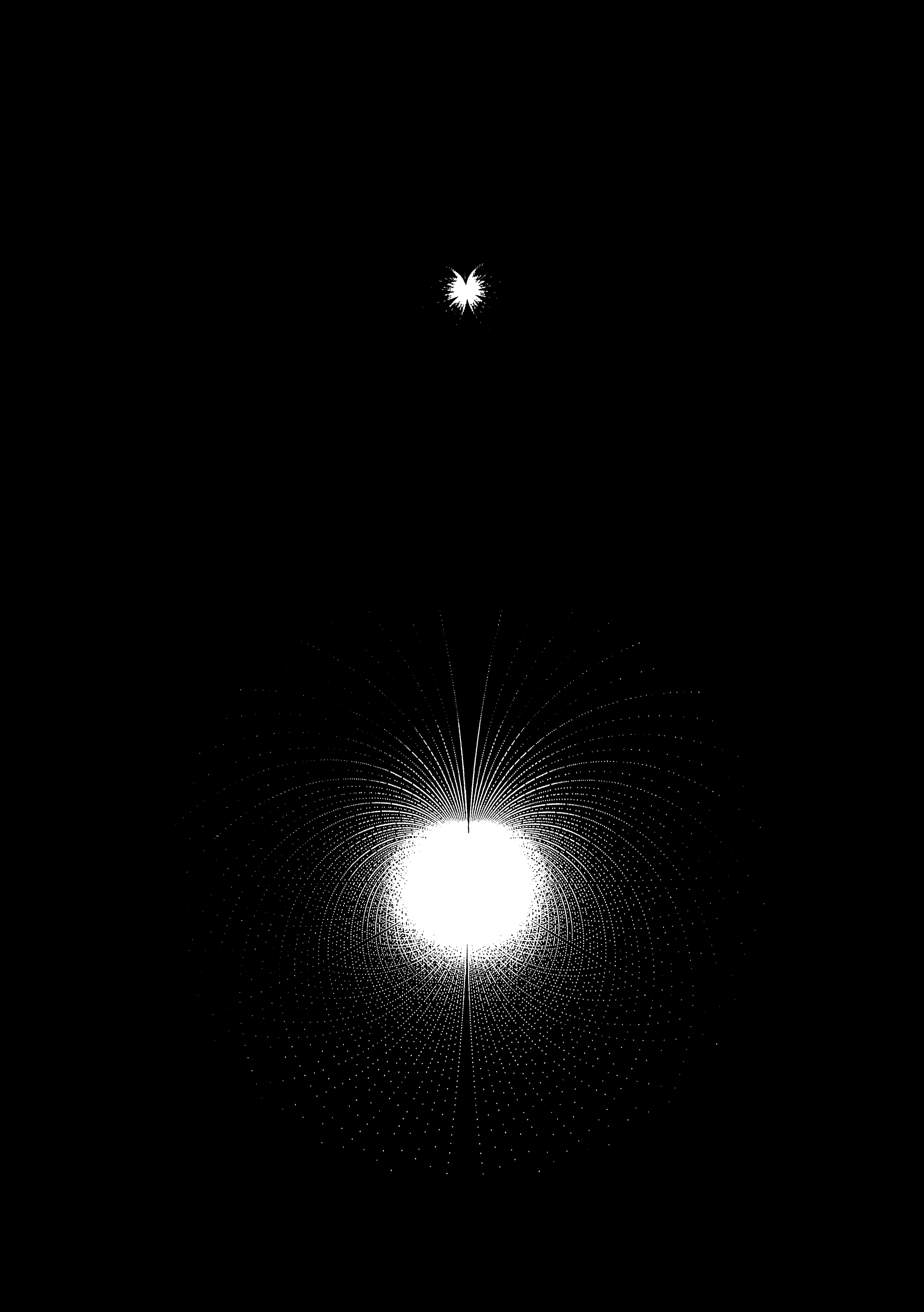


Image 23

The Distribution of the Prime Numbers up to 30000

Above: From the viewpoint of the number 210 (2 x 3 x 5 x 7)

Below: From the viewpoint of the number 211 (prime number)

Conceived and drawn by Peter Neubäcker

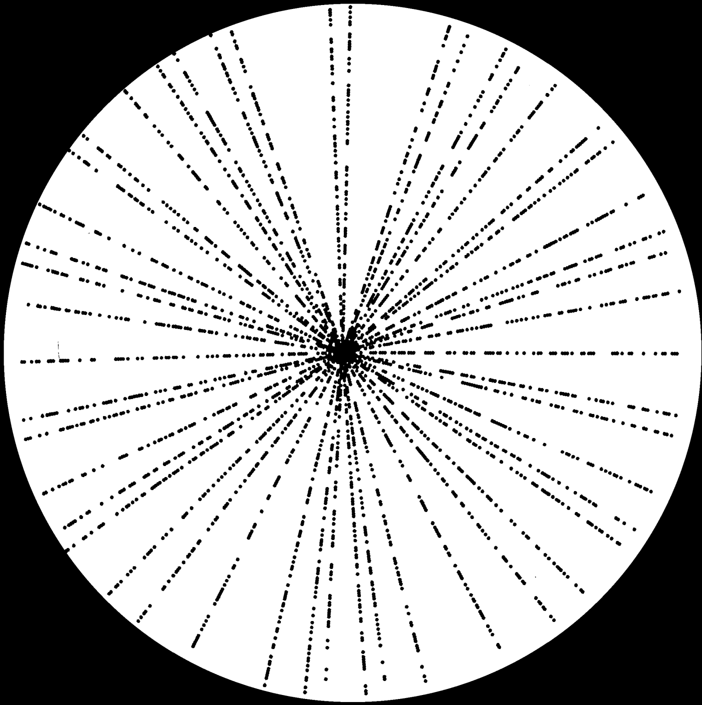
This and the two following images relate to the distribution of the prime numbers. The first question to be asked here is, what does a problem of number theory such as the distribution of prime numbers have to do with harmonics in the narrower sense?

The musical intervals are built from numbers - thus intervals that are built from the same prime numbers are closely connected with each other. The first interval-building number is two - it only ever leads to octaves. The next is three - the entire series of fifths can be built from it, which actually leads to an endless supply of tones, consisting only of fifth ratios. Until the Renaissance, this was the only tone structure used in the Western world. Then the prime number five emerged in music as the next step, leading to a more complex supply of tones, which includes harmony structures through the relationships of thirds in the sense of triads. In modern times the next prime number, seven, is also applied; but at present, its musical potentials have hardly been explored or integrated.

In studying harmonics, the entire world of numbers becomes a symbolic image for the world, in which each number plays its specific role - this role can be explored through the factorization of all numbers, which always leads to the prime numbers and thus reveals the relationships of the numbers. By listening to these numbers, one can also approach their nature more closely through perception. This is harder with the higher prime numbers, since for cultural reasons no specific musical experience is yet connected with them. But since all prime numbers manifest qualities with their own character in the world of numbers, their distribution is of interest in principle.

The distribution of the prime numbers is irregular. No mathematical law has yet been discovered with which prime numbers can be clearly predicted - probably no such law can be found - but certain structures in their distribution can be determined. Such structures are illustrated from various viewpoints in the image shown.

Images 23 and 24 show the distribution of prime numbers from the “viewpoint” of individual numbers - mathematically expressed, the arrangement is the “module of a given number.”



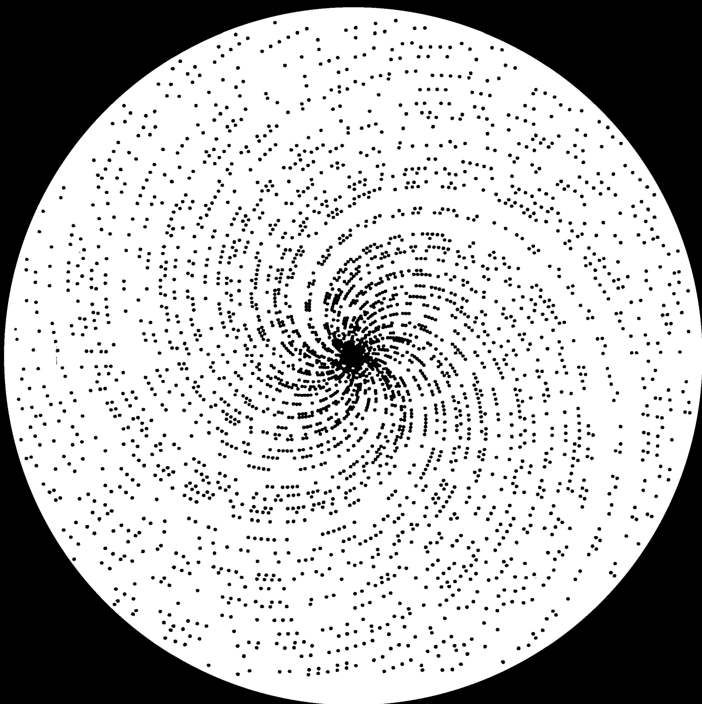


Image 24

The Distribution of Prime Numbers up to 10000

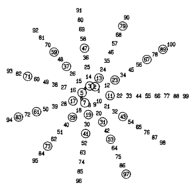
from the Viewpoint of the Numbers 200 to 228

(in increments of 0.2)

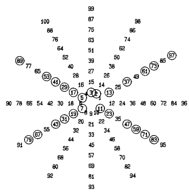
Conceived and drawn by Peter Neubäcker

When all natural numbers are arranged so that they lie upon concentric circles, and the same quantity of numbers is on each circle, then the number of rays emerging corresponds to the number used as a basis. For the numbers 11 and 12, for example, these two images would be produced:

11

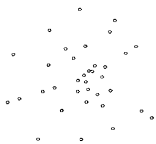


12

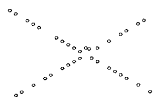


The prime numbers here are indicated by circles. Thus it is shown that for the number 12, prime numbers can only lie upon four of the 12 rays, since all numbers on the second ray are divisible by two, all on the third are divisible by three, all on the fourth are divisible by four, and so on - because these numbers are factors of 12. For the image of the number 11, on the other hand, prime numbers can lie on all rays (except for the 11th ray, on which all numbers are multiples of 11), since 11 itself is a prime number. If the composite numbers are removed and only the prime numbers left, these images result for 11 and 12:

11



12



From the viewpoint of the number 12, the prime numbers appear very orderly, but from the viewpoint of 11 completely disorderly. In this manner Images 23 and 24 emerge for higher numbers: the number 210, as a product of 2x3x5x7, puts the numbers in order to a very high degree, but the adjacent number 211 is a prime number which generates no order in itself; instead the order of the number 210 still appears here. Image 24 shows that every number has its own “face” - partly generated of itself, partly shaped by its neighbors.

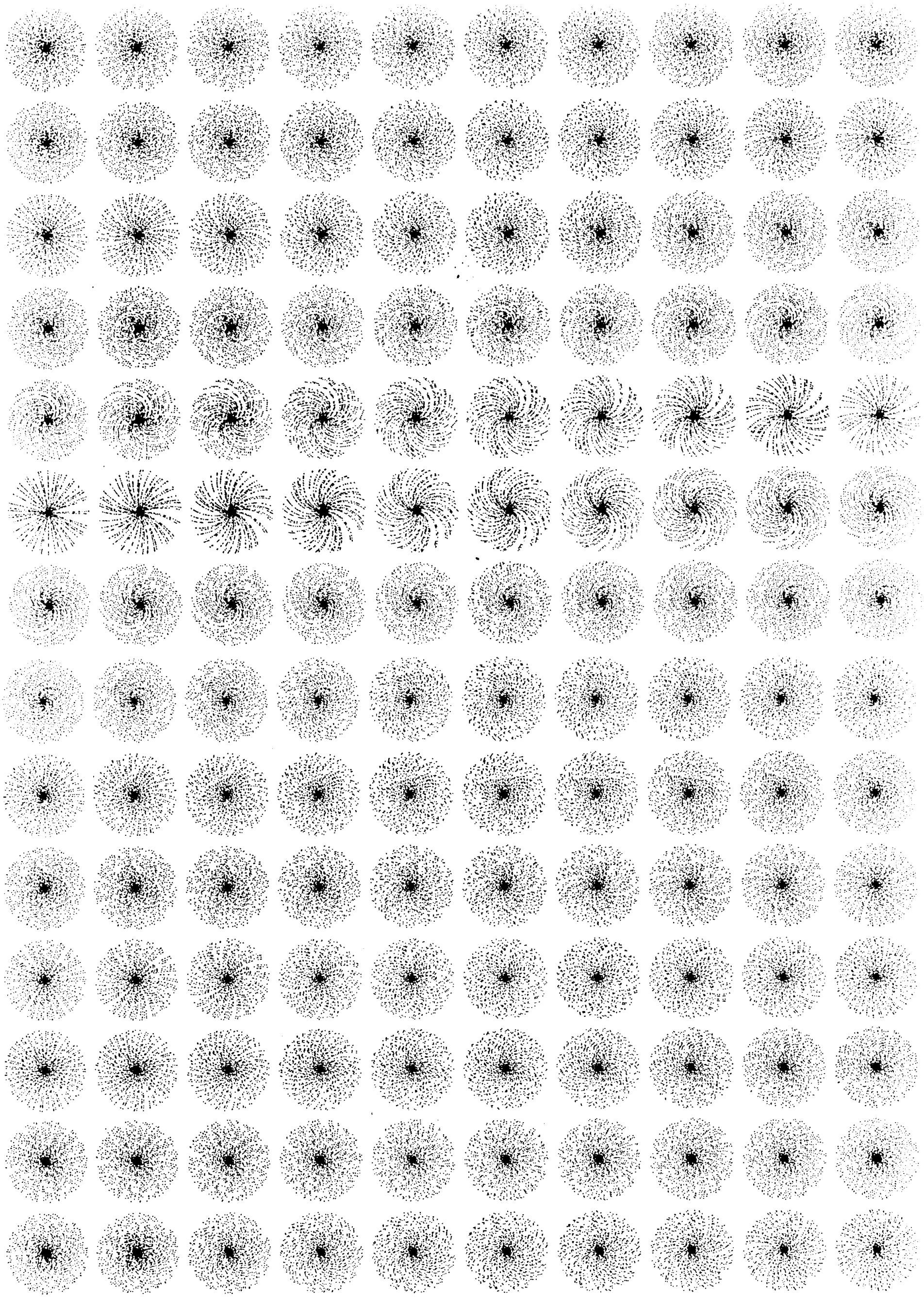


Image 25

The Distribution of the Prime Numbers According to the Pythagorean Polarity

of Square Numbers and Rectangular Numbers

Above: numbers up to 2500

Below: numbers up to 100000

Conceived and drawn by Peter Neubäcker

The ordering principle of this image is different from those of the two previous images: for the Pythagoreans, polarity was a central concept, and they counted ten such polarities, so that concepts emerged such as limited and unlimited, odd and even, male and female, etc. The tenth of these polarities is the opposition of square and rectangular numbers.

Why precisely these two concepts should be polarities was not entirely understandable. But when all numbers are arranged in accordance with that polarity, it becomes clear that the prime numbers become ordered by this arrangement to a high degree. In the images shown here, all natural numbers are arranged in a spiral, so that in each revolution a square number is encountered - thus the series of rectangular numbers appears precisely on the opposite, i.e. the numbers that are the product of two consecutive numbers, such as $6 = 2 \times 3$, $12 = 3 \times 4$, $20 = 4 \times 5$, etc. In the upper image, all numbers up to 2500 are shown, and the prime numbers are indicated by white circles. In the lower image the numbers up to 100000 are shown; the composite numbers are omitted here, and only the prime numbers indicated by white dots.

Thus it can be seen that all prime numbers arrange themselves in curves, which approach lines parallel to the ray of the rectangular numbers. In each case such a ray represents the sum of a given number with all rectangular numbers, and can therefore be represented by the expression $(n \times (n+1)) + m$. For certain values of m, an especially large quantity of prime numbers is yielded, e.g. for $m = 17$ or even more for $m = 41$. The expression $n \times (n+1) + 41$ has already been described by Leonhard Euler as an especially fruitful prime number formula. The numbers 17 and 41 are prime numbers themselves - so it can be said that prime numbers that generate many new prime numbers are especially “fruitful.”

It is unlikely that the prime number-ordering function of this polarity was known to the Pythagoreans - it more likely emerged from the finding that the square numbers are always sums of all odd numbers and the rectangular numbers are always sums of all even numbers - but here it can be seen again that the truth is always found in testimonies from many different levels at the same time.